Permutations and Combinations **Marks: 60**

ANSWER KEY

Maths

Q.1 A	Q.2 A	Q.3 B	Q.4 C	Q.5 B	Q.6 B	Q.7 B	Q.8 C
Q.9 A	Q.10 A	Q.11 C	Q.12 A	Q.13 C	Q.14 D	Q.15 B	Q.16 B
Q.17 D	Q.18 D	Q.19 A	Q.20 C	Q.21 D	Q.22 D	Q.23 B	Q.24 B
Q.25 C	Q.26 C	Q.27 B	Q.28 D	Q.29 C	Q.30 C		

Maths

Q.1 The number of ways in which the letters of the world MACHINE can be arranged such that the vowels may occupy only odd position, is

Correct option: (A)

There are 7 letters in the word 'MACHINE' and the odd places are 1^{st}, 3^{rd}, 5^{th} and 7^{th}.

3 vowels A, I and E can occupy odd places in $^4P_3$ ways and then the 4 consonants can occupy remaining places in $^4P_4$ ways.

$\therefore$ Required number of ways = $^4P_3 \times {^4P_4}$

= 4! × 4! = 576

Q.2 The number of arrangements of the letters of the word APPLE in which two P's do not appear adjacently is

Correct option: (A)

Required number of arrangements

= (Total number of arrangements) – (Number of arrangements in which P's are together)

= $\frac{5!}{2!} - 4!$

= 60 – 24 = 36

Q.3

In how many ways 3 letters can be posted in 4 letter-boxes, if all the letters are not posted in the same letter-box?

Correct option: (B)

Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box.

Hence, required ways = 60

Q.4 The sum of the digits in the unit place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is

Correct option: (C)

Required sum = 3! (3+4+5+6) = 6 × 18 = 108.

[If we fix 3 of the unit place, other three digits can be arranged in 3! ways similarly for 4, 5, 6.]

Q.5 There are 10 chairs in a room on which 7 persons are to be seated, out of which one is a guest with one specific chair. In how many ways can they sit ?

Correct option: (B)

A specific chair is fixed for a specific guest.

Remaining 6 chairs can be selected in $^9C_6$ ways and then 6 persons can be arranged in 6! ways.

Total no. of ways = $^9C_6 \times 6!$ or 60480

Q.6 If $\frac{1}{12!} + \frac{1}{13!} = \frac{x}{14!}$, then x is equal to

Correct option: (B)

$\frac{1}{12!} + \frac{1}{13!} = \frac{x}{14!}$

$\therefore \frac{13}{13 \times 12!} + \frac{1}{13!} = \frac{x}{14!}$

$\Rightarrow \frac{13+1}{13!} = \frac{x}{14!}$

$\Rightarrow \frac{14}{13!} = \frac{x}{14 \times 13!}$

$\Rightarrow x = 196$

Q.7 If $^nP_7 = 12.\ ^nP_5$, then n =

Correct option: (B)

$\frac{n!}{(n-7)!} \times \frac{(n-5)!}{n!} = 12$

$\Rightarrow (n-5)(n-6) = 12 \Rightarrow n = 9$ or 2

But n = 2 is not acceptable.

$\therefore$ n = 9

Q.8 Value of r for which $^{15}C_{r+3} = {^{15}C_{2r-6}}$ is

Correct option: (C)

Either r + 3 = 2r – 6

or r + 3 + 2r – 6 = 15

$\Rightarrow$ r = 9 or r = 6

Q.9 How many words can be formed using any three letters of the word EDUCATION?

Correct option: (A)

Required number of words = $^9P_3 = 504$.

Q.10 In a class there are 10 boys and 8 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways the teacher can make this selection?

Correct option: (A)

There are 10 + 8 = 18 ways of selecting either a boy or a girl.

Q.11 $^nC_r + 2^nC_{r-1} + ^nC_{r-2} =$

Correct option: (C)

$^nC_r + 2\,^nC_{r-1} + ^nC_{r-2} = ^nC_r + ^nC_{r-1} + ^nC_{r-1} + ^nC_{r-2}$

$= ^{n+1}C_r + ^{n+1}C_{r-1}$

$= ^{n+2}C_r$

Q.12 The number of ways in which 5 boys and 3 girls can be seated in a row so that each girl sits in between two boys [Kerala (Engg.) 2002]

Correct option: (A)

5 boys can be seated in 5! ways B × B × B × B × B. Three girls can be seated in places marked '×' in $^4P_3$ ways.

∴ Total ways = $5!.^4P_3$ = 2880

Q.13 If $^{12}P_r$ = 1320, then r is equal to

Correct option: (C)

Since, $^{12}P_3$ = 1320

∴ r = 3

Q.14 If $\frac{(17-n)!}{(15-n)!}$ = 132, then n =

Correct option: (D)

$\frac{(17-n)!}{(15-n)!} = 132$

$\Rightarrow \frac{(17-n)(16-n)(15-n)!}{(15-n)!} = 132$

⇒ (17 – n) (16 – n) = 12 × 11

⇒ n = 5

Q.15 How many numbers greater than hundred and divisible by 5 can be made from the digits 3, 4, 5, 6, if no digit is repeated

Correct option: (B) 12

Numbers which are divisible by 5 have '5' fixed in extreme right place

3 Digit Numbers	4 Digit Numbers
H T U × × 5 $^3P_2$ ways $= \frac{3!}{1!} = 3 \times 2 = 6$	Th H T U × × × 5 $^3P_3$ ways $= \frac{3!}{0!} = 3 \times 2 = 6$

⇒ Total ways = 6 + 6 = 12

Q.16 The letters of the word PENCIL are arranged in all possible ways. The number of ways in which N always occur next to E is

Correct option: (B)

We have 5 different letters: P, C, I, L and EN, which can be arranged in $^5P_5$ = 5! ways.

Q.17 Everybody in a room shakes hand with everybody else. The total number of hand shakes is 91. The total number of persons in the room is

Correct option: (D)

$^nC_2 = 91$

⇒ n(n–1) = 182 ⇒ n = 14

Q.18 Out of 7 consonants and 4 vowels, the number of words consiting of 3 consonants and 2 vowels are

Correct option: (D)

3 consonants can be choosen from 7 consonants in $^7C_3$ ways.

2 vowels can be choosen from 4 vowels in $^4C_2$ ways.

The 3 consonants and 2 vowels can be arrange in $^5P_5$ = 5! ways.

∴ Required number of words formed = $\left(^7C_3 \times ^4C_2\right) \times 5!$ = 25200

Q.19 The number of 7 digit numbers which can be formed using the digits 1, 2, 3, 2, 3, 3, 4 is

Correct option: (A)

Required number = $\frac{7!}{3!\,2!} = \frac{5040}{6 \times 2}$ = 420

Q.20 In how many ways can 7 persons be selected from 5 officers and 6 constables, if at least two officers are to be included

Correct option: (C)

Required number of ways

$= ^5C_2 \times ^6C_5 + ^5C_3 \times ^6C_4 + ^5C_4 \times ^6C_3 + ^5C_5 \times ^6C_2$

= 10 × 6 + 10 × 15 + 5 × 20 + 1 × 15 = 325

Q.21 A letter lock has 4 rings each containing 9 different letters. What is the maximum number of false trials that can be made before the lock is opened?

Correct option: (D)

Each ring can be adjusted in 9 different ways.

∴ 4 rings can be arranged in 9 × 9 × 9 × 9 = 6561 ways.

Out of these 6561 attempts, only one attempt is successful.
Hence, the maximum number of false trials = 6561 – 1 = 6560

Q.22 The number of different ways of preparing a garland using 6 distinct white roses and 5 distinct red roses such that no two red roses come together, is
Correct option: (D)

_ R _ R _ R_ R _R _

5 red roses can be arranged in 5! ways.
6 white roses can be arranged in 6 places in ${}^6P_6$ ways.

$\therefore$ Total no. of ways = $5! \times {}^6P_6$
$= 5! \times 6!$
$= 86400$

Q.23 Three different prizes are to be distributed in a class of 20 boys. In how many ways can this be done, if a boy is eligible to get any number of prizes.
Correct option: (B)

If a student is eligible to get any number of prizes, each prize can be given to any of the 20 boys in 20 ways.

$\therefore$ Total number of ways = $20 \times 20 \times 20$

Q.24 The number of words which can be made out of the letters of the word MOBILE when consonants always occupy odd places is
Correct option: (B)

The word MOBILE has three even places and three odd places. It has 3 consonants and 3 vowels.
In three odd places, we have to fix up 3 consonants which can be done in ${}^3P_3$ ways.
Now, remaining three places we have to fix up 3 vowels which can be done in ${}^3P_3$ ways.

$\therefore$ The total number of ways = ${}^3P_3 \times {}^3P_3 = 36$

Q.25 The number of ways in which 8 different pearls can be arranged to form a necklace is
Correct option: (C)

Required number of ways

$= \frac{1}{2}(8-1)! \quad \ldots$

$\left[\begin{array}{l}\text{Since, clockwise and anticlockwise} \\ \text{are same in case of ring}\end{array}\right]$

$= \frac{7!}{2}$

= 2520

Q.26 The number of arrangements of the letters of the word SALOON, if the two O's do not come together, is
Correct option: (C)

Total number of arrangements are $\frac{6\,!}{2\,!} = 360$

The number of ways in which O's come together = 5! = 120 .
Hence, required number of ways = 360 – 120 = 240.

Q.27 A regular polygon has 20 sides. The number of triangles that can be drawn by using the vertices but not using the sides are
Correct option: (B)

Total triangles = ${}^{20}C_3$
Triangles with two sides of the polygon = 20 (formed by 3 consecutive vertices)
Triangles with one side of the polygon = 20×16

$\therefore$ Required number of triangles

$= {}^{20}C_3 - 20 - 20 \times 16$

$= \frac{20 \times 19 \times 18}{1 \times 2 \times 3} - 20 - 16 \times 20$

$= 20\,(57 - 1 - 16)$
$= 20 \times 40$
$= 800$

Q.28 Which of the following is not correct?
Correct option: (D) ${}^nC_r + {}^nC_{r-1} = {}^nC_{r+1}$

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ ${}^nC_{r+1}$

Q.29 The number of ways in which the letters of the word TRIANGLE can be arranged such that two vowels do not occur together is [MP PET 2010]
Correct option: (C)

.T.R.N.G.L.

Three vowels can be arranged at 6 places in $^6P_3$ ways = 120 ways. Hence, the required number of arrangements = 120 × 5! = 14400

Q.30 **If $\frac{1}{^4C_n} = \frac{1}{^5C_n} + \frac{1}{^6C_n}$, then n =**

Correct option: (C)

We have, $\frac{1}{^4C_n} = \frac{1}{^5C_n} + \frac{1}{^6C_n}$

$$\Rightarrow \frac{n!(4-n)!}{4!} = \frac{n!(5-n)!}{5!} + \frac{n!(6-n)!}{6!}$$

$$\Rightarrow \frac{(4-n)!}{4!} = \frac{(5-n)!}{5!} + \frac{(6-n)!}{6!}$$

Multiplying both sides by $\frac{4!}{(4-n)!}$, we get

$$1 = \frac{5-n}{5} + \frac{(6-n)(5-n)}{6 \times 5}$$

$$\Rightarrow 1 = \frac{30 - 6n + 30 - 11n + n^2}{30}$$

$\Rightarrow 30 = 60 - 17n + n^2$

$\Rightarrow n^2 - 17n + 30 = 0$

$\Rightarrow (n - 15)(n - 2) = 0$

$\Rightarrow n = 15$ or $n = 2$

But, n = 15 does not satisfy the given condition.

$\therefore$ n = 2