Complex Numbers **Marks: 60**

ANSWER KEY

Maths

Q.1 A	Q.2 A	Q.3 A	Q.4 D	Q.5 B	Q.6 A	Q.7 D	Q.8 C
Q.9 A	Q.10 B	Q.11 B	Q.12 B	Q.13 D	Q.14 B	Q.15 A	Q.16 C
Q.17 C	Q.18 A	Q.19 D	Q.20 B	Q.21 D	Q.22 A	Q.23 C	Q.24 B
Q.25 B	Q.26 D	Q.27 A	Q.28 B	Q.29 A	Q.30 B		

Maths

Q.1 $\dfrac{(\cos 2\theta - i\sin 2\theta)^4(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$

=

Correct option: (A)

$$\frac{(\cos 2\theta - i\sin 2\theta)^4\,(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$$

$$= \frac{\left[(\cos\theta + i\sin\theta)^{-2}\right]^4\left[(\cos\theta + i\sin\theta)^4\right]^{-5}}{\left[(\cos\theta + i\sin\theta)^3\right]^{-2}\left[(\cos\theta + i\sin\theta)^{-3}\right]^{-9}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-8}(\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{-6}\,(\cos\theta + i\sin\theta)^{27}}$$

$$= (\cos\theta + i\sin\theta)^{-8-20+6-27}$$

$= (\cos\theta + i\sin\theta)^{-49}$

$= \cos 49\theta - i\sin 49\theta$

Q.2 If $a = \sqrt{2i}$, then which of the following is correct?

Correct option: (A)

$$a = \sqrt{2i} = \sqrt{2}\; i^{1/2} \;=\; \sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/2}$$

=

$$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = 1 + i$$

Q.3 Modulus of $\left(\frac{5\,+\,3i}{5\,-\,3i}\right)$ is

Correct option: (A)

$$\frac{5+3i}{5-3i} = \frac{5+3i}{5-3i} \times \frac{5+3i}{5+3i}$$

$$= \frac{25 - 9 + 30i}{34}$$

$$= \frac{8}{17} + \frac{15i}{17}$$

$\therefore$ Modulus = $\sqrt{\left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2} = 1$

Q.4 $i^2 + i^3 + \ldots + i^{4000}$ =

Correct option: (D)

$i^2 + i^3 + \ldots + i^{4000}$

$= i + i^2 + i^3 + \ldots + i^{4000} - i$

$= (i + i^2 + i^3 + i^4) + \ldots + (i^{3097} + i^{3098} + i^{3099} + i^{4000}) - i$

$= 0 + \ldots + 0 - i$

$= -i$

Q.5 If z is purely real and Re (z) < 0, then Arg (z) is

Correct option: (B)

If z is purely real and Re(z) < 0, it means z is a negative real number. In the complex plane, negative real numbers lie on the negative real axis, which makes an angle of π radians with the positive real axis. Therefore, the argument of z is π.

Q.6 If $x = 1 - i\sqrt{3}$, then $x^3 - x^2 + 2x + 4$ =

Correct option: (A)

$x = 1 - i\sqrt{3}$

$\therefore (x-1)^2 = \left(-i\sqrt{3}\right)^2 \Rightarrow x^2 - 2x + 4 = 0$

$\therefore x^3 - x^2 + 2x + 4 = (x^2 - 2x + 4)(x + 1)$

$= (0)\,(x + 1) = 0$

Q.7 $\left(\dfrac{1}{1\,-\,2i} + \dfrac{3}{1\,+\,i}\right)\left(\dfrac{3\,+\,4i}{2\,-\,4i}\right)$ =

Correct option: (D)

$$\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$$

=

$$\left(\frac{1+2i}{1^2+2^2} + \frac{3-3i}{1^2+1^2}\right)\left(\frac{6-16+12i+8i}{2^2+4^2}\right)$$

$$= \left(\frac{2+4i+15-15i}{10}\right)\left(\frac{-1+2i}{2}\right)$$

$$= \frac{(17-11i)(-1+2i)}{20} = \frac{5+45i}{20} = \frac{1}{4} + \frac{9}{4}i$$

Q.8 If i = $\sqrt{-1}$, then 4 + 5$\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{334}$ + 3$\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{365}$ =

Correct option: (C)

$$4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$$

$= 4 + 5\omega^{334} + 3\omega^{365}$

$= 4 + 5\omega + 3\omega^2$

$= 4 + 5\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + 3\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$= i\sqrt{3}$

Q.9 The value of $(1 + i)^6 + (1 - i)^6$ is

Correct option: (A)

$(1 + i)^6 + (1 - i)^6 = [(1 + i)^2]^3 + [(1 - i)^2]^3$

$= (2i)^3 + (-2i)^3$

$= (8 - 8)i^3$

$= 0$

Q.10 The value of $\frac{1}{1+\omega} + \frac{1}{1+\omega^2} =$

Correct option: (B)

$$\frac{1 + \omega^2 + 1 + \omega}{(1 + \omega)(1 + \omega^2)} = \frac{1}{(-\omega^2)(-\omega)} = \frac{1}{\omega^3} = 1$$

Q.11 a + ib form of the complex number $\frac{1 + 3i}{2 + 3i}$ is

Correct option: (B)

$$\frac{1 + 3i}{2 + 3i} = \frac{(1 + 3i)(2 - 3i)}{(2 + 3i)(2 - 3i)} = \frac{2 + 3i + 9}{4 + 9}$$

$$= \frac{11}{13} + \frac{3}{13}i$$

Q.12 If a > 0 and $z = \frac{(1 + i)^2}{a + i}$, $(i = \sqrt{-1})$ has magnitude $\frac{2}{\sqrt{5}}$, then $\bar{z}$ is equal to

Correct option: (B)

$$z = \frac{(1 + i)^2}{a + i} = \frac{2i}{a + i}$$

$$= \frac{2i(a - i)}{(a + i)(a - i)}$$

$$= \frac{2 + 2ai}{a^2 + 1} \quad \ldots(i)$$

$$|z| = \frac{2}{\sqrt{5}} \Rightarrow \frac{4}{(a^2 + 1)^2} + \frac{4a^2}{(a^2 + 1)^2} = \frac{4}{5}$$

$\Rightarrow 20 + 20a^2 = 4(a^4 + 2a^2 + 1)$

$\Rightarrow 4a^4 - 12a^2 - 16 = 0$

$\Rightarrow a^4 - 3a^2 - 4 = 0$

$\Rightarrow (a^2 - 4)(a^2 + 1) = 0$

$\Rightarrow a^2 = 4$ and $a^2 = -1$

$\Rightarrow a = 2 \quad \ldots[\because a > 0]$

$\therefore$ (i) $\Rightarrow z = \frac{2}{5} + \frac{4}{5}i$

$\therefore \bar{z} = \frac{2}{5} - \frac{4}{5}i$

Q.13 The square roots of – 2i are

Correct option: (D)

Let $x + iy = \sqrt{-2i}$

$\Rightarrow x^2 - y^2 + 2xyi = -2i$

$\therefore x^2 - y^2 = 0$ and $2xy = -2$

Solving these equations, we get

$x = 1, y = -1$ and $x = -1, y = 1$

$\therefore$ Square roots are $1 - i, -1 + i$.

Q.14 The value of $(1 + i)^5 (1 - i)^7$ is

Correct option: (B)

$(1 + i)^5 (1 - i)^7 = [(1 + i)(1 - i)]^5 (1 - i)^2$

$= (1 + 1)^5 (1 - 2i - 1)$

$= 32(-2i) = -64i$

Q.15 If $a_n = i^{n^2}$, $n \in N$, then

$a_1 + a_3 + a_5 + \ldots + a_{75} =$ ________.

Correct option: (A)

$a_n = i^{n^2}$ $\therefore a_1 = i, a_3 = i^9 = i, a_5 = i^{25} = i, \ldots, a_{75} = i_{75}$

$= i^{5625} = i$

$\therefore a_1 + a_3 + a_5 + \ldots + a_{75} = i + i + \ldots 38$ times $= 38i$.

Q.16 $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be real, if θ =

Correct option: (C)

Let $z = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$

$= \left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right) + i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$

Since it is real, therefore $\text{Im}(z) = 0$

$\Rightarrow \frac{8\sin\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in Z$

Q.17 $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ will be purely imaginary, if θ =

Correct option: (C)

$\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ will be purely imaginary, if the real part vanishes, i.e., $\frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} = 0$

$\Rightarrow 3 - 4\sin^2\theta = 0$ (only if θ be real)

$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} = \sin\left(\pm\frac{\pi}{3}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\pm\frac{\pi}{3}\right)$

$= n\pi \pm \frac{\pi}{3}$, where n is an integer

Q.18 If $z_1 = 7 - i$ and $z_2 = 3 + 4i$, then $\bar{z}_1 - \bar{z}_2 =$

Correct option: (A)

$\bar{z}_1 = 7 + i$ and $\bar{z}_2 = 3 - 4i$

$\therefore \bar{z}_1 - \bar{z}_2 = (7 + i) - (3 - 4i)$

$= 4 + 5i$

Q.19 If $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x + iy$, then

Correct option: (D)

$\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2$, we get

$\begin{vmatrix} 3i+2 & 0 & 0 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix}$

$= (3i + 2)(9i^2 + 9)$

$= (3i + 2)(-9 + 9)$

$= 0$

$\therefore x = 0, y = 0$

Q.20 If $Z_1 = 2 + i$ and $Z_2 = 3 - 4i$ and $\frac{\overline{Z_1}}{\overline{Z_2}} = a +$ bi, then the value of $-7a + b$ is (where $i = \sqrt{-1}$ and $a, b \in R$)

Correct option: (B)

$Z_1 = 2 + i, \quad Z_2 = 3 - 4i$

$\overline{Z_1} = 2 - i \quad , \quad \overline{Z_2} = 3 + 4i$

$\frac{\overline{Z_1}}{\overline{Z_2}} = \frac{2 - i}{3 + 4i}$

$= \frac{(2 - i)(3 - 4i)}{(3 + 4i)(3 - 4i)}$

$= \frac{6 - 8i - 3i + 4i^2}{(3)^2 - (4i)^2}$

$= \frac{6 - 11i - 4}{9 + 16}$ …$[i^2 = -1]$

$a + bi = \frac{2 - 11i}{25}$

$\therefore a = \frac{2}{25}, b = \frac{-11}{25}$

Now $-7a + b = -7\left(\frac{2}{25}\right) - \frac{11}{25}$

$= \frac{-14 - 11}{25}$

$= \frac{-25}{25} = -1$

Q.21 If $z = 1 - i$, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$)

Correct option: (D)

Given, $z = 1 - i$ and $i = \sqrt{-1}$

Squaring on both sides, we get

$z^2 = (1 - i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$

Since, it is a multiplicative identity, therefore multiplicative inverse of z^2 =

$\frac{1}{-2i} \times \frac{i}{i} = \frac{i}{-2i^2} = \frac{i}{2}$

Q.22 If ω is a complex cube root of unity, then $\omega^{149} + \omega^{150} + \omega^{151} =$

Correct option: (A)

$\omega^{149} + \omega^{150} + \omega^{151} = \omega^{149}(1 + \omega + \omega^2) = \omega^{149}(0) = 0$

Q.23 If $z^2 + z + 1 = 0$ then $\left(z^3 + \frac{1}{z^3}\right)^2 + \left(z^4 + \frac{1}{z^4}\right)^2 =$ where $z = w =$ complex cube root of unity

Correct option: (C)

Z is a complex cube root of unity

$\therefore z^3 = 1$ …(i)

and $1 + z + z^2 = 0$

$\therefore z^2 + 1 = -z$ …(ii)

Consider,

$\left(z^3 + \frac{1}{z^3}\right)^2 + \left(z^4 + \frac{1}{z^4}\right)^2$

$= \left(1 + \frac{1}{1}\right)^2 + \left(z^3 \cdot z + \frac{1}{z^3 \cdot z}\right)^2$

$= 4 + \left[z + \frac{1}{z}\right]^2$ …[From (i)]

$= 4 + \left[\frac{z^2+1}{z}\right]^2$

$= 4 + \left[\frac{-z}{z}\right]^2$ …[From (ii)]

$= 4 + (-1)^2$

$= 5$

Q.24 If $\frac{1}{(1+i)^3}$ = a + ib, then

Correct option: (B)

$\frac{1}{(1+i)^3} = \frac{1}{1+3i+3i^2+i^3}$

$= \frac{1}{2(-1+i)}$

$= \frac{(-1-i)}{2(-1+i)(-1-i)}$

$= \frac{-1-i}{2(1-i^2)}$

$= \frac{-1-i}{4}$

$\therefore$ a = $\frac{-1}{4}$ and b = $\frac{-1}{4}$

Q.25 The value of

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1 =$$

Correct option: (B)

$$\frac{i^{584}(i^8+i^6+i^4+i^2+1)}{i^{574}(i^8+i^6+i^4+i^2+1)} - 1$$

$$= \frac{i^{584}}{i^{574}} - 1$$

$= i^{10} - 1 = -1 - 1 = -2$

Q.26 The locus of the points represented by |z + 3|–|z – 3| = 6, where z is a complex number, is ....

Correct option: (D)

$|z + 3| - |z - 3| = 6$

$\Rightarrow |x + iy + 3| - |x + iy - 3| = 6$

$\Rightarrow |x + 3 + iy| - |x - 3 + iy| = 6$

$\Rightarrow \sqrt{(x+3)^2+y^2} - \sqrt{(x-3)^2+y^2} = 6$

$\Rightarrow \sqrt{(x+3)^2+y^2} = 6 + \sqrt{(x-3)^2+y^2}$

$\Rightarrow (x+3)^2 + y^2 = \left(\sqrt{(x-3)^2+y^2} + 6\right)^2$

$\Rightarrow y = 0$

It is a equation of X–axis

Q.27 If α and β are imaginary cube roots of unity, then value of α + β + $\frac{1}{\alpha\beta}$ is

Correct option: (A)

It is given that,

α and β are imaginary cube roots of unity.

Let α = ω and β = ω^2.

Now, $\alpha + \beta + \frac{1}{\alpha\beta}$

$= \omega + \omega^2 + \frac{1}{(\omega)(\omega^2)}$

$= (-1) + \frac{1}{\omega^3}$ …[$\because 1 + \omega + \omega^2 = 0$]

$= -1 + 1 = 0$ …[$\because \omega^3 = 1$]

Q.28 If p + iq = $\sqrt{\frac{a+ib}{c+id}}$, where p, q, a, b, c, d ∈ R, then $(p^2 + q^2)^2$ =

Correct option: (B)

$p + iq = \sqrt{\frac{a+ib}{c+id}}$

$\Rightarrow |p+iq|^2 = \frac{|a+ib|}{|c+id|}$

$\Rightarrow \left(\sqrt{p^2+q^2}\right)^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$\Rightarrow (p^2+q^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Q.29 Modulus of $\left(\frac{3+2i}{3-2i}\right)$ is

Correct option: (A)

$\left(\frac{3+2i}{3-2i}\right) = \left(\frac{3+2i}{3-2i}\right)\left(\frac{3+2i}{3+2i}\right)$

$= \frac{9-4+12i}{13} = \frac{5}{13} + i\left(\frac{12}{13}\right)$

Modulus $= \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = 1$

Q.30 $\frac{(\cos 3\theta - i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^{-3}}{(\cos 5\theta + i\sin 5\theta)^{-2}(\cos 5\theta - i\sin 5\theta)^{-7}} =$

Correct option: (B)

$\frac{(\cos 3\theta - i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^{-3}}{(\cos 5\theta + i\sin 5\theta)^{-2}(\cos 5\theta - i\sin 5\theta)^{-7}}$

$= \frac{[(\cos\theta + i\sin\theta)^{-3}]^2[(\cos\theta + i\sin\theta)^4]^{-3}}{[(\cos\theta + i\sin\theta)^5]^{-2}[(\cos\theta + i\sin\theta)^{-5}]^{-7}}$

$= \frac{(\cos\theta + i\sin\theta)^{-6}(\cos\theta + i\sin\theta)^{-12}}{(\cos\theta + i\sin\theta)^{-10}(\cos\theta + i\sin\theta)^{35}}$

$= (\cos\theta + i\sin\theta)^{-6-12+10-35}$

$= (\cos\theta + i\sin\theta)^{-43}$

$= \cos 43\theta - i \sin 43\theta$