Limits Continuity and Differentiation **Marks: 220**

ANSWER KEY

Maths

Q.1 B	Q.2 C	Q.3 D	Q.4 C	Q.5 C	Q.6 A	Q.7 C	Q.8 B
Q.9 A	Q.10 B	Q.11 D	Q.12 A	Q.13 C	Q.14 A	Q.15 C	Q.16 C
Q.17 D	Q.18 B	Q.19 B	Q.20 B	Q.21 C	Q.22 D	Q.23 A	Q.24 C
Q.25 D	Q.26 C	Q.27 C	Q.28 C	Q.29 B	Q.30 C	Q.31 C	Q.32 A
Q.33 C	Q.34 A	Q.35 A	Q.36 D	Q.37 A	Q.38 D	Q.39 A	Q.40 B
Q.41 D	Q.42 A	Q.43 C	Q.44 B	Q.45 B	Q.46 C	Q.47 A	Q.48 D
Q.49 C	Q.50 B	Q.51 B	Q.52 A	Q.53 A	Q.54 B	Q.55 D	Q.56 D
Q.57 C	Q.58 C	Q.59 C	Q.60 B	Q.61 D	Q.62 B	Q.63 C	Q.64 A
Q.65 B	Q.66 B	Q.67 B	Q.68 B	Q.69 C	Q.70 C	Q.71 C	Q.72 A
Q.73 A	Q.74 D	Q.75 B	Q.76 C	Q.77 A	Q.78 B	Q.79 D	Q.80 A
Q.81 B	Q.82 B	Q.83 C	Q.84 C	Q.85 A	Q.86 C	Q.87 D	Q.88 A
Q.89 C	Q.90 D	Q.91 C	Q.92 B	Q.93 B	Q.94 D	Q.95 C	Q.96 C
Q.97 A	Q.98 A	Q.99 B	Q.100 A	Q.101 A	Q.102 D	Q.103 D	Q.104 A
Q.105 B	Q.106 C	Q.107 B	Q.108 B	Q.109 B	Q.110 C		

Maths

Q.1 The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ **w. r. t.** $\sqrt{1-x^2}$ **at** $x=\frac{1}{2}$ **is**

Correct option: (B)

Let $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ and $z=\sqrt{1-x^2}$

$\therefore y = \cos^{-1}(2x^2 - 1)$

Put $x = \cos\theta$, then $\theta = \cos^{-1}x$

$\therefore y = \cos^{-1}(2\cos^2\theta - 1)$

$= \cos^{-1}(\cos 2\theta) = 2\theta$

$\therefore y = 2\cos^{-1}x$

$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$ and $\frac{dz}{dx} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{2}{x}$

$\therefore \left(\frac{dy}{dz}\right)_{\left(x=\frac{1}{2}\right)} = 4$

Q.2 $\lim\limits_{x\to 0}\dfrac{\sin x+\log(1-x)}{x^2}$ **is equal to**

Correct option: (C)

Applying L–Hospital's rule, we get

$$\lim_{x\to 0}\frac{\cos x-\frac{1}{1-x}}{2x} = \lim_{x\to 0}\frac{-\sin x-\frac{1}{(1-x)^2}}{2} = -\frac{1}{2}$$

Alternate Method:

$$\lim_{x\to 0}\frac{\sin x+\log\ (1-x)}{x^2}$$

$$= \lim_{x\to 0}\frac{\left(x-\frac{x^3}{3\,!}+\frac{x^5}{5\,!}-\ldots\right)}{x^2} + \lim_{x\to 0}\frac{\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\ldots\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{\frac{-x^2}{2}-x^3\left(\frac{1}{3\,!}+\frac{1}{3}\right)-\frac{x^4}{4}\ldots}{x^2} = -\frac{1}{2}$$

Q.3 The value of $\lim\limits_{x\to 7}\dfrac{2-\sqrt{x-3}}{x^2-49}$ **is**

Correct option: (D)

Applying L–Hospital's rule, we get

$$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x\to 7}\frac{0-\frac{1}{2\sqrt{x-3}}}{2x}$$

$$= \lim_{x\to 7}\frac{-1}{4x\sqrt{x-3}} = \frac{-1}{4\,(7)(2)}$$

$$= \frac{-1}{56}$$

Q.4 If f(*x*) = $\dfrac{\left(e^{kx}-1\right)^2\sin x}{x^3}$**,** $x \neq 0$

= 4, $\quad$ $x = 0$

is continuous at $x = 0$**, then k =**

Correct option: (C)

Since, f(x) is continuous at $x = 0$.

$\therefore f(0) = \lim\limits_{x\to 0} f(x)$

$$\Rightarrow 4 = \lim_{x\to 0}\frac{\left(e^{kx}-1\right)^2\sin x}{x^3}$$

$$\Rightarrow 4 = \lim_{x\to 0}\left[k^2\times\frac{\left(e^{kx}-1\right)^2}{k^2x^2}\cdot\frac{\sin x}{x}\right]$$

$\Rightarrow 4 = k^2$

$\Rightarrow k = \pm 2$

Q.5 $\lim\limits_{x\to -2}\dfrac{\sin^{-1}(x+2)}{x^2+2x}$ **is equal to**

Correct option: (C)

Applying L–Hospital's rule, we get

$$\lim_{x\to -2}\frac{\sin^{-1}(x+2)}{x^2+2x} = \lim_{x\to -2}\frac{\left(\frac{1}{\sqrt{1-(x+2)^2}}\right)}{2x+2}$$

$$= \frac{1}{-4+2} = -\frac{1}{2}$$

Q.6 The value of $\lim\limits_{x\to\infty}\dfrac{ax^2+bx+c}{dx^2+ex+f}$ **is**

Correct option: (A)

$$\lim_{x\to\infty}\frac{ax^2+bx+c}{dx^2+ex+f} = \lim_{x\to\infty}\frac{a+\frac{b}{x}+\frac{c}{x^2}}{d+\frac{e}{x}+\frac{f}{x^2}}$$

$$= \frac{a+0+0}{d+0+0} = \frac{a}{d}$$

Q.7 If f(*x*) is continuous at $x = 0$**, where f(*x*) =** $\dfrac{\left(e^{3x}-1\right)\cdot\sin x}{x^2}$ **; then f(0) =**

Correct option: (C)

Since f(x) is continuous at x = 0.

$\therefore$ $f(0) = \lim_{x\to 0} f(x)$

$$= \lim_{x\to 0} \frac{(e^{3x}-1)\sin x}{x^2}$$

$$= 3\left(\lim_{x\to 0} \frac{e^{3x}-1}{3x}\right)\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$

$\therefore$ f(0) = 3

Q.8 **The value of** $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{\frac{1}{x}}$ **is**

Correct option: (B)

$$\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{\frac{1}{x}} = \lim_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left(1+\frac{2\tan x}{1-\tan x}\right)^{\frac{1}{x}}$$

$$= e^{\lim_{x\to 0}\frac{2\tan x}{1-\tan x}\cdot\frac{1}{x}}$$

$$= e^{\lim_{x\to 0}\frac{2}{1-\tan x}\times\frac{\tan x}{x}}$$

$= e^2$

Q.9 **If f(x) is continuous in [–2, 2], where f(x) =** $\begin{cases} x+a, & x<0 \\ x, & 0\le x<1 \\ b-x, & x\ge 1\end{cases}$**, then**

Correct option: (A)

Since, f(x) is continuous in [–2, 2].

$\therefore$ it is continuous at x = 0 and x = 1.

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{x\to 0^-}(x+a) = \lim_{x\to 0^+} x$$

$\Rightarrow$ a = 0

Also, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$$\Rightarrow \lim_{x\to 1^-} x = \lim_{x\to 1^+}(b-x)$$

$\Rightarrow$ 1 = b – 1

$\Rightarrow$ b = 2

Q.10 $\lim_{x\to\infty}\frac{\sin x}{x} =$

Correct option: (B)

Let $x = \frac{1}{y}$ or $y = \frac{1}{x}$,

so that $x \to \infty, y \to 0$

$\therefore \lim_{x\to\infty}\left(\frac{\sin x}{x}\right) = \lim_{y\to 0}\left(y.\sin\frac{1}{y}\right)$

$= \lim_{y\to 0} y \times \lim_{y\to 0} \sin\frac{1}{y} = 0$

Q.11 $\lim_{x\to 0}\frac{xe^x - \tan x}{x}$ **is equal to**

Correct option: (D)

$\lim_{x\to 0}\frac{xe^x - \tan x}{x} = \lim_{x\to 0}\left(e^x - \frac{\tan x}{x}\right)$

$= e^0 - 1$

$= 1 - 1 = 0$

Q.12 **If log(x + y) = log xy + 3, then** $\frac{dy}{dx}$ **=**

Correct option: (A)

Given:

log(x + y) = log x + log y + 3

Differentiating w.r.t. 'x', we get

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$$\frac{1}{x+y}-\frac{1}{x} = \left(\frac{1}{y}-\frac{1}{x+y}\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{x-x-y}{x(x+y)} = \left(\frac{x+y-y}{y(x+y)}\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2} = -\left(\frac{y}{x}\right)^2$$

Q.13 **The points of discontinuity of tan x are**

Correct option: (C)

Let f(x) = tan x

The point of discontinuity of f(x) are those points where tan x is infinite.

i.e., tan $x = \infty$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$, n $\in$ I

Q.14 $\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{3n^2} =$

Correct option: (A)

$$\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{3n^2} = \lim_{n\to\infty}\frac{n(n+1)}{2}\cdot\frac{1}{3n^2}$$

$$= \frac{1}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$= \frac{1}{6}(1) = \frac{1}{6}$$

Alternate Method:

$$\lim_{n\to\infty} \frac{1+2+3+\ldots+n}{3n^2} =$$

$$\frac{1}{3}\lim_{n\to\infty} \frac{1+2+3+\ldots+n}{n^2}$$

$$= \frac{1}{3}\cdot\frac{1}{1+1} \quad \ldots$$

$$\left[\lim_{x\to\infty} \frac{1^\alpha+2^\alpha+3^\alpha+\ldots+x^\alpha}{x^{\alpha+1}} = \frac{1}{\alpha+1}\right]$$

$$= \frac{1}{6}$$

Q.15 **The function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by**

$$f(x) = \begin{cases} \frac{1}{x}\log\left(\frac{1+3x}{1-2x}\right) & , \quad x \neq 0 \\ k & , \quad x = 0 \end{cases}$$

is continuous at x = 0, then k is

Correct option: (C)

f is continuous at $x = 0$.

$\therefore$ $f(0) = \lim_{x\to 0} f(x)$

$$\therefore \quad k = \lim_{x\to 0}\left(\frac{1}{x}\log(1+3x) - \frac{1}{x}\log(1-2x)\right)$$

$$= \lim_{x\to 0}\left(\frac{3\ \log(1+3x)}{3x} + \frac{2\ \log(1-2x)}{-2x}\right)$$

$= 3 + 2 = 5$

Q.16

$$\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1+cosx}{1-cosx}}\right] =$$

Correct option: (C)

$$\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+cosx}{1-cosx}}\right)$$

$$= \frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}\right)$$

$$= \frac{d}{dx}\left(\tan^{-1}\left(\cot\frac{x}{2}\right)\right)$$

$$= \frac{d}{dx}\left(\tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right)\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{2} - \frac{x}{2}\right)$$

$$= -\frac{1}{2}$$

Q.17 **If $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$, then $\frac{dy}{dx}$ at x = 1 is**

Correct option: (D)

$$\Rightarrow x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$$

$$\Rightarrow \log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$$

$$\Rightarrow \tan(\log x) = \frac{y-x^2}{x^2}$$

$\Rightarrow y = x^2 \tan(\log x) + x^2$

Differentiating w.r.t.x, we get

$$\frac{dy}{dx} = 2x\ \tan(\log x) + x^2\frac{\sec^2(\log x)}{x} + 2x$$

$= x \sec^2(\log x) + 2x\tan(\log x) + 2x$

$$\therefore \left.\frac{dy}{dx}\right|_{x=1} = 1 + 2 = 3$$

Q.18 $$\lim_{x\to 5}\frac{\sqrt{2-2\cos(x^2-12x+35)}}{(x-5)} = \ldots$$

Correct option: (B)

$$\lim_{x\to 5}\frac{\sqrt{2-2\cos(x^2-12x+35)}}{x-5}$$

$$= \lim_{x\to 5}\frac{\sqrt{2[1-\cos(x^2-12x+35)]}}{x-5}$$

$$= \lim_{x\to 5}\frac{\sqrt{2\left(2\sin^2\left(\frac{x^2-12x+35}{2}\right)\right)}}{x-5}$$

$$= \lim_{x\to 5}\frac{2\sin\left(\frac{x^2-12x+35}{2}\right)}{x-5}$$

$$= \lim_{x\to 5}\frac{\sin\left[\frac{(x-5)(x-7)}{2}\right]}{\frac{(x-5)(x-7)}{2}} \times (x-7)$$

$= 1 \times (5 - 7)$

$= -2$

Q.19 The function f(t) = $\frac{1}{t^2+t-2}$ where t = $\frac{1}{x-1}$ is discontinuous at

Correct option: (B)

f(t) = $\frac{1}{t^2+t-2} = \frac{1}{(t+2)(t-1)}$

f(t) is not defined at t = –2 and t = 1.

t = –2

⇒ $\frac{1}{x-1}$ = –2

⇒ $x = \frac{1}{2}$

t = 1

⇒ $\frac{1}{x-1}$ = 1

⇒ x = 2

∴ The function f(t) is discontinuous at $x = \frac{1}{2}$ and x = 2.

Q.20

If f(x) is continuous at x = 0, where f(x) = $\begin{cases} x^2+a, & x \geq 0 \\ 2\sqrt{x^2+1}+b, & x<0 \end{cases}$ **and** $f\left(\frac{1}{2}\right) = 2$, **then the values of a and b are respectively**

Correct option: (B)

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + a$$

⇒ 2 = $\frac{1}{4} + a$ ⇒ a = $\frac{7}{4}$....(i)

Since, f(x) is continuous at x = 0.

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{x\to 0^-}\left(2\sqrt{x^2+1}+b\right) = \lim_{x\to 0^+}\left(x^2+a\right)$$

$$\Rightarrow 2\sqrt{0+1}+b = 0+a$$

⇒ 2 + b = $\frac{7}{4}$....[From (i)]

⇒ b = $-\frac{1}{4}$

Q.21 If x = 2 cos θ – cos 2θ and y = 2 sin θ – sin 2θ, then $\frac{dy}{dx}$ =

Correct option: (C)

x = 2 cos θ – cos 2θ and y = 2 sin θ – sin 2θ

∴ $\frac{dx}{d\theta}$ = –2 sin θ + 2 sin 2θ and

$\frac{dy}{d\theta}$ = 2 cos θ – 2 cos 2θ

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

$$= \frac{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}} = \tan\frac{3\theta}{2}$$

Q.22 Let k be a non-zero real number.

If f(x) = $\begin{cases} \frac{(e^x-1)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}, x \neq 0 \\ 12, x = 0 \end{cases}$

is a continuous function, then the value of k is

Correct option: (D)

f(x) is continuous at x = 0.

∴ f(0) = $\lim_{x\to 0} f(x)$

$$\Rightarrow 12 = \lim_{x\to 0}\frac{(e^x-1)^2}{\sin\left(\frac{x}{k}\right)\cdot\log\left(1+\frac{x}{4}\right)}$$

$$\Rightarrow 12 = \lim_{x\to 0}\frac{\frac{(e^x-1)^2}{x^2}}{\frac{\sin\left(\frac{x}{k}\right)}{x}\cdot\log\frac{\left(1+\frac{x}{4}\right)}{x}}$$

$$\Rightarrow 12 = \frac{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^2}{\lim_{x\to 0}\frac{\sin\frac{x}{k}}{\frac{x}{k}k}\cdot\log\lim_{x\to 0}\left(1+\frac{x}{4}\right)^{\frac{1}{x}}}$$

$$\Rightarrow 12 = \frac{k\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^2}{\lim_{x\to 0}\frac{\sin\frac{x}{k}}{\frac{x}{k}}\cdot\log\lim_{x\to 0}\left[\left(1+\frac{x}{4}\right)^{\frac{4}{x}}\right]^{\frac{1}{4}}}$$

$\Rightarrow 12 = \dfrac{k}{1 \times \log e^{\frac{1}{4}}}$ ... $\left[\begin{array}{l} \lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = 1 \\ \lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1 \\ \lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e \end{array}\right]$

$\Rightarrow 12 = \dfrac{k}{\frac{1}{4}}$

$\Rightarrow 12 = 4k$

$\Rightarrow k = 3$

Q.23 If f (x) = $\log_{(1-9x)}(1 + 9x)$, $x \neq 0$

= k, x = 0 is continuous at x = 0,

then the value of k is

Correct option: (A)

Since, f(x) is continuous at x = 0.

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow k = \lim_{x\to 0} \log_{(1-9x)}(1 + 9x)$

$\Rightarrow k = \lim_{x\to 0} \frac{\log(1+9x)}{\log(1-9x)}$

$\Rightarrow k = \dfrac{\lim_{x\to 0} \frac{\log(1+9x)}{9x} \times 9}{\lim_{x\to 0} \frac{\log(1-9x)}{-9x} \times -9} = \dfrac{9}{-9}$

$\Rightarrow k = -1$

Q.24 The value of $\lim\limits_{x \to 3} \dfrac{x^5 - 243}{x^2 - 9}$ is

Correct option: (C)

$\lim\limits_{x \to 3} \dfrac{x^5 - 243}{x^2 - 9} = \lim\limits_{x \to 3} \dfrac{x^5 - 3^5}{x^2 - 3^2} = \dfrac{5}{2}(3)^{5-2} = \dfrac{135}{2}$

$\ldots\left[\because \lim\limits_{x\to a} \dfrac{x^m - a^m}{x^n - a^n} = \dfrac{m}{n} a^{m-n}\right]$

Q.25 $\lim_{x\to 1} \frac{1+\log x - x}{1-2x+x^2} =$

Correct option: (D)

Applying L–Hospital's rule, we get

$\lim\limits_{x\to 1} \frac{1+\log x - x}{1-2x+x^2} = \lim\limits_{x\to 1} \frac{\frac{1}{x}-1}{-2+2x} = \lim\limits_{x\to 1} \frac{1-x}{2x(x-1)} = \lim\limits_{x\to 1} \frac{-1}{2x} = -\frac{1}{2}$

Q.26 $\lim\limits_{x\to 0} \dfrac{\sin x^2(1-\cos x^2)}{x^6}$ is equal to

Correct option: (C)

$\lim\limits_{x\to 0} \dfrac{\sin x^2(1-\cos x^2)}{x^6} = \lim\limits_{x\to 0} \dfrac{\sin x^2}{x^2} \cdot \dfrac{1-\cos x^2}{x^4}$

$= \lim\limits_{x\to 0} \dfrac{1-\cos x^2}{x^4}$

$= \lim\limits_{x\to 0} \dfrac{\sin x^2 . 2x}{4x^3}$

$= \lim\limits_{x\to 0} \dfrac{\sin x^2}{2x^2} = \dfrac{1}{2}$

Q.27 The value of $\lim\limits_{x\to 2} \dfrac{e^{3x-6} - 1}{\sin(2-x)}$ is

Correct option: (C)

$\lim\limits_{x\to 2} \dfrac{e^{3x-6} - 1}{\sin(2-x)} = \lim\limits_{x\to 2} \dfrac{e^{3(x-2)} - 1}{3(x-2)} \times \dfrac{1}{\frac{\sin(2-x)}{-3(2-x)}}$

$= 1 \times -3$

$\ldots \left[\because \lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = 1, \lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1\right]$

$= -3$

Q.28 If f(x) = $(\sec^2 x)^{\cot^2 x}$, $x \neq 0$

= k , x = 0

is continuous at x = 0, then k is equal to

Correct option: (C)

Since, f(x) is continuous at x = 0.

$\therefore f(0) = \lim\limits_{x\to 0} f(x)$

$\Rightarrow k = \lim\limits_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\Rightarrow k = \lim\limits_{x\to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

$\Rightarrow k = e$

Q.29 If y = f (ax^2 + b), then $\dfrac{dy}{dx}$ is equal to

Correct option: (B)

$y = f(ax^2 + b)$

$\therefore \dfrac{dy}{dx} = f'(ax^2 + b) . \dfrac{d}{dx}(ax^2 + b) = 2ax\, f'(ax^2 + b)$

Q.30 $\lim\limits_{x\to 0} \dfrac{\tan 2x - x}{3x - \sin x} =$

Correct option: (C)

$\lim\limits_{x\to 0} \dfrac{\tan 2x - x}{3x - \sin x} = \lim\limits_{x\to 0} \left\{ \dfrac{\frac{2\tan 2x}{2x} - 1}{3 - \frac{\sin x}{x}} \right\} = \dfrac{1}{2}$

Alternate Method:

Applying L–Hospital's rule, we get

$\lim\limits_{x\to 0} \dfrac{\tan 2x - x}{3x - \sin x} = \lim\limits_{x\to 0} \dfrac{2\sec^2 2x - 1}{3 - \cos x}$

$= \dfrac{2-1}{3-1} = \dfrac{1}{2}$

Q.31 If f(x) = $|x| + |x-1|$, then

Correct option: (C)

Given, f(x) = $|x| + |x-1|$

$$\therefore f(x) = \begin{cases} -x-(x-1), & if\, x<0 \\ x-(x-1), & if\, 0 \le x < 1 \\ x+(x-1), & if\, x \ge 1 \end{cases}$$

$$\therefore f(x) = \begin{cases} -2x+1\ , & if\, x<0 \\ 1\ , & if\, 0 \le x < 1 \\ 2x-1\ , & if\, x \ge 1 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} (-2x+1) = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} 1 = 1$$

f(0) = 1

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$\therefore$ f(x) is continuous at x = 0.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} 1 = 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (2x-1) = 1$$

f (1) = 2(1) – 1 = 1

$$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$\therefore$ f(x) is continuous at x = 1.

Q.32 If f(x) is continuous at x = a, where

f(x) = $\dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$, for $x \neq$ a,

then f(a) =

Correct option: (A)

Since f(x) is continuous at x = a.

$\therefore$ f(a) = $\lim_{x\to a}$ f(x)

$$= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$$

$$= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x+a}.\sqrt{x-a}}$$

$$= \lim_{x\to a} \frac{1}{\sqrt{x+a}} \left(\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x-a}}\right)$$

$$= \frac{1}{\sqrt{2a}} \lim_{x\to a} \left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x-a}} + 1\right)$$

$$= \frac{1}{\sqrt{2a}} \lim_{x\to a} \left[\frac{(\sqrt{x})^2-(\sqrt{a})^2}{\sqrt{x-a}(\sqrt{x}+\sqrt{a})} + 1\right]$$

$$= \frac{1}{\sqrt{2a}} \lim_{x\to a} \left(\frac{x-a}{\sqrt{x-a}(\sqrt{x}+\sqrt{a})} + 1\right)$$

$$= \frac{1}{\sqrt{2a}} \lim_{x\to a} \left[\frac{\sqrt{x-a}}{\sqrt{x}+\sqrt{a}} + 1\right]$$

$$= \frac{1}{\sqrt{2a}} (0+1)$$

$$= \frac{1}{\sqrt{2a}}$$

Q.33 The value of $\lim_{x \to 1} \dfrac{x+x^2+\ldots+x^n-n}{x-1}$ is

Correct option: (C)

$$\lim_{x \to 1} \frac{x+x^2+\ldots+x^n-n}{x-1}$$

$$= \lim_{x \to 1} \frac{(x-1)+(x^2-1^2)+(x^3-1^3)+\ldots+(x^n-1^n)}{x-1}$$

$$= \lim_{x \to 1} \left(\frac{x-1}{x-1}+\frac{x^2-1^2}{x-1}+\frac{x^3-1^3}{x-1}+\ldots+\frac{x^n-1^n}{x-1}\right)$$

$= 1 + 2\,(1)^{2-1} + 3\,(1)^{3-1} + \ldots + n\,(1)^{n-1}$

$= 1 + 2 + 3 + \ldots + n$

$= \dfrac{n\,(n+1)}{2}$

Alternate method:

Applying L–Hospital's Rule, we get

$$\lim_{x \to 1} (1 + 2x + \ldots\ldots + nx^{n-1})$$

$= 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$

Q.34 $\lim_{n\to\infty} \dfrac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n} =$

Correct option: (A)

$$\lim_{n\to\infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n}$$

$$= \lim_{n\to\infty} \frac{6.2^n - 20.5^n}{5.2^n + 7.5^n} = \lim_{n\to\infty} \frac{5^n[6(2/5)^n - 20]}{5^n[5(2/5)^n + 7]}$$

$$= -\frac{20}{7} \quad ....\left[\because n \to \infty, \left(\frac{2}{5}\right)^n \to 0\right]$$

Q.35 $\lim_{x\to 1} \frac{1 - x^{-\frac{2}{3}}}{1 - x^{-\frac{1}{3}}}$ **is equal to**

Correct option: (A)

$$\lim_{x\to 1} \frac{1 - x^{-\frac{2}{3}}}{1 - x^{-\frac{1}{3}}} = \lim_{x\to 1} \frac{1^2 - \left(x^{-\frac{1}{3}}\right)^2}{1 - x^{-\frac{1}{3}}}$$

$$= \lim_{x\to 1} \frac{\left(1 + x^{-\frac{1}{3}}\right)\left(1 - x^{-\frac{1}{3}}\right)}{1 - x^{-\frac{1}{3}}}$$

$$= \lim_{x\to 1} \left(1 + x^{-1/3}\right) = 2$$

Alternate method:

Apply L– Hospital's Rule.

Q.36 **If** $f(x) = \begin{cases} x + \lambda, & x < 3 \\ 4, & x = 3 \\ 3x - 5, & x > 3 \end{cases}$ **is continuous at** $x = 3$**, then** $\lambda =$

Correct option: (D)

Since, f(x) is continuous at x = 3.

$\therefore$ f(3) = $\lim_{x\to 3^-} f(x)$

$\Rightarrow$ 4 = 3 + λ

$\Rightarrow$ λ = 1

Q.37 $\lim_{x\to\frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x} =$

Correct option: (A)

$$\lim_{x\to\pi/4} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$$

$$= \lim_{x\to\pi/4} \frac{\left[(\cos x + \sin x)^2\right]^{\frac{5}{2}} - (2)^{\frac{5}{2}}}{(1 + \sin 2x) - 2}$$

$$= \lim_{x\to\pi/4} \frac{(1 + \sin 2x)^{\frac{5}{2}} - 2^{\frac{5}{2}}}{(1 + \sin 2x) - 2}$$

$= \lim_{y\to 2} \frac{y^{\frac{5}{2}} - 2^{\frac{5}{2}}}{y - 2}$, where $y = 1 + \sin 2x$

$$= \frac{5}{2} \times 2^{\frac{5}{2} - 1} = 5\sqrt{2}$$

Q.38 **If** $f(x) = \begin{cases} mx + 1, & x \leqslant \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$**, is continuous at** $x = \frac{\pi}{2}$ $(\mathrm{m}, n \in \mathbb{Z})$ **then**

Correct option: (D)

Since f(x) is continuous at $x = \frac{\pi}{2}$.

$$\therefore \lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^+} f(x)$$

$$\therefore \lim_{x\to\frac{\pi}{2}} (mx + 1) = \lim_{x\to\frac{\pi}{2}} (\sin x + n)$$

$$\therefore \frac{\pi}{2}m + 1 = \sin\frac{\pi}{2} + n$$

$$\therefore n = \frac{m\pi}{2}$$

Q.39 **The value of** $\lim_{a\to 0} \frac{\sin a - \tan a}{\sin^3 a}$ **will be**

Correct option: (A)

$$\lim_{a\to 0} \frac{\sin a - \tan a}{\sin^3 a} = \lim_{a\to 0} \frac{\sin a - \frac{\sin a}{\cos a}}{\sin^3 a}$$

$$= \lim_{a\to 0} \frac{\cos a - 1}{\sin^2 a \cos a}$$

$$= \lim_{a\to 0} \frac{-(1 - \cos a)}{(1 - \cos^2 a)(\cos a)}$$

$$= \lim_{a\to 0} \left[-\frac{1}{(1 + \cos a)\cos a}\right] = \frac{-1}{2}$$

Q.40 **If the function**

$$f(x) = \begin{cases} -2\sin x & , \text{ if } x \leq \frac{-\pi}{2} \\ A\sin x + B & , \text{ if } \frac{-\pi}{2} < x < \frac{\pi}{2} \\ \cos x & , \text{ if } x \geq \frac{\pi}{2} \end{cases}$$

is continuous everywhere, then the values of A **and** B **are respectively**

Correct option: (B)

Since f(x) is continuous everywhere.

$\therefore$ f(x) is continuous at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

$$\therefore \lim_{x\to\frac{-\pi}{2}^-} f(x) = \lim_{x\to\frac{-\pi}{2}^+} f(x)$$

$$\Rightarrow \lim_{x\to\frac{-\pi}{2}} (-2\sin x) = \lim_{x\to\frac{-\pi}{2}} (A\sin x + B)$$

$\Rightarrow$ –2 (–1) = A(–1) + B

$\Rightarrow$ –A + B = 2 ....(i)

Also, $\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} f(x)$

$\Rightarrow \lim_{x\to\frac{\pi}{2}} (A\sin x + B) = \lim_{x\to\frac{\pi}{2}} (\cos x)$

⇒ A(1) + B = 0

⇒ A + B = 0 ....(ii)

From (i) and (ii), we get

A = –1, B = 1

Q.41 **If f(*x*) =** $\begin{cases} \frac{x^2-9}{x-3}+a, & x>3 \\ 5 & ,x=3 \\ 2x^2+3x+b, & x<3 \end{cases}$ **is**

continuous at *x* = 3, then

Correct option: (D)

Since, f(*x*) is continuous at *x* = 3.

$\therefore \lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = f(3)$

$\lim_{x\to3^-} f(x)$= f(3)

$\Rightarrow \lim_{x\to3^-} (2x^2 + 3x + b) = 5$

$\Rightarrow 2(3)^2 + 3(3) + b = 5$

⇒ b = – 22

Also, $\lim_{x\to3^+}$ f(*x*) = f(3)

$\Rightarrow \lim_{x\to3^+} \left(\frac{x^2-9}{x-3}+a\right) = 5$

⇒ (3 + 3 + a) = 5

⇒ a = –1

Q.42 **If f(*y*) =** $\begin{cases} \frac{(e^{7x}-1).\sin x}{x^2}, & for\ x\neq 0 \\ 7 & , for\ x=0 \end{cases}$**,**

then

Correct option: (A)

$\lim_{x\to0} f(x) = \lim_{x\to0} \frac{(e^{7x}-1)}{7x} \times 7 \times \frac{\sin x}{x}$

= log e × 7 × 1 = 7

and f(0) = 7

∴ f(*x*) is continuous at *x* = 0.

Q.43 **If f(x) =** $\frac{4\sin\pi x}{5x}$**, for x ≠ 0**

= 2k, for *x* = 0

is continuous at *x* = 0, then the value of k is

Correct option: (C)

Since f(*x*) is continuous at *x* = 0.

$\Rightarrow f(0) = \lim_{x\to0} f(x)$

$\Rightarrow 2k = \lim_{x\to0} \frac{4\sin\pi x}{5x}$

$\Rightarrow 2k = \frac{4\pi}{5}\left(\lim_{x\to0}\frac{\sin\pi x}{\pi x}\right)$

$\Rightarrow 2k = \frac{4\pi}{5}(1)$

$\Rightarrow k = \frac{2\pi}{5}$

Q.44 **The derivative of** $\tan^{-1}\left(\sqrt{1+x^2}-1\right)$ **is**

Correct option: (B)

Let $y = \tan^{-1}\left(\sqrt{1+x^2}-1\right)$

Differentiating w.r.t. *x*, we get

$\frac{dy}{dx} =$

$$\frac{1}{1+\left(\sqrt{1+x^2}-1\right)^2} \times \frac{d}{dx}\left(\sqrt{1+x^2}-1\right)$$

$$= \frac{1}{1+2+x^2-2\sqrt{1+x^2}} \times \frac{1}{2\sqrt{1+x^2}} \times 2x$$

$$= \frac{x}{\left(\sqrt{1+x^2}\right)\left(x^2-2\sqrt{1+x^2}+3\right)}$$

Q.45 **The second order derivative of** $\frac{e^x+1}{e^x}$ **is**

Correct option: (B)

Let $y = \frac{e^x+1}{e^x}$

$\Rightarrow y = 1 + \frac{1}{e^x}$

$\therefore \frac{dy}{dx} = -e^{-x}$

$\therefore \frac{d^2y}{dx^2} = e^{-x} = \frac{1}{e^x}$

Q.46 **For *x* > 0,**

$\lim_{x\to0}\left\{(\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}\right\} is$

Correct option: (C)

$\lim_{x\to0}\left\{(\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}\right\}$

$= \lim_{x\to0} (\sin x)^{\frac{1}{x}} + \lim_{x\to0}\left(\frac{1}{x}\right)^{\sin x}$

$= 0 + \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin\ x} = \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin\ x}$

Let $l = \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin\ x}$,then

$\log l = \log \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin\ x}$

$\Rightarrow \log l = \lim_{x\to 0}\ (-\ \sin\ x\ \log\ x)$

$\Rightarrow \log l = -\lim_{x\to 0}\ \frac{\log\ x}{cosec\, x}$

$\Rightarrow \log l = -\lim_{x\to 0}\ \frac{\frac{1}{x}}{-\,cosec\, x\, cot\, x} = \lim_{x\to 0}\ \frac{\sin^2 x}{x\, cos\, x}$

$\Rightarrow \log l = \lim_{x\to 0}\ \frac{\tan\ x}{x}\ \times \sin\ x = 1 \times 0 = 0$

$\Rightarrow l = e^0 = 1$

Q.47 **Let f(x) =** $\begin{cases} \frac{\sin \pi x}{5x} & ; \quad x \neq 0 \\ k & ; \quad x = 0 \end{cases}$**. If f($x$) is continuous at x = 0, then k =**

Correct option: (A)

Since, f(x) is continuous at x = 0.

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow k = \lim_{x\to 0} \frac{\sin \pi\, x}{5x}$

$\Rightarrow k = \lim_{x\to 0} \left(\frac{\sin \pi\, x}{\pi x}\right).\frac{\pi}{5}$

$\Rightarrow k = (1).\ \frac{\pi}{5}$

$\Rightarrow k = \frac{\pi}{5}$

Q.48 $\frac{d^2x}{dy^2}$ **equals**

Correct option: (D)

$\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$

$\therefore \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\ \frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2}.\frac{d}{dx}\left(\frac{dy}{dx}\right).\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$

Q.49 **Let** $f(x) = e^x, g(x) = \sin^{-1}x$ **and** $h(x) = f(g(x))$**, then** $\left(\frac{h'(x)}{h(x)}\right)^2$ **is equal to**

Correct option: (C)

h(x) = f(g(x))

= f($\sin^{-1}x$)

$\therefore$ h(x) = $e^{\sin^{-1}x}$

Differentiating w.r.t. x, we get

$h'(x) = e^{\sin^{-1}x}\cdot\frac{d}{dx}(\sin^{-1}x)$

$= e^{\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}}$

Now, $\frac{h'(x)}{h(x)} = \frac{e^{\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}}}{e^{\sin^{-1}x}} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \left(\frac{\mathbf{h'(x)}}{\mathbf{h(x)}}\right)^2 = \frac{1}{1-\mathbf{x}^2}$

Q.50 **If f(x) =** $\begin{cases} \frac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x}}, & x>0 \end{cases}$ **is continuous at x = 0, then**

Correct option: (B)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin\ (a+1)x + \sin x}{x}$

$= \lim_{x\to 0^-}\left[\frac{\sin\ (a+1)x}{(a+1)x}\times(a+1)+\frac{\sin\ x}{x}\right]$

= a + 1 + 1

= a + 2

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x}}$

$= \lim_{x\to 0^+} \frac{\sqrt{x}\left(\sqrt{1+bx}-1\right)}{b\sqrt{x}}$

$= \lim_{x\to 0^+} \dfrac{\sqrt{1+bx}-1}{b} = \dfrac{0}{b} = 0$, if b

≠ 0

Since, f(x) is continuous at x = 0.

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

⇒ a + 2 = 0 = c

⇒ a = –2, c = 0

∴ a = –2, b ≠ 0 and c = 0

Q.51 If f(x) = e^x, g(x) = $\sin^{-1}x$ and h(x) = f(g(x)), then $\dfrac{h'(x)}{h(x)}$ is

Correct option: (B)

h(x) = f(g(x))

= f($\sin^{-1}x$)

∴ h(x) = $e^{\sin^{-1}x}$

Differentiating w.r.t. x, we get

$h'(x) = e^{\sin^{-1}x} \cdot \dfrac{d}{dx}(\sin^{-1}x)$

$= e^{\sin^{-1}x} \cdot \dfrac{1}{\sqrt{1-x^2}}$

Now, $\dfrac{h'(x)}{h(x)} = \dfrac{e^{\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}}}{e^{\sin^{-1}x}}$

$= \dfrac{1}{\sqrt{1-x^2}}$

Q.52 If y = sec($\tan^{-1} x$), then $\dfrac{dy}{dx}$ is

Correct option: (A)

$y = \sec(\tan^{-1}x)$

$\therefore \dfrac{dy}{dx} = \sec(\tan^{-1}x).\tan(\tan^{-1}x).\dfrac{d}{dx}(\tan^{-1}x)$

$= \sec(\tan^{-1}x)\tan(\tan^{-1}x).\dfrac{1}{1+x^2}$

$= \sqrt{1+x^2}.\dfrac{x}{1+x^2}$

$\ldots \left[\because \tan^{-1}x = \sec^{-1}\sqrt{1+x^2}\right]$

$= \dfrac{x}{\sqrt{1+x^2}}$

Q.53 $\lim_{y\to 0} \dfrac{(x+y)\sec(x+y)-x\sec x}{y} =$

Correct option: (A)

$\lim_{y\to 0} \dfrac{(x+y)\sec(x+y)-x\sec x}{y}$

$= \lim_{y\to 0}\left\{\dfrac{x[\sec(x+y)-\sec x]}{y} + \sec(x+y)\right\}$

$= \lim_{y\to 0}\left[\dfrac{x}{y}\cdot\dfrac{\cos x-\cos(x+y)}{\cos(x+y)\cos x}\right] + \lim_{y\to 0}\sec(x+y)$

$= \lim_{y\to 0}\left[\dfrac{x}{y}\cdot\dfrac{2\sin\left(x+\frac{y}{2}\right)\sin\left(\frac{y}{2}\right)}{\cos(x+y)\cos x}\right] + \sec x$

$= \lim_{y\to 0}\left[\dfrac{x\sin\left(x+\frac{y}{2}\right)}{\cos(x+y).\ \cos x}\cdot\dfrac{\sin\left(\frac{y}{2}\right)}{\frac{y}{2}}\right] + \sec x$

$= \left(\dfrac{x\sin x}{\cos x.\ \cos x}\cdot 1\right) + \sec x$

= x tan x sec x + sec x = sec x(x tan x + 1)

Q.54 $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right) =$

Correct option: (B)

$\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$

$= \lim_{x\to\infty}\dfrac{x^2+x+1-x^2-1}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}$

$= \lim_{x\to\infty}\dfrac{1}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}}$

$= \dfrac{1}{1+1} = \dfrac{1}{2}$

Q.55 Let f be a twice differentiable function such that $f''(x) = -f(x)$, $f'(x) = g(x)$ and $h(x) = [f(x)]^2 + [g(x)]^2$. If h(5) = 1, then h(10) is ______.

Correct option: (D)

h(x) = $[f(x)]^2 + [g(x)]^2$

∴ $h'(x) = 2[f(x)]\,f'(x) + 2[g(x)]\,g'(x)$

$= 2[-f''(x)]\,f'(x) + 2[f'(x)]\cdot f''(x)$

= 0

∴ h(x) is a constant function.

∴ h(5) = 1 ⇒ h (10) = 1

Q.56 **If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$,**

then $\frac{dy}{dx}$ is

Correct option: (D)

$y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$= \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\therefore y = \frac{\pi}{2} \ldots \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\therefore \frac{dy}{dx} = 0$

Q.57 **Let $A = \lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$, then**

$\log_e A =$

Correct option: (C)

$A = \lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$

$= e^{\lim_{x\to 0}\frac{1}{2x}\tan^2\sqrt{x}} \quad \ldots$

$\left[\begin{array}{l} If \lim_{x\to a} f(x) = 1 \ and \lim_{x\to a} g(x) = \infty, \\ then \lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]} \end{array}\right]$

$= e^{\lim_{x\to 0}\frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2}$

$\therefore \ A = e^{\frac{1}{2}}$

$\Rightarrow \log_e A = \frac{1}{2}$

Q.58 **If f(x) = $\begin{cases} \frac{1+\cos x}{\pi - x} & ; \quad x \neq \pi \\ \lambda & ; \quad x = \pi \end{cases}$ is**

continuous at $x = \pi$, then the value of λ is

Correct option: (C)

Since, f(x) is continuous at $x = \pi$.

$\therefore f(\pi) = \lim_{x\to\pi} f(x)$

$\therefore \lambda = \lim_{x\to\pi}\frac{1+\cos x}{\pi - x}$

Applying L′ Hospital's rule on R.H.S, we get

$\lambda = \lim_{x\to\pi}\frac{-\sin x}{-1}$

$\Rightarrow \lambda = \sin \pi = 0$

Q.59 $\lim_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}} =$

Correct option: (C)

$\lim_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}} =$

$\lim_{x\to\infty}\left[\frac{8+\frac{5}{x}+\frac{3}{x^2}}{2-\frac{7}{x}-\frac{5}{x^2}}\right]^{\left(\frac{4+\frac{3}{x}}{8-\frac{1}{x}}\right)}$

$= \left(\frac{8}{2}\right)^{\left(\frac{4}{8}\right)} = 4^{\left(\frac{1}{2}\right)} = 2$

Q.60 **If $y = (\tan^{-1} x)^2$ then $(x^2+1)^2\frac{d^2y}{dx^2}$ +**

$2x(x^2+1)\frac{dy}{dx}$ =

Correct option: (B)

$y = (\tan^{-1} x)^2$

$\therefore \frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}$

$\Rightarrow \frac{dy}{dx}(1+x^2) = 2\tan^{-1} x$

$\therefore \frac{dy}{dx}(2x) + (1+x^2)\frac{d^2y}{dx^2} = \frac{2}{1+x^2}$

$\Rightarrow (x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

Q.61 $\lim_{x\to 0}\frac{\sin\left(x^2+5x\right)}{x} =$

Correct option: (D)

$\lim_{x\to 0}\frac{\sin\left(x^2+5x\right)}{x} =$

$\lim_{x\to 0}\frac{\sin\left(x^2+5x\right)}{x(x+5)} \times (x+5)$

$= 1(0+5) = 5$

Q.62 **Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation**

$\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$

where a > –1 then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{a\to 0^+}\beta(a)$ respectively are

Correct option: (B)

Let A = 1+ a

∴ When $a \to 0^+$, $A \to 1^+$

∴ Given function is written as

$$\left(A^{\frac{1}{3}}-1\right)x^2+\left(A^{\frac{1}{2}}-1\right)x+\left(A^{\frac{1}{6}}-1\right)=0$$

∴
$$\left(\frac{A^{\frac{1}{3}}-1}{A-1}\right)x^2+\left(\frac{A^{\frac{1}{2}}-1}{A-1}\right)x+\left(\frac{A^{\frac{1}{6}}-1}{A-1}\right)=0$$

Taking $\lim\limits_{x\to 0^+}$ on both sides, we get

∴ $\frac{1}{3}x^2+\frac{1}{2}x+\frac{1}{6}=0$

∴ $2x^2 + 3x + 1 = 0$

∴ $x = -1$ or $\frac{-1}{2}$

i.e. $\lim\limits_{x\to 0^+}\alpha(a) = -1$ and $\lim\limits_{x\to 0^+}\beta(a) = \frac{-1}{2}$

Q.63 **If f(x) = $\frac{\sqrt{x+3}-2}{x^3-1}$, $x \neq 1$, is continuous at x = 1, then f(1) is**

Correct option: (C)

Since, f(x) is continuous at x = 1.

∴ $f(1) = \lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{x^3-1}$

$= \lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{x^3-1^3}\times\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$

$= \lim\limits_{x\to 1}\frac{x-1}{(x-1)(x^2+x+1)\left(\sqrt{x+3}+2\right)} =$

$\frac{1}{3(4)} = \frac{1}{12}$

Q.64 **Differential coefficient of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}x$ is**

Correct option: (A)

Let $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ and $z = \sin^{-1}x$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\therefore y = \tan^{-1}\left(\frac{\sin\ \theta}{1+\ \cos\ \theta}\right)$

$= \tan^{-1}\left(\tan\ \frac{\theta}{2}\right) = \frac{\theta}{2}$

$= \frac{\sin^{-1}x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$ and $\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{1}{2}$

Q.65 **If** $f(x)=\begin{cases}\frac{5}{2}-x\,,\ when\ x<2\\ 1\quad,\ when\ \ x=2\\ x-\frac{3}{2},\ when\ x>2\end{cases}$, **then**

Correct option: (B)

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2}\left(\frac{5}{2}-x\right) = \frac{1}{2}$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2}\left(x-\frac{3}{2}\right) = \frac{1}{2}$ and f(2) = 1

$\therefore \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) \neq f(2)$

∴ f(x) is discontinuous at x = 2.

Q.66 $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ **is equal to**

Correct option: (B)

$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

$= \lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$= \lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$= \lim_{x\to\infty}\frac{\sqrt{1+x^{-1/2}}}{\sqrt{1+\sqrt{x^{-1}+x^{-3/2}}}+1} = \frac{1}{2}$

Q.67 $\lim\limits_{x\to 0}\frac{|x|}{|x|+x^2}$ **=**

Correct option: (B)

Let $f(x) = \frac{|x|}{|x|+x^2}$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}\frac{-x}{-x+x^2} = \lim\limits_{x\to 0^-}\frac{-1}{-1+x} =$

1

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}\frac{x}{x+x^2} = \lim\limits_{x\to 0^+}\frac{1}{1+x} = 1$

Here, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} \frac{|x|}{|x| + x^2} = 1$

Q.68 $\lim_{x \to 0} \frac{2 \log (1 + x) - \log (1 + 2x)}{x^2}$ **is equal to**

Correct option: (B)

$\lim_{x \to 0} \frac{2 \log (1 + x) - \log (1 + 2x)}{x^2}$

$= \lim_{x \to 0} \frac{\log \left\{\frac{(1 + x)^2}{1 + 2x}\right\}}{x^2}$

$= \lim_{x \to 0} \frac{\log \left(1 + \frac{x^2}{1 + 2x}\right)}{\frac{x^2}{1 + 2x}} \times \frac{1}{1 + 2x} = 1$

Q.69 **If f(x) =** $\frac{e^x - e^{\sin x}}{2(x - \sin x)}$**,** $x \neq 0$ **is continuous at** $x = 0$**, then f(0) =**

Correct option: (C)

For f(x) to be continuous at $x = 0$,

$f(0) = \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \frac{e^x - e^{\sin x}}{2(x - \sin x)}$

$= \frac{1}{2} \lim_{x \to 0} e^{\sin x} \left(\frac{e^{x - \sin x} - 1}{x - \sin x}\right)$

$= \frac{1}{2} \times e^0 \times 1 \ldots \left[\because \lim_{x \to 0} \frac{e^x - 1}{x} = 1\right]$

$= \frac{1}{2}$

Q.70 $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ **is**

Correct option: (C)

$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$

$= \lim_{x \to 0} \frac{x(\tan 2x - 2 \tan x)}{(2 \sin^2 x)^2}$

$= \lim_{x \to 0} \frac{x(\tan 2x - 2 \tan x)}{4 \sin^4 x}$

$$= \frac{1}{4} \lim_{x \to 0} \frac{x \left\{ \begin{array}{l} \left(2x + \frac{1}{3}(2x)^3 + \frac{2}{15}(2x)^5 + \ldots\right) \\ -2\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\right) \end{array} \right\}}{x^4\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \ldots\right)^4}$$

$= \frac{1}{4}\left(\frac{8}{3} - \frac{2}{3}\right) = \frac{2}{4} = \frac{1}{2}$

Q.71 **If f(x) =** $\begin{cases} \frac{1}{2}\sin x^2, & x \neq 0 \\ 0, & x = 0 \end{cases}$**, then**

Correct option: (C)

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{2} \sin x^2 = 0 = f(0)$

$\therefore$ f(x) is continuous at $x = 0$.

Q.72 **If f(x) =** $\frac{\tan(x^2 - x)}{x}$**,** $x \neq 0$**, is continuous at** $x = 0$**, then f(0) is**

Correct option: (A)

Since, f(x) is continuous at $x = 0$.

$\therefore f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\tan (x^2 - x)}{x}$

$= \lim_{x \to 0} \frac{\tan [x(x - 1)]}{x(x - 1)} \times (x - 1) = 1 \times (-1) = -1$

Q.73 $\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} =$

Correct option: (A)

$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2} \frac{x^5 - 2^5}{x^3 - 2^3}$

$= \frac{5}{3}(2)^{5-3} \quad \ldots\left[\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}\right]$

$= \frac{5 \times 4}{3} = \frac{20}{3}$

Q.74 **If f(x) =** $\sin^{-1}\left(\frac{2x}{1 + x^2}\right) + \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$**,** $x \in (1, \infty)$**, then f ′(x) =**

Correct option: (D)

Given f(x) =

$\sin^{-1}\left(\frac{2x}{1 + x^2}\right) + \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$

Put $x = \tan \theta$,

$\Rightarrow \theta = \tan^{-1} x$

$\therefore$ f(x) =

$\sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) + \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$

$= \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta)$

$= 2\theta + 2\theta$

$\therefore$ f(x) = 4θ

$\therefore$ f(x) = $4\tan^{-1}x$

$f'(x) = \dfrac{4}{1+x^2}$

Q.75 **If** $f(x) = \begin{cases} \frac{x^2 - 49}{x - 7} ; & if\ x \neq 7 \\ 2x + k ; & otherwise \end{cases}$**, is continuous at x = 7, then k =**

Correct option: (B)

Since, f(x) is continuous at x = 7.

$\therefore f(7) = \lim_{x\to 7} f(x)$

$\Rightarrow 2(7) + k = \lim_{x\to 7} \frac{x^2 - 49}{x - 7}$

$\Rightarrow 14 + k = \lim_{x\to 7} \frac{(x + 7)(x - 7)}{x - 7}$

$\Rightarrow 14 + k = \lim_{x\to 7} (x + 7)$

$\Rightarrow 14 + k = 14 \Rightarrow k = 0$

Q.76 **If f(x) = $\frac{1 + \cos x}{(\pi - x)^2}$, when $x \neq \pi$ and f(π) = λ, then f(x) will be continuous function at $x = \pi$, when λ =**

Correct option: (C)

Since, f(x) is continuous at $x = \pi$.

$\therefore f(\pi) = \lim_{x\to\pi} f(x)$

$\Rightarrow \lambda = \lim_{x\to\pi} \frac{1 + \cos x}{(\pi - x)^2}$

Applying L′ Hospital rule on R.H.S., we get

$\lambda = \lim_{x\to\pi} \frac{-\sin x}{-2(\pi - x)}$

Applying L′ Hospital rule on R.H.S., we get

$\lambda = \lim_{x\to\pi} \frac{-\cos x}{-2(-1)}$

$\Rightarrow \lambda = \frac{-\cos \pi}{2} = \frac{1}{2}$

Q.77 **If x = a$\left(\cos t + \log\tan\frac{t}{2}\right)$, y = a sint, then $\frac{dy}{dx}$ =**

Correct option: (A)

$$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2}\right)$$

$$= a\left(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right) = a\left(\frac{\cos^2 t}{\sin t}\right)$$

= a cos t cot t

and $\frac{dy}{dt} = a\cos t$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{\cot t} = \tan t$$

Q.78 **If** $f(x) = \begin{cases} \frac{x^2-9}{x-3} ; & if\ x \neq 3 \\ 2x + k ; & otherwise \end{cases}$**, is continuous at x = 3, then k =**

Correct option: (B)

Since, f(x) is continuous at x = 3.

$\therefore f(3) = \lim_{x\to 3} f(x)$

$$\Rightarrow 2(3) + k = \lim_{x\to 3} \frac{x^2 - 9}{x - 3}$$

$$\Rightarrow 6 + k = \lim_{x\to 3} \frac{(x+3)(x-3)}{x-3}$$

$$\Rightarrow 6 + k = \lim_{x\to 3} (x+3)$$

$\Rightarrow 6 + k = 6 \Rightarrow k = 0$

Q.79 **If u = log** $\left(\sqrt{x-1} - \sqrt{x+1}\right)$ **and** $v = \sqrt{x+1} + \sqrt{x-1}$ **then** $\frac{du}{dv} = \ldots.$

Correct option: (D)

$u = \log\left(\sqrt{x-1} - \sqrt{x+1}\right)$

$\therefore$

$$\frac{du}{dx} = \frac{1}{\sqrt{x-1} - \sqrt{x+1}}\cdot\frac{d}{dx}\left(\sqrt{x-1} - \sqrt{x+1}\right)$$

$$= \frac{1}{\sqrt{x-1} - \sqrt{x+1}}\left(\frac{1}{2\sqrt{x-1}} - \frac{1}{2\sqrt{x+1}}\right)$$

$$= \frac{1}{\sqrt{x-1} - \sqrt{x+1}}\left(\frac{\sqrt{x+1} - \sqrt{x-1}}{2\sqrt{x^2-1}}\right)$$

$$= \frac{-1}{2\sqrt{x^2-1}}$$

$v = \sqrt{x+1} + \sqrt{x-1}$

$$\therefore \frac{dv}{dx} = \frac{1}{2\sqrt{x+1}} + \frac{1}{2\sqrt{x-1}}$$

$$= \frac{\sqrt{x-1} + \sqrt{x+1}}{2\sqrt{x^2-1}}$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{x-1} + \sqrt{x+1}} = \frac{-1}{v}$$

Q.80 **If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then the value of $\frac{dy}{dx}$ =**

Correct option: (A)

$y = \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

$= \pi/2$

$\therefore \frac{dy}{dx} = 0$

Q.81 **If the function**

$f(x) = \frac{\log 10 + \log(0.1+2x)}{2x}$, if $x \neq 0$

$= k$, if $x = 0$

is continuous at $x = 0$, then k + 2 =

Correct option: (B)

Since f(x) is continuous at x = 0.

$\therefore$ $f(0) = \lim_{x\to 0} \frac{\log 10 + \log(0.1+2x)}{2x}$

$\Rightarrow k = \lim_{x\to 0} \frac{\log(1+20x)}{2x}$

$\Rightarrow k = \frac{1}{2}(20)$ $\quad \ldots\left[\because \lim_{x\to 0} \frac{\log(1+kx)}{x} = k\right]$

$\Rightarrow$ k = 10

$\Rightarrow$ k + 2 = 12

Q.82 $\lim_{x\to 0} \frac{\sqrt{a+x}-\sqrt{a-x}}{4x}$ =

Correct option: (B)

$\lim_{x\to 0} \frac{\sqrt{a+x}-\sqrt{a-x}}{4x}$ =

$\lim_{x\to 0} \frac{2x}{4x\left(\sqrt{a+x}+\sqrt{a-x}\right)}$

$= \frac{1}{2}\lim_{x\to 0} \frac{1}{\sqrt{a+x}+\sqrt{a-x}}$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{a+0}+\sqrt{a-0}}$

$= \frac{1}{4\sqrt{a}}$

Q.83 **If $y = px^3 + \frac{q}{x^2}$, then $\frac{d^2y}{dx^2}$ =**

Correct option: (C)

$y = px^3 + \frac{q}{x^2}$ ...(i)

$\therefore \frac{dy}{dx} = 3px^2 - \frac{2q}{x^3}$

$\therefore \frac{d^2y}{dx^2} = 6px + \frac{6q}{x^4}$

$= \frac{6}{x^2}\left(px^3 + \frac{q}{x^2}\right) = \frac{6y}{x^2}$ ...[From (i)]

Q.84 $\lim_{\theta\to\frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta}$ =

Correct option: (C)

Applying L–Hospital's rule, we get

$\lim_{\theta\to\frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta} = \lim_{\theta\to\frac{\pi}{2}} \frac{-1}{-\operatorname{cosec}^2\theta} = 1$

Q.85 **The function f(x) = sin|x| is**

Correct option: (A)

Let g(x) = |x| and h(x) = sin x.

Then, f(x) = (hog) (x) for all $x \in$ R.

As both g and h are continuous functions on R.

$\therefore$ f(x) is also continuous for all $x \in$ R.

Q.86 **If the function**

$$f(x) = \begin{cases} 1+\sin\frac{\pi x}{2}, & \text{for } -\infty < x \leq 1 \\ ax+b, & \text{for } 1 < x < 3 \\ 6\tan\frac{\pi x}{12}, & \text{for } 3 \leq x < 6 \end{cases}$$

is continuous in the interval (–∞, 6), then the values of a and b are respectively

Correct option: (C)

Since, f(x) is continuous in (–∞, 6).

$\therefore$ it is continuous at x = 1 and x = 3.

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\Rightarrow \lim_{x\to 1^-}\left(1+\sin\frac{\pi x}{2}\right) = \lim_{x\to 1^+}(ax+b)$

$\Rightarrow 1+\sin\frac{\pi}{2} = a+b$

$\Rightarrow$ a + b = 2 ...(i)

Also, $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\Rightarrow \lim_{x\to 3^-}(ax+b) = \lim_{x\to 3^+}\left(6\tan\frac{\pi x}{12}\right)$

$\Rightarrow 3a+b = 6\tan\frac{3\pi}{12}$

$\Rightarrow$ 3a + b = 6 ...(ii)

From (i) and (ii), we get a = 2, b = 0

Q.87 $\lim_{x\to\infty}\left(\frac{x+8}{x+1}\right)^{x+5} = \ldots$

Correct option: (D)

$$\lim_{x\to\infty}\left(\frac{x+8}{x+1}\right)^{x+5} = \frac{x^{x+5}\left(1+\frac{8}{x}\right)^{x+5}}{x^{x+5}\left(1+\frac{1}{x}\right)^{x+5}}$$

$$= \frac{\left(1+\frac{8}{x}\right)^{x}\left(1+\frac{8}{x}\right)^{5}}{\left(1+\frac{1}{x}\right)^{x}\left(1+\frac{1}{x}\right)^{5}}$$

$$= \frac{e^8\cdot e^5}{e(1)^5} = e^7$$

Q.88 If $y = 2\left[1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots\right]$, then $\frac{d^2y}{dx^2}$ is equal to

Correct option: (A)

We know that,

$e^x = 1 + x + \frac{x^2}{2!}+\frac{x^3}{3!}+ \ldots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}+\ldots$

$\therefore e^x + e^{-x} = 2\left[1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots\right]$

i.e., $y = e^x + e^{-x}$

$\frac{dy}{dx} = e^x - e^{-x}$

$\frac{d^2y}{dx^2} = e^x + e^{-x} = y$

Q.89 $\lim_{x\to 2}\left(\frac{5^x+5^{3-x}-30}{5^{3-x}-5^{\frac{x}{2}}}\right) =$

Correct option: (C)

Let $L = \lim_{x\to 2}\left(\frac{5^x+5^{3-x}-30}{5^{3-x}-5^{\frac{x}{2}}}\right)$

$$= \lim_{x\to 2}\left(\frac{5^x+\frac{5^3}{5^x}-30}{\frac{5^3}{5^x}-(5^x)^{\frac{1}{2}}}\right)$$

Let $t = 5^x$

$\therefore \quad x \to 2 \Rightarrow t \to 25$

$$\therefore \quad L = \lim_{t\to 25}\left(\frac{t+\frac{125}{t}-30}{\frac{125}{t}-\sqrt{t}}\right)$$

$$= \lim_{t\to 25}\left(\frac{t^2-30t+125}{25\sqrt{25}-t\sqrt{t}}\right)$$

$$= \lim_{t\to 25}\left(\frac{(t-25)(t-5)}{25^{\frac{3}{2}}-t^{\frac{3}{2}}}\right)$$

$$= \lim_{t\to 25}\frac{t-5}{-\left(\frac{25^{\frac{3}{2}}-t^{\frac{3}{2}}}{25-t}\right)}$$

$$= \frac{25-5}{-\frac{3}{2}(25)^{\frac{1}{2}}}$$

$$= \frac{-8}{3}$$

Q.90 The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is

Correct option: (D)

Let $y = \sin^2 x$ and $z = \cos^2 x$

$\therefore \frac{dy}{dx} = \sin 2x$ and $\frac{dz}{dx} = -\sin 2x$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = -1$

Q.91 $y = (1 + x)(1 + x^2)(1 + x^4) \ldots \ldots \ldots (1 + x^{2n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is

Correct option: (C)

$y = (1 + x)(1 + x^2)(1 + x^4) \ldots (1 + x^{2n}) \quad \ldots$(i)

Taking 'log' on both sides, we get

$\log y = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \ldots + \log(1 + x^{2n})$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\ldots+\frac{2n\times x^{2n-1}}{1+x^{2n}}$$

…(ii)

At $x = 0$, (i) $\Rightarrow y = 1$

$\therefore$ (ii) $\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = 1 + 0 + 0 + \ldots + 0 = 1$

Q.92 $\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$ **equals**

Correct option: (B)

$$\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2} = \lim_{x\to 0}\frac{\sin\left(\pi - \pi\cos^2 x\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{\pi\sin^2 x}\times\frac{\pi\sin^2 x}{x^2} = \pi$$

Q.93 $f(x) = \frac{x^2+x-2}{x^2-3x+2}$ is discontinuous at x =

Correct option: (B)

$f(x)$ is discontinuous, when $x^2 - 3x + 2 = 0$

i.e., $(x - 1)(x - 2) = 0 \Rightarrow x = 1, x = 2$

Q.94 **If f (x) = $\frac{1}{x-2}$, then the function f [f (x)] is discontinuous at x equal to ______**

Correct option: (D)

f (f(x)) = $\frac{1}{\frac{1}{x-2}-2} = \frac{x-2}{5-2x}$

∴ f (f(x)) is discontinuous at x = $\frac{5}{2}$.

Q.95

$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$ =

Correct option: (C)

Applying L–Hospital's rule, we get

$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} = \lim_{x\to 4} \frac{\frac{1}{2\sqrt{5+x}}}{-\frac{1}{2\sqrt{5-x}}}$

$= -\lim_{x\to 4} \frac{\sqrt{5-x}}{\sqrt{5+x}}$

$= \frac{-1}{\sqrt{9}} = -\frac{1}{3}$

Q.96 **If f(x) = [x], for x ∈ (–1, 2), then f is discontinuous at (where [x] represents floor function)**

Correct option: (C)

f(x) = [x] for x ∈ (–1, 2)

The floor function is discontinuous at all integral values in (–1, 2).

The integral values in (–1, 2) are 0 and 1.

Hence, f(x) is discontinuous at x = 0, 1.

Q.97 **Let f(x) =** $\begin{cases} 5^{\frac{1}{x}} ; & x<0 \\ \lambda[x] ; & x \ge 0,\ \lambda \in R \end{cases}$ **, then at x = 0**

Correct option: (A)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 5^{\frac{1}{x}} = \lim_{h\to 0} 5^{-\frac{1}{h}} = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \lambda[x] = 0$, for all λ ∈ R

f(0) = λ(0) = 0

∴ f is continuous at x = 0, whatever λ may be.

Q.98 **The value of $\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$ is**

Correct option: (A)

Applying L–Hospital's rule, we get

$\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3} = \lim_{x\to 0} \frac{\frac{2x}{2\sqrt{x^2+1}}}{\frac{2x}{2\sqrt{x^2+9}}}$

$= \lim_{x\to 0} \frac{\sqrt{x^2+9}}{\sqrt{x^2+1}} = 3$

Q.99 **If f(x) =** $\begin{cases} \frac{3\sin x-\sqrt{3}\cos x}{6x-\pi}, & x \ne \frac{\pi}{6} \\ a, & x = \frac{\pi}{6} \end{cases}$ **is continuous at x = $\frac{\pi}{6}$, then a =**

Correct option: (B)

Since, f(x) is continuous at x = $\frac{\pi}{6}$,

∴ $\lim_{x\to \frac{\pi}{6}} f(x) = f\left(\frac{\pi}{6}\right)$

⇒ $\lim_{x\to \frac{\pi}{6}} \frac{3\sin x-\sqrt{3}\cos x}{6x-\pi} = a$

Applying L'Hospital rule to L.H.S, we get

$\lim_{x\to \frac{\pi}{6}} \frac{3\cos x+\sqrt{3}\sin x}{6} = a$

⇒ $\frac{3\left(\frac{\sqrt{3}}{2}\right)+\sqrt{3}\left(\frac{1}{2}\right)}{6} = a$

⇒ $\frac{4\sqrt{3}}{12} = a \Rightarrow a = \frac{1}{\sqrt{3}}$

Q.100 **If the function** $f(x) = \begin{cases} 3x-8, & \text{if } 0<x\le 2 \\ x^2+3bx, & \text{if } 2<x<3 \end{cases}$ **is continuous at every point of its domain, then the value of b is**

Correct option: (A)

Since, f(x) is continuous at every point of its domain.

So, it is continuous at x = 2.

∴ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

⇒ $\lim_{x\to 2} (3x-8) = \lim_{x\to 2} (x^2+3bx)$

⇒ –2 = 4 + 6b

⇒ b = –1

Q.101 $\lim_{x\to 1}\left[\frac{\sqrt{x}-1}{\log x}\right] =$

Correct option: (A)

$\lim_{x\to 1}\left[\frac{\sqrt{x}-1}{\log x}\right] = \lim_{x\to 1}\frac{\frac{1}{2\sqrt{x}}}{\left(\frac{1}{x}\right)}$ …[By L-Hospital's Rule]

$= \frac{1}{2}$

Q.102 **If f(x) = $\frac{e^{x^2}-\cos x}{x^2}$, for $x \neq 0$ is continuous at x = 0, then value of f(0) is**

Correct option: (D)

For f(x) to be continuous at x = 0,

$f(0) = \lim_{x\to 0} f(x)$

$= \lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$

$= \lim_{x\to 0}\frac{e^{x^2}-1+1-\cos x}{x^2}$

$= \lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{1-\cos x}{x^2}$

$= 1 + \frac{1}{2} = \frac{3}{2}$

Q.103 **If x = a sec^2 θ, y = a tan^2 θ then $\frac{d^2y}{dx^2}$ =**

Correct option: (D)

$x = a\sec^2\theta,\ y = a\tan^2\theta$

$\therefore \frac{dx}{d\theta} = 2a\sec^2\theta\tan\theta$

and $\frac{dy}{d\theta} = 2a\tan\theta\sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\tan\theta\ \sec^2\theta}{2a\sec^2\theta\ \tan\theta} = 1$

$\therefore \frac{d^2y}{dx^2} = 0$

Q.104 $\frac{d}{dx}\left[\tan^{-1}\left(\frac{2}{x^{-1}-x}\right)\right] =$

Correct option: (A)

Let $y = \tan^{-1}\left(\frac{2}{x^{-1}-x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$

Q.105 **If $y = \tan^{-1}\left(\sqrt{x^2+y^2}\right) + \cot^{-1}\left(\sqrt{x^2+y^2}\right)$, then $\frac{dy}{dx}$ =**

Correct option: (B)

Given: $y = \tan^{-1}\left(\sqrt{x^2+y^2}\right) + \cot^{-1}\left(\sqrt{x^2+y^2}\right)$

$y = \frac{\pi}{2}$ …$\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \frac{dy}{dx} = 0$

Q.106 **For $x \in$ R, f(x) = |log 2 – sin x| and g(x) = f(f(x)), then**

Correct option: (C)

Given: f(x) = |log 2 – sin x| and g(x) = f(f(x))

As $x \to 0$, log 2 > sin x

$\therefore$ f(x) = log 2 – sin x and f(0) = log 2

$\Rightarrow$ f ′(x) = –cos x and f ′(0) = –1

Now, g(x) = f(f(x))

$\Rightarrow$ g ′(x) = f ′(f(x)) · f ′(x)

$\Rightarrow$ g ′(0) = f ′(f(0)) · f ′(0)

$\Rightarrow$ g ′(0) = f ′(log 2) (–1)

$\Rightarrow$ g ′(0) = –(–cos (log 2))

$\Rightarrow$ g ′(0) = cos (log 2)

Q.107 **Function f(x) = $\frac{1-\cos 8x}{32x^2}$, where $x \neq 0$ and f(x) = k, where x = 0 is a continuous function at x = 0, then the value of k will be**

Correct option: (B)

Since, f(x) is continuous at x = 0.

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow k = \lim_{x\to 0}\frac{1-\cos 8x}{32x^2} = \lim_{x\to 0}\frac{2\sin^2 4x}{32x^2}$

$\Rightarrow k = \lim_{x\to 0}\frac{\sin^2 4x}{16x^2} = \left(\lim_{x\to 0}\frac{\sin 4x}{4x}\right)^2 = (1)^2$

$\Rightarrow$ k = 1

Q.108 $\frac{d}{dx}\left(\cos^2 x^2 - \sin^2 x^2\right) =$

Correct option: (B)

$\frac{d}{dx}\left(\cos^2 x^2 - \sin^2 x^2\right) = \frac{d}{dx}\left(\cos\ 2x^2\right)$

$= -\sin(2x^2)$.

$\frac{d}{dx}\left(2x^2\right)$

$= -\sin(2x^2).\ 4x$

$= -4x\sin(2x^2)$

Q.109 $\lim_{x\to 0} x^x =$

Correct option: (B)

$\lim\limits_{x \to 0} x^x = e^{\lim\limits_{x \to 0} x(x-1)}$

$= e^0 = 1$

Q.110 **If $y = \log(\log(\log x^3))$, then $\frac{dy}{dx}$ is**

Correct option: (C)

$y = \log(\log(\log x^3))$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{\log(\log x^3)} \cdot \frac{d}{dx}[\log(\log x^3)]$$

$$= \frac{1}{\log(\log x^3)} \cdot \frac{1}{\log x^3} \cdot \frac{d}{dx}(\log x^3)$$

$$= \frac{1}{\log(\log x^3)} \cdot \frac{1}{3\log x} \cdot \frac{1}{x^3} .3x^2$$

$$= \frac{1}{x \log x \log(\log x^3)}$$