Functions **Marks: 80**

ANSWER KEY

Maths

Q.1 C	Q.2 B	Q.3 D	Q.4 D	Q.5 B	Q.6 A	Q.7 B	Q.8 C
Q.9 C	Q.10 B	Q.11 B	Q.12 C	Q.13 B	Q.14 A	Q.15 A	Q.16 D
Q.17 B	Q.18 A	Q.19 D	Q.20 C	Q.21 C	Q.22 B	Q.23 D	Q.24 B
Q.25 C	Q.26 D	Q.27 C	Q.28 A	Q.29 A	Q.30 A	Q.31 C	Q.32 B
Q.33 B	Q.34 B	Q.35 B	Q.36 A	Q.37 C	Q.38 D	Q.39 C	Q.40 C

Maths

Q.1 If f(x) = $\frac{3x+2}{5x-3}$, $x \in$ R – $\left\{\frac{3}{5}\right\}$, then

Correct option: (C)

Given: f(x) = $\frac{3x+2}{5x-3}$, $x \in$ R – $\left\{\frac{3}{5}\right\}$

Let y = f(x) = $\frac{3x+2}{5x-3}$

$\Rightarrow y(5x - 3) = 3x + 2$

$\Rightarrow 5xy - 3y - 3x = 2$

$\Rightarrow x(5y - 3) = 2 + 3y$

$\Rightarrow x = \frac{2+3y}{5y-3}$

$\Rightarrow f^{-1}(y) = \frac{2+3y}{5y-3}$ …[∵ f(x) = y]

$\Rightarrow f^{-1}(x) = \frac{2+3x}{5x-3} = f(x)$

Hence, $f^{-1}(x) = f(x)$

Q.2 The range of the function f(x) = $\frac{x^2}{x^2+1}$ is

Correct option: (B)

Let $y = \frac{x^2}{x^2+1}$

$\Rightarrow yx^2 + y = x^2$

$\Rightarrow x^2 (y - 1) + y = 0$

$\Rightarrow x^2 = \frac{y}{1-y}$

For x to be real,

$y(1 - y) \geq 0$ and $1 - y \neq 0$

$\Rightarrow y(y - 1) \leq 0$ and $y \neq 1$

$\Rightarrow 0 \leq y < 1$

Q.3 If [x] denotes the greatest integer less than or equal to x, then the range of the function f(x) = [x] – x is

Correct option: (D)

Since, $[x] \leq x < [x] + 1$

$\Rightarrow 0 \leq x - [x] < 1$

$\Rightarrow 0 \geq -(x - [x]) > -1$

$\Rightarrow 0 \geq [x] - x > -1$

$\therefore R_f = (-1, 0]$

Q.4 Let f : I → I be defined by f(x) = x^6 , then

Correct option: (D)

Let $x_1, x_2 \in$ I such that $f(x_1) = f(x_2)$

$\Rightarrow x_1 = \pm x_2$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

∴ f is not one-one.

Consider an element 2 in the co-domain I.

There does not exist any x in domain I such that f(x) = 2.

∴ f is not onto.

Q.5 The domain of the function f(x) = $\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

Correct option: (B)

To define f(x), $9 - x^2 > 0 \Rightarrow |x| < 3$

$\Rightarrow -3 < x < 3$, ...(i)

and $-1 \leq (x - 3) \leq 1$

$\Rightarrow 2 \leq x \leq 4$ ...(ii)

From (i) and (ii), $2 \leq x < 3$ i.e., [2, 3).

Q.6 Domain of the function $\frac{\sqrt{2+x}-\sqrt{2-x}}{x}$ is

Correct option: (A)

$2 + x \geq 0$

$\Rightarrow x \geq -2;$

$2 - x \geq 0$

$\Rightarrow x \leq 2;$

$x \neq 0$

Hence, domain is $[-2, 2] - \{0\}$.

Q.7 The composite map fog of the functions f : R → R, f(x) = cos x and g : R → R, g(x) = x^3 is

Correct option: (B)

$(fog)(x) = f(g(x)) = f(x^3) = \cos x^3$

$(fog)(-x) = \cos(-x)^3$

$= \cos(-x^3)$

$= \cos x^3$

$= (fog)(x)$

Q.8 If $x \neq 3$ and $f(x) = \frac{x+3}{x-3}$ is a real function, then f(f(f(4))) is

Correct option: (C)

Here, $f(4) = \frac{4+3}{4-3} = 7$

$\therefore f(f(4)) = f(7) = \frac{7+3}{7-3} = \frac{10}{4} = \frac{5}{2}$

$\therefore f(f(f(4))) = f\left(\frac{5}{2}\right) = \frac{\frac{5}{2}+3}{\frac{5}{2}-3} = -11$

Q.9 The domain of the function $\sqrt{\log(x^2-5x+5)}$ is

Correct option: (C)

The function $f(x) = \sqrt{\log(x^2-5x+5)}$ is defined, when $\log(x^2 - 5x + 5) \geq 0$

$\Rightarrow x^2 - 5x + 5 \geq 1 \Rightarrow x^2 - 5x + 4 \geq 0 \Rightarrow (x-4)(x-1) \geq 0$

This inequality holds, if $x \leq 1$ or $x \geq 4$.

Hence, the domain of the function will be $(-\infty, 1] \cup [4, \infty)$.

Q.10 The values of b and c for which the identity f (x + 1) – f (x) = 8x + 3 is satisfied, where $f(x) = bx^2 + cx + d$, are

Correct option: (B)

Given that $f(x) = bx^2 + cx + d$

$f(x+1) - f(x) = 8x + 3$

$\Rightarrow b(x+1)^2 + c(x+1) + d - (bx^2 + cx + d) = 8x + 3$

$\Rightarrow 2bx + b + c = 8x + 3$

$\Rightarrow 2b = 8 \Rightarrow b = 4$

Also, $b + c = 3 \Rightarrow c = -1$

Q.11 Let f : R → R satisfy f(x) f(y) = f (xy) for all real numbers x and y. If f(3) = 6, then $f\left(\frac{1}{3}\right) =$

Correct option: (B)

Since, $f(x) f(y) = f(xy)$

$\therefore f(1).f(3) = f(3)$

$\therefore f(1).6 = 6$

$\therefore f(1) = 1$ ...(i)

Also, $f(3).f\left(\frac{1}{3}\right) = f(1)$

$\therefore 6 \times f\left(\frac{1}{3}\right) = 1$ ...[From (i)]

$\therefore f\left(\frac{1}{3}\right) = \frac{1}{6}$

Q.12 If [x] is greatest integer function and 2[2x – 5] – 1 = 7, then x lies in

Correct option: (C)

Given: $2[2x-5] - 1 = 7$

$\Rightarrow 2[2x-5] = 8$

$\Rightarrow [2x-5] = 4$

$\Rightarrow 4 \leq 2x - 5 < 5$

$\Rightarrow 9 \leq 2x < 10$

$\Rightarrow \frac{9}{2} \leq x < 5$

$\Rightarrow x \in \left[\frac{9}{2}, 5\right)$

Q.13 If $f(x) = x^2 - 3x + 4$ and f(x) = f(2x + 1), then x =

Correct option: (B)

$f(x) = x^2 - 3x + 4$ …(i)

$f(2x+1)$

$= (2x+1)^2 - 3(2x+1) + 4$

$= 4x^2 + 4x + 1 - 6x - 3 + 4$

$\therefore f(2x+1) = 4x^2 - 2x + 2$ …(ii)

$f(x) = f(2x+1)$ …[Given]

$x^2 - 3x + 4 = 4x^2 - 2x + 2$ …[From (i) and (ii)]

$\Rightarrow 3x^2 + x - 2 = 0$

$\Rightarrow (x+1)(3x-2) = 0$

$\Rightarrow x = -1, \frac{2}{3}$

Q.14 The domain of $\cos^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ is

Correct option: (A)

$y = \cos^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$

$\therefore -1 \leq \left[\log_2\left(\frac{x}{2}\right)\right] \leq 1$

$\therefore \frac{1}{2} \leq \frac{x}{2} \leq 2$

$\therefore 1 \leq x \leq 4$

$\therefore x \in [1, 4]$

Q.15 If f be the greatest integer function and g be the modulus function, then (gof) $\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right) =$

Correct option: (A)

Given, (gof)$\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right)$

$= g\left\{f\left(\frac{-5}{3}\right)\right\} - f\left\{g\left(\frac{-5}{3}\right)\right\}$

$= g(-2) - f\left(\frac{5}{3}\right) = 2 - 1 = 1$

Q.16 If $f(x) = \frac{x}{2-x}$, ~$g(x) = \frac{x+1}{x+2}$, then (gogof) $(x) =$

Correct option: (D)

f(x) = $\frac{x}{2-x}$, g(x) = $\frac{x+1}{x+2}$

(gof) (x) = $\frac{\frac{x}{2-x}+1}{\frac{x}{2-x}+2}$

(gogof) (x) = $\frac{\frac{\frac{x}{2-x}+1}{\frac{x}{2-x}+2}+1}{\frac{\frac{x}{2-x}+1}{\frac{x}{2-x}+2}+2}$

= $\frac{\frac{x+2-x}{x+4-2x}+1}{\frac{x+2-x}{x+4-2x}+2}$

= $\frac{\frac{2}{4-x}+1}{\frac{2}{4-x}+2}$

= $\frac{2+4-x}{2+8-2x}$

= $\frac{6-x}{10-2x}$

Q.17 If the function f(x) = cos²x + cos² $\left(\frac{\pi}{3}+x\right)$ – cos x cos $\left(\frac{\pi}{3}+x\right)$ is constant (independent of x), then the value of this constant is

Correct option: (B)

f(x) = $\frac{1}{2}$(1 + cos 2x) + $\frac{1}{2}\left[1+\cos\left(\frac{2\pi}{3}+2x\right)\right]$

– $\frac{2\cos x\ \cos\left(\frac{\pi}{3}+x\right)}{2}$

= 1 + $\frac{1}{2}\left[\cos 2x+\cos\left(\frac{2\pi}{3}+2x\right)\right.$

$\left.-\cos\left(2x+\frac{\pi}{3}\right)-\cos\left(\frac{\pi}{3}\right)\right]$

= $\frac{3}{4}+\frac{1}{2}\left[\cos\ 2x\ +\ cos\left(2x+\frac{2\pi}{3}\right)\right.$

$\left.-\cos\left(2x+\frac{\pi}{3}\right)\right]$

=

$\frac{3}{4}+\frac{1}{2}\left[\cos\ 2x\ -\ 2\ \sin\left(2x\ +\ \frac{\pi}{2}\right)\sin\left(\frac{\pi}{6}\right)\right]$

=

$\frac{3}{4}+\frac{1}{2}\left[\cos\ 2x-2\sin\left(\frac{\pi}{2}+2x\right).\frac{1}{2}\right]$

= $\frac{3}{4}+\frac{1}{2}[\cos\ 2x-\cos\ 2x]=\frac{3}{4}$

Q.18 Domain of the function f (x) = $\sqrt{\frac{x}{1+x}}$ is

Correct option: (A)

For domain, take $\frac{x}{1\ +\ x}$ ≥ 0

∴ D_f = (– ∞, – 1) ∪ [0, ∞)

Q.19 If f(x) = $\frac{\alpha x}{x\ +\ 2}$, x ≠ –2, for what value of α is f(f(x)) = x

Correct option: (D)

f(x) = $\frac{\alpha x}{x\ +\ 2}$;

f(f(x)) = $f\left(\frac{\alpha x}{x\ +\ 2}\right)=\frac{\alpha\left(\frac{\alpha x}{x\ +\ 2}\right)}{\frac{\alpha x}{x\ +\ 2}+2}$

But f(f(x)) = x

∴ $\frac{\alpha^2 x}{\alpha x\ +\ 2x\ +\ 4}=x$

In L.H.S., Put α = –2,

∴ $\frac{(-2)^2 x}{(-2)x\ +\ 2x\ +\ 4}=\frac{4x}{-2x\ +\ 2x\ +\ 4}=x;$

∴ α = –2

Q.20 A = {11, 12, 13, 14}, B = {12, 13, 14, 15, 16, 17} are two sets and function f : A → B is defined by f(x) = x + 3; ∀x ∈ A, then the function f is

Correct option: (C)

f(x) = f(y)

⇒ x + 3 = y + 3 ⇒ x = y

∴ Function f is one-one.

But for 12 ∈ B, there is no pre-image in A.

∴ Function f is not onto.

Q.21 A = {1, 2, 3, 4}, B = {1, 2, 3, 4, 5, 6} are two sets and function f : A → B is defined by f(x) = x + 2; ∀ x ∈ A, then the function f is

Correct option: (C)

f(x) = f(y)

⇒ x + 2 = y + 2 ⇒ x = y

∴ Function f is one-one

Q.22 If f : R → R, g : R → R are two functions defined by f(x) = 2x – 3, g(x) = x^3 + 5, then $(fog)^{-1}$ (x) =

Correct option: (B)

f(x) = 2x – 3 and g(x) = x^3 + 5

(fog)(x) = f(g(x))

= 2[g(x)] – 3

= 2(x^3 + 5) – 3

= 2x^3 + 7

Let y = (fog)(x) = 2x^3 + 7

⇒ $\frac{y-7}{2} = x^3$

⇒ $x = \left(\frac{y-7}{2}\right)^{\frac{1}{3}}$

⇒ $(\text{fog})^{-1}(y) = \left(\frac{y-7}{2}\right)^{\frac{1}{3}}$ …[∵ $(\text{fog})(x) = y$]

⇒ $(\text{fog})^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

Q.23 If f(x) = [8x] – 3, where [x] is greatest integer function of x, then f(π) = (where π = 3.14)

Correct option: (D)

f(π) = [8π] – 3

= [8 × 3.14] – 3 …[π = 3.14]

= [25.12] – 3

= 25 – 3

∴ f(π) = 22

Q.24 The domain of definition of f (x) = $\sqrt{\frac{1 - |x|}{2 - |x|}}$ is

Correct option: (B)

$\frac{1-|x|}{2-|x|} \geq 0$

⇒ $\frac{|x|-1}{|x|-2} \geq 0$

⇒ |x| ≤ 1 as |x| > 2

⇒ x ∈ (–∞, –2) ∪ (2, ∞) ∪ [–1, 1]

Q.25 If f(x) = $\frac{4x+7}{7x-4}$, then the value of f{f[f(2)]} =

Correct option: (C)

Given: f(x) = $\frac{4x+7}{7x-4}$

$f(2) = \frac{4(2)+7}{7(2)-4} = \frac{15}{10} = \frac{3}{2}$

Now,

$f[f(2)] = f\left(\frac{3}{2}\right) = \frac{4\left(\frac{3}{2}\right)+7}{7\left(\frac{3}{2}\right)-4} = \frac{26}{13} = 2$

Also,

f{f[f(2)]} = f(2) = $\frac{3}{2}$

Q.26 The domain of the f(x) = exp($\sqrt{x^2 - 3x + 2}$) is

Correct option: (D)

f(x) = exp($\sqrt{x^2 - 3x + 2}$)

⇒ $x^2 - 3x + 2 \geq 0$

⇒ (x – 1) (x – 2) ≤ 0

∴ D_f = [1, 2]

Q.27 The range of the function, f(x) = $\frac{1+x^2}{x^2}$ is

Correct option: (C)

f(x) is defined for all x ∈ R – {0}.

So, dom(f) = R – {0}

Let $y = \frac{1+x^2}{x^2}$

⇒ $x = \pm\sqrt{\frac{1}{y-1}}$

For x to be real, y – 1 > 0 ⇒ y ∈ (1, ∞)

Q.28 f: R→R; g: R → R are two functions such that f(x) = 2x – 3, g(x) = x^3 + 5, then (fog)$^{-1}$(–9) is

Correct option: (A)

We have, f(x) = 2x – 3, g(x) = x^3 + 5

fog(x) = f(g(x))

= 2g(x) – 3

= 2(x^3 + 5) – 3 = 2x^3 + 7

Let (fog) (x) = y = 2x^3 + 7

y = 2x^3 + 7

⇒ y – 7 = 2x^3

⇒ $x^3 = \frac{y-7}{2}$

⇒ $x = \left(\frac{y-7}{2}\right)^{\frac{1}{3}}$

∴ $(\text{fog})^{-1}(y) = \left(\frac{y-7}{2}\right)^{\frac{1}{3}}$

$\therefore\ (fog)^{-1}(-9) = \left(\dfrac{-9-7}{2}\right)^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = -2$

Q.29 If f(x) = $\begin{cases} x, & if\, x\, is\, rational \\ 0, & if\, x\, is\, irrational \end{cases}$ and g(x) = $\begin{cases} 0, & if\, x\, is\, rational \\ x, & if\, x\, is\, irrational \end{cases}$, then f – g is

Correct option: (A)

Here, (f – g) (x) = f(x) – g(x)

∴ (f – g) (x) =
$\begin{cases} x - 0 = x, & if\, x\, is\, rational \\ 0 - x = -x, & if\, x\, is\, irrational \end{cases}$

Let k = f – g

Let x, y be any two distinct real numbers.

Then, $x \neq y$

⇒ $-x \neq -y$

Now, $x \neq y$

⇒ k(x) ≠ k(y)

⇒ (f – g) (x) ≠ (f – g) (y)

⇒ f – g is one-one.

Let y be any real number

If y is a rational number, then

k(y) = y

⇒ (f – g) (y) = y

If y is an irrational number , then

k(– y) = y

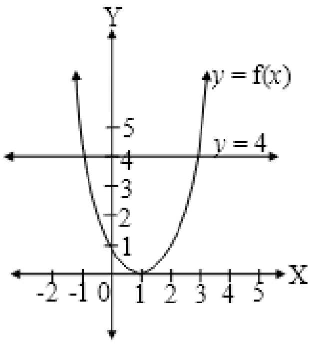
⇒ (f – g) (– y) = y

Thus, every y ∈ R (co-domain) has its pre-image in R (domain)

∴ f – g : R → R is onto.

Hence, f – g is one-one and onto.

Q.30 From the graph below, find x for which f (x) = 4



Correct option: (A)

The graph shows that the function f(x) intersects the line y = 4 at two points: x = 3 and x = -1.

Therefore, the correct answer is 3 and -1.

Q.31 The range of the function f(x) = $\dfrac{x^2-3x+2}{x^2+x-6}$ is

Correct option: (C)

f(x) is defined for $x^2 + x - 6 \neq 0$, i.e., $x \neq -3, 2$

∴ Dom(f) = R – {– 3, 2}

Let $y = \dfrac{x^2-3x+2}{x^2+x-6} = \dfrac{x-1}{x+3}$

⇒ $x = \dfrac{3y+1}{y-1}$

x is real for $y - 1 \neq 0$, i.e., $y \neq 1$

Hence, range(f) = R – {1}

Q.32 If f : A → B is a bijection and g : B → A is the inverse of f, then fog is equal to [BCECE 2014]

Correct option: (B) I_B

Self Explanatory

Q.33 Range of the function f(x) = $\dfrac{1}{3x+2}$ is

Correct option: (B)

Dom(f) = R – $\left\{-\dfrac{2}{3}\right\}$

For Range(f), let y = f (x) = $\dfrac{1}{3x+2}$

∴ $3x + 2 = \dfrac{1}{y}$ ⇒ $x = \dfrac{1}{3}\left(\dfrac{1}{y} - 2\right)$

∴ x is real if $y \neq 0$.

Hence, R_f = R – {0}

Q.34

A function f is said to be even, if

Correct option: (B)

A function f is said to be even if f(-x) = f(x) for all values of x. In other words, the function is symmetric about the y-axis.

Q.35 For f(x) = [x], where [x] is the greatest integer function, which of the following is true, for every x ∈ R.

Correct option: (B)

We know that, x = [x] + {x}, for all x ∈ R, where [x] is the greatest integer function of x and {x} is fractional part function of x

Also, $0 < \{x\} < 1$

$\Rightarrow 0 < x - [x] < 1$

$\therefore \quad x - [x] < 1$

$\therefore \quad x < [x] + 1$

Q.36 If $f : [1, \infty) \to [2, \infty)$ is given by f $(x) = x + \dfrac{1}{x}$ then $f^{-1}(x)$ equals

Correct option: (A)

$f(x) = x + \dfrac{1}{x}$

let $y = x + \dfrac{1}{x}$

$\therefore \quad xy = x^2 + 1$

$\therefore \quad x^2 - xy + 1 = 0$

$\therefore \quad x = \dfrac{y \pm \sqrt{y^2 - 4}}{2}$

$\therefore \quad x = \dfrac{y + \sqrt{y^2 - 4}}{2} \qquad \ldots[\because x \in [1, \infty)]$

$\therefore \quad f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

Q.37 If f(x) = x^2 + 1, then f[f(x)] =

Correct option: (C)

$f(x) = x^2 + 1$

$f[f(x)] = [f(x)]^2 + 1$

$= (x^2 + 1)^2 + 1$

$= x^4 + 2x^2 + 1 + 1$

$= x^4 + 2x^2 + 2$

Q.38 The domain and range of the relation R given by R =
$\left\{(x, y)/y = x + \dfrac{6}{x}, x, y \in N \text{ and } x < 6\right\}$

Correct option: (D)

Given: R =
$\left\{(x, y)/y = x + \dfrac{6}{x}, x, y \in N \text{ and } x < 6\right\}$

For $x = 1$, $y = 1 + \dfrac{6}{1} = 7$

For $x = 2$, $y = 2 + \dfrac{6}{2} = 5$

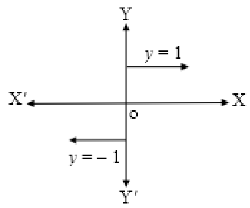
For $x = 3$, $y = 3 + \dfrac{6}{3} = 5$

For $x = 4$, $y = 4 + \dfrac{6}{4} = 5.5 \notin N$

For $x = 5$, $y = 5 + \dfrac{6}{5} = 6.2 \notin N$

$\therefore$ Domain = {1, 2, 3} and Range = {5, 7}

Q.39



is the graph of

Correct option: (C)

The graph represents the signum function.

$\therefore sgn(x) = 1 \; for \; x > 0$

$= 0 \; for \; x = 0$

$= \; -1 \; for \; x < 0$

Q.40 If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are the roots of the equation $f(x) = 0$ then the value of $4m + 5n$ is

Correct option: (C)

$f(x) = 2x^3 + mx^2 - 13x + n$

2, 3 are the roots of the given equation f(x) = 0.

$\Rightarrow f(2) = 0$

$\Rightarrow 2(2)^3 + m(2)^2 - 13(2) + n = 0$

$\Rightarrow 16 + 4m - 26 + n = 0$

$\Rightarrow 4m + n = 10 \quad \ldots(i)$

Also, $f(3) = 0$

$\Rightarrow 2(3)^3 + m(3)^2 - 13(3) + n = 0$

$\Rightarrow 54 + 9m - 39 + n = 0$

$\Rightarrow 9m + n = -15 \quad \ldots(ii)$

Subtracting (i) from (ii), we get

$5m = -25$

$\Rightarrow m = -5$

Substituting m = – 5 in equation (i), we get

$4(-5) + n = 10$

$\Rightarrow n = 10 + 20$

$\Rightarrow n = 30$

$\therefore \quad 4m + 5n = 4(-5) + 5(30)$

$= -20 + 150 = 130$