Conics **Marks: 150**

ANSWER KEY

Maths

Q.1 B	Q.2 D	Q.3 A	Q.4 B	Q.5 B	Q.6 A	Q.7 D	Q.8 D
Q.9 C	Q.10 A	Q.11 C	Q.12 C	Q.13 C	Q.14 D	Q.15 C	Q.16 C
Q.17 B	Q.18 B	Q.19 A	Q.20 C	Q.21 B	Q.22 A	Q.23 A	Q.24 B
Q.25 C	Q.26 B	Q.27 C	Q.28 B	Q.29 C	Q.30 B	Q.31 B	Q.32 D
Q.33 B	Q.34 B	Q.35 D	Q.36 A	Q.37 C	Q.38 C	Q.39 A	Q.40 C
Q.41 D	Q.42 D	Q.43 C	Q.44 B	Q.45 A	Q.46 C	Q.47 C	Q.48 A
Q.49 D	Q.50 D	Q.51 A	Q.52 B	Q.53 B	Q.54 D	Q.55 D	Q.56 C
Q.57 B	Q.58 C	Q.59 B	Q.60 C	Q.61 D	Q.62 D	Q.63 A	Q.64 C
Q.65 B	Q.66 C	Q.67 A	Q.68 A	Q.69 C	Q.70 A	Q.71 B	Q.72 B
Q.73 A	Q.74 A	Q.75 B					

SOLUTION

Maths

Q.1

For what value of k, the points (0, 0), (1, 3), (2, 4) and (k, 3) are con-cyclic?

Correct option: (B)

The equation of circle through points (0, 0), (1, 3) and (2, 4) is

$x^2 + y^2 – 10x = 0$

Point (k, 3) will be on the circle, if

$k^2 + 9 – 10k = 0$

$\Rightarrow k^2 – 10k + 9 = 0$

$\Rightarrow k^2 – 9k – k + 9 = 0$

$\Rightarrow (k – 1)(k – 9) = 0$

$\Rightarrow k = 1$ or $k = 9$

Q.2 The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K, then the angle which KP subtends at its focus is

Correct option: (D)

Here, $P(at^2, 2at)$ and $S(a, 0)$.

If the tangent at P, $ty = x + at^2$, meets the directrix

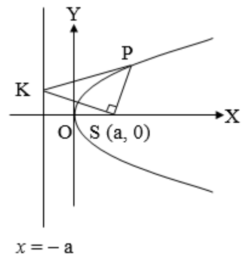
$x = –a$ at K, then $K = \left(-a, \dfrac{at^2 - a}{t}\right)$

Now, m_1 = slope of SP $= \dfrac{2at}{a(t^2 - 1)}$

and m_2 = slope of SK $= \dfrac{a(t^2 - 1)}{-2at}$

$\Rightarrow m_1 m_2 = –1$

$\therefore \quad \angle PSK = 90$



Q.3 Equation of tangent to the circle $x^2 + y^2 = 10$ at the point with abscissa 1 is

Correct option: (A)

Abscissa = 1

Hence, given equation of circle reduces to

$y^2 = 9$

$\Rightarrow y = \pm 3$

$\therefore$ Equation of tangent at

$(1, \pm 3)$ to $x^2 + y^2 = 10$ is $x(1) + y(\pm 3) = 10$

Q.4 Equation of tangent to $2x^2 + 3y^2 = 5$ and perpendicular to $3x + 2y + 7 = 0$ is

Correct option: (B)

We have, $2x^2 + 3y^2 = 5$

$\Leftrightarrow \dfrac{x^2}{\left(\frac{5}{2}\right)} + \dfrac{y^2}{\left(\frac{5}{3}\right)} = 1$ (Ellipse)

and $3x + 2y + 7 = 0$

$\Rightarrow$ slope of the given line $= -\dfrac{3}{2}$

$\Rightarrow$ slope of tangent $= \dfrac{2}{3}$

$\therefore$ Equation of tangents are

$y = \dfrac{2}{3}x \pm \sqrt{\dfrac{5}{2} \times \dfrac{4}{9} + \dfrac{5}{3}}$

$= \dfrac{2}{3}x \pm \sqrt{\dfrac{10 + 15}{9}}$

$= \dfrac{2}{3}x \pm \dfrac{5}{3}$

$\therefore$ Tangents are $2x – 3y + 5 = 0$ or $2x – 3y – 5 = 0$

Q.5 The lines $2x – 5y = 11$ and $x – 3y = 7$ are diameters of the circle of area 25π square units. The equation of the circle is

Correct option: (B)

Given equations of diameters are

$2x - 5y = 11$ …(i)

$x - 3y = 7$ …(ii)

On solving (i) and (ii), we get

$x = -2, y = -3$

∴ Centre of circle is (–2, –3).

Area = πr^2

⇒ $25\pi = \pi r^2$

⇒ r = 5 units

∴ The equation of the circle is

$(x + 2)^2 + (y + 3)^2 = 5^2$

⇒ $x^2 + y^2 + 4x + 6y - 12 = 0$

Q.6 A man running round a race-course notes that the sum of the distance of two flag-posts from him is always 10 metres and the distance between the flag-posts is 8 metres. The area of the path he encloses in square metres is

Correct option: (A)

Here, 2a = 10m and 2ae = 8m

∴ $e = \frac{4}{5}$, a = 5m

Now, $b^2 = a^2(1 - e^2) = 9$

⇒ b = 3

Thus, required area = πab = 15π sq. metre

Q.7 The number of common tangents that can be drawn to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 6x + 2y + 1 = 0$ is ______

Correct option: (D)

$x^2 + y^2 - 6x = 0$

C_1 (3, 0), $r_1 = 3$

$x^2 + y^2 + 6x + 2y + 1 = 0$

$C_2 = (-3, -1)$, $r_2 = \sqrt{9 + 1 - 1} = 3$

$C_1C_2 = \sqrt{(-3-3)^2 + (-1-0)^2} = \sqrt{37}$

∴ $C_1C_2 > r_1 + r_2$

∴ Number of tangents = 4

Q.8 If t is a parameter, then $x = a\left(t + \frac{1}{t}\right), y = b\left(t - \frac{1}{t}\right)$ represents

Correct option: (D)

Given, $x = a\left(t + \frac{1}{t}\right)$

⇒ $\frac{x}{a} = t + \frac{1}{t}$ ….(i)

and $y = b\left(t - \frac{1}{t}\right)$

⇒ $\frac{y}{b} = t - \frac{1}{t}$ ….(ii)

Squaring and subtracting equation (ii) from (i), we get

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = t^2 + 2 + \frac{1}{t^2} - t^2 + 2 - \frac{1}{t^2}$

⇒ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 4$

which represents a hyperbola.

Q.9 If the eccentricity of the two ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are equal, then the value of $\frac{a}{b}$ is

Correct option: (C)

According to the given condition,

$\sqrt{1 - \frac{25}{169}} = \sqrt{1 - \frac{b^2}{a^2}}$

⇒ $\frac{144}{169} = 1 - \frac{b^2}{a^2}$

⇒ $\frac{b^2}{a^2} = \frac{25}{169}$

⇒ $\frac{b}{a} = \frac{5}{13}$ ….[∵ a > 0, b > 0]

⇒ $\frac{a}{b} = \frac{13}{5}$

Q.10 The difference of the focal distance of any point on the hyperbola $9x^2 - 16y^2 = 144$, is

Correct option: (A)

The hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

∴ Difference of focal distance = 2a = 8

Q.11 The equation of parabola whose vertex and focus are (0, 4) and (0, 2) respectively, is

Correct option: (C)

Vertex = (0, 4), focus = (0, 2)

⇒ a = 2

Hence, equation of parabola is

$(x - 0)^2 = -4 \times 2(y - 4)$

i.e., $x^2 + 8y = 32$

Q.12 The line $y = 6x + 1$ touches the parabola $y^2 = 24x$. The co-ordinates of a point P on this line from which the tangent to $y^2 = 24x$ is perpendicular to this $y = 6x + 1$ is

Correct option: (C)

Slope of $y = 6x + 1$ is 6

∴ slope of line perpendicular to $y = 6x + 1$ is $\frac{-1}{6}$

Equation of tangent to $y^2 = 24x$ is

$y = mx + \frac{a}{m}$

$\Rightarrow y = \frac{-1}{6}x - 36$

$\Rightarrow \frac{x}{6} + y = -36$

Point (–6, –35) satisfies the above equation

∴ Option (C) is correct answer.

Q.13 The length of the axes of the conic $9x^2 + 4y^2 - 6x + 4y + 1 = 0$ are

Correct option: (C)

$9x^2 + 4y^2 - 6x + 4y + 1 = 0$

$\Rightarrow 9\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + 4\left(y^2 + y + \frac{1}{4}\right) = -1$

$+ 1 + 1$

$\Rightarrow 9\left(x - \frac{1}{3}\right)^2 + 4\left(y + \frac{1}{2}\right)^2 = 1$

$\Rightarrow \frac{\left(x - \frac{1}{3}\right)^2}{\frac{1}{9}} + \frac{\left(y + \frac{1}{2}\right)^2}{\frac{1}{4}} = 1$

Here, $a = \frac{1}{3}$, $b = \frac{1}{2}$

$\Rightarrow 2a = \frac{2}{3}$, $2b = 1$

∴ Length of axes are $1, \frac{2}{3}$.

Q.14 The equation of the tangent to the parabola $y^2 = 9x$ which goes through the point (4, 10), is

Correct option: (D)

Given that $y^2 = 9x$. Here, $a = \frac{9}{4}$.

Now, $y^2 = 9x$ is

$y = mx + \frac{\frac{9}{4}}{m}$

If this tangent passes through the point (4, 10), then $10 = 4m + \frac{9}{4m}$

⇔ $(4m - 9)(4m - 1) = 0$

⇔ $m = \frac{9}{4}, \frac{1}{4}$

Equations of tangents are

$4y = x + 36$ and $4y = 9x + 4$

i.e., $x - 4y + 36 = 0$ and $9x - 4y + 4 = 0$.

Q.15 The distance of the point ′θ′ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from a focus is

Correct option: (C)

Focal distance of any point P (x, y) on the ellipse is equal to SP = a + ex.

Here, $x = a\cos\theta$

∴ SP = a + ae cos θ

= a(1 + e cos θ)

Q.16 The equation of a circle passing through the vertex and the extremities of the latus rectum of the parabola $y^2 = 8x$ is

Correct option: (C)

For parabola, $y^2 = 8x$

⇒ 4a = 8

⇒ a = 2

vertex of $y^2 = 8x$ is O ≡ (0, 0)

Now, end points of latus rectum are

$L(a, 2a); L'(a, -2a) \Rightarrow L(2, 4); L'(2, -4)$

∴ the circle passes through the points (0,0), (2,4) and (2,–4).

All the three points are satisfied by the option (C).

∴ Option (C) is the correct answer.

Q.17 The slopes of the common tangents of the hyperbolae $\frac{x^2}{9} - \frac{y^2}{16} = 1$ and $\frac{y^2}{9} - \frac{x^2}{16} = 1$ are

Correct option: (B)

Given hyperbolae are

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \quad \text{...(i)}$$

and $$\frac{x^2}{16} - \frac{y^2}{9} = -1 \quad \text{...(ii)}$$

The equation of any tangent to (i) is

$y = mx \pm \sqrt{9m^2 - 16}$

If it touches (ii), then

$9m^2 - 16 = 9 - 16m^2$ …

$$\left[\begin{array}{l}\text{If } y = mx + c \text{ touches } \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1, \\ \text{then } c^2 = b^2 - a^2m^2\end{array}\right]$$

$\Rightarrow$ m = ± 1

Q.18 Eccentricity of the ellipse whose latus rectum is equal to the distance between two focus points, is

Correct option: (B)

We have, $\frac{2b^2}{a}$ = 2ae

$\Rightarrow b^2 = a^2e$

$\Rightarrow e = \frac{b^2}{a^2}$ ….(i)

Also, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e^2 = 1 - e$ ….[From (i)]

$\Rightarrow e^2 + e - 1 = 0$

$\therefore$ $e = \frac{-1 \pm \sqrt{5}}{2}$

$\therefore$ $e = \frac{\sqrt{5} - 1}{2}$

Q.19 The latus rectum of the hyperbola $9x^2 - 16y^2 + 72x - 32y - 16 = 0$ is

Correct option: (A)

Given, equation of hyperbola is

$9x^2 - 16y^2 + 72x - 32y - 16 = 0$

$\Rightarrow 9(x^2 + 8x) - 16(y^2 + 2y) - 16 = 0$

$\Rightarrow 9(x + 4)^2 - 16(y + 1)^2 = 144$

$\Rightarrow \frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = 1$

$\therefore$ Latus rectum = $\frac{2b^2}{a} = 2 \times \frac{9}{4} = \frac{9}{2}$

Q.20 Equation x = a cos θ, y = b sin θ (a > b) represent a conic section whose eccentricity e is given by

Correct option: (C)

Q.21 The equation of tangents to the circle $x^2 + y^2 = 4$ which are parallel to $x + 2y + 3 = 0$ are

Correct option: (B)

Equation of line parallel to $x + 2y + 3 = 0$ is $x + 2y + k = 0$.

But it is tangent to the circle $x^2 + y^2 = 4$, then

$\left|\frac{k}{\sqrt{1+4}}\right| = 2$

$\Rightarrow k = \pm 2\sqrt{5}$

Hence, the required equations are $x + 2y = \pm 2\sqrt{5}$

Q.22 The equation of the tangent to the parabola $y^2 = 4ax$ at point $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ is

Correct option: (A)

Equation of the tangent to the parabola,

$y^2 = 4ax$ at (x_1, y_1) is $yy_1 = 2a(x + x_1)$

$\Rightarrow y.\frac{2a}{t} = 2a\left(x + \frac{a}{t^2}\right)$

$\Rightarrow \frac{y}{t} = \left(x + \frac{a}{t^2}\right)$

$\Rightarrow \frac{y}{t} = \frac{t^2x + a}{t^2}$

$\Rightarrow ty = t^2x + a$

Q.23 The length of transverse axis of the hyperbola $3x^2 - 4y^2 = 32$ is

Correct option: (A)

The given equation can be written as $\frac{x^2}{\frac{32}{3}} - \frac{y^2}{8} = 1$

$\Rightarrow \frac{x^2}{\left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2} - \frac{y^2}{(2\sqrt{2})^2} = 1$

$\Rightarrow a = \frac{4\sqrt{2}}{\sqrt{3}}$

$\therefore$ Length of transverse axis

$= 2a = 2 \times \frac{4\sqrt{2}}{\sqrt{3}} = \frac{8\sqrt{2}}{\sqrt{3}}$

Q.24 The values of m for which the line $y = mx + 2$ becomes a tangent to the hyperbola $4x^2 - 9y^2 = 36$ is

Correct option: (B)

$4x^2 - 9y^2 = 36$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{4} = 1$

Here, $c = 2$, $a^2 = 9$, $b^2 = 4$

$\therefore \; 2^2 = 9m^2 - 4$

$\Rightarrow m^2 = \frac{8}{9}$

$\Rightarrow m = \pm\frac{2\sqrt{2}}{3}$

Q.25 The length of the latus rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is

Correct option: (C)

The given equation of parabola is $x^2 - 4x - 8y + 12 = 0$

$\Rightarrow x^2 - 4x = 8y - 12$

$\Rightarrow x^2 - 4x + 4 = 8y - 12 + 4$

$\Rightarrow (x - 2)^2 = 8(y - 1)$

Hence, the length of latus rectum = 4a = 8.

Q.26 In the parabola $y^2 = 6x$, the equation of the chord through vertex and negative end of latus rectum, is

Correct option: (B)

Vertex ≡ (0, 0).

End points of latus rectum are (a, ± 2a).

Here $a = \frac{6}{4} = \frac{3}{2}$

Therefore, –ve end of latus rectum is

$\left(\frac{3}{2}, -3\right) = A$

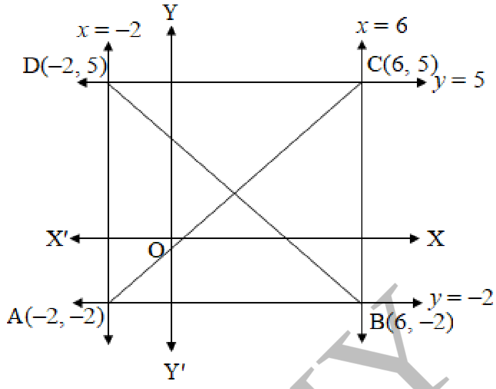
Slope of the chord through O and A $= -\frac{3}{\frac{3}{2}} = -2$

Equation of the chord: $y = -2x$ or $y + 2x = 0$.

Q.27 If the sides of a rectangle are given by the equations $x = -2, x = 6, y = -2, y = 5$, then the equation of the circle, drawn on the diagonal of this rectangle as its diameter, is

Correct option: (C)

The given equations of the sides are $x = -2$, $x = 6$, $y = -2$ and $y = 5$.



Here, the diagonals AC and BD of rectangle ABCD are diameters of the circle passing through the vertices A, B, C and D.

Considering diagonal AC with end points A(–2, –2) and C(6, 5), we get

Equation of circle in diameter form as,

$(x - 6)(x + 2) + (y - 5)(y + 2) = 0$

$\Rightarrow x^2 - 4x - 12 + y^2 - 3y - 10 = 0$

$\Rightarrow x^2 + y^2 - 4x - 3y - 22 = 0$

Q.28 The length of latus rectum of the parabola whose focus is at (1, –2) and directrix is the line $x + y + 3 = 0$ is

Correct option: (B)

Distance between focus and directrix

$= \left|\frac{1 - 2 + 3}{\sqrt{2}}\right|$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

Since latus rectum is two times the distance between focus and directrix.

$\therefore$ Length of latus rectum $= 2\sqrt{2}$ units

Q.29 The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$, if

Correct option: (C)

We have, $lx + my + n = 0$

$\therefore \; y = -\frac{l}{m}x - \frac{n}{m}$

Condition for above line to be tangent to $y^2 = 4ax$ is

$$-\frac{n}{m} = \frac{am}{-l}$$

$\Rightarrow nl = am^2$

Q.30 The length of the latus rectum of the hyperbola $3x^2 - y^2 = 4$ is

Correct option: (B)

Length of latus rectum = $\frac{2b^2}{a}$

$= \frac{2 \times 4}{\frac{2}{\sqrt{3}}} = 4\sqrt{3}$

Q.31 If the latus rectum of an ellipse be equal to half of its minor axis, then its eccentricity is

Correct option: (B)

Given, $\frac{2b^2}{a} = b$

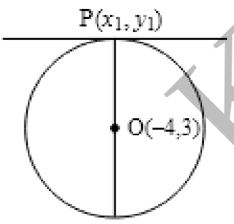
$\Rightarrow \frac{b}{a} = \frac{1}{2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$

Hence, e = $\sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{3}}{2}$

Q.32 If a circle, whose centre is (– 4, 3) touches the straight line $3x - 4y - 11 = 0$, then the co-ordinates of the point of contact are

Correct option: (D)



Let point of contact be $P(x_1, y_1)$.

This point lies on line $3x_1 - 4y_1 = 11$ ...(i)

Gradient of OP = $m_1 = \frac{y_1 - 3}{x_1 + 4}$

Gradient of $3x - 4y - 11 = 0$ is $m_2 = \frac{3}{4}$

The two lines are perpendicular.

$\therefore \left(\frac{y_1 - 3}{x_1 + 4}\right)\left(\frac{3}{4}\right) = -1$

$\Rightarrow 4x_1 + 3y_1 = -7$ …(ii)

On solving equations (i) and (ii), we get

$(x_1, y_1) = \left(\frac{1}{5}, \frac{-13}{5}\right)$

Q.33 If the length of the tangent segment from the point (5, 1) to the circle $x^2 + y^2 + 6x + ky - 3 = 0$ is 7, then k equals

Correct option: (B)

Length of tangent segment

$= \sqrt{5^2 + 1^2 + 6(5) + k(1) - 3}$

$\Rightarrow 7 = \sqrt{53 + k}$

$\Rightarrow 49 = 53 + k$

$\Rightarrow k = -4$

Q.34 The equation of the circle which touches both axes and whose centre is (x_1, y_1) is

Correct option: (B)

The equation of circle with centre (x_1, y_1) is

$(x - x_1)^2 + (y - y_1)^2 = r^2$

Since the circle touches both the axes,

$x_1 = y_1 = r$

$\therefore (x - x_1)^2 + (y - x_1)^2 = x_1^2$

$\Rightarrow x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$

Q.35 If the eccentricity of an ellipse be $\frac{1}{\sqrt{2}}$, then its latus rectum is equal to its

Correct option: (D)

We have, e = $\frac{1}{\sqrt{2}}$

$\therefore$ Latus rectum = $\frac{2b^2}{a} = \frac{2}{a} \times a^2\left(1 - e^2\right)$

$= 2a\left(1 - \frac{1}{2}\right) = a$

i.e., semi-major axis

Q.36 The directrix of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is

Correct option: (A)

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$\Rightarrow a^2 = 9, b^2 = 4$

e = $\frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{9 + 4}}{3} = \frac{\sqrt{13}}{3}$

Directrix of hyperbola is $x = \pm\frac{a}{e}$

$\Rightarrow x = \pm\frac{3}{\frac{\sqrt{13}}{3}}$ $\Rightarrow x = \pm\frac{9}{\sqrt{13}}$

Q.37 The equation of the tangent to the ellipse $x^2 + 16y^2 = 16$ making an angle of 60° with X-axis is

Correct option: (C)

We have m = tan 60° = $\sqrt{3}$ = slope of the tangent

Equation of tangent in slope form, is $y = mx \pm \sqrt{a^2m^2+b^2}$

$\Rightarrow y = \sqrt{3}x \pm \sqrt{1+3(16)}$ $\Rightarrow y = \sqrt{3}x \pm 7$

Q.38 If a bar of given length moves with its extremities on two fixed straight lines at right angles, then the locus of any point on bar marked on the bar describes a/an

Correct option: (C)

Q.39 If the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{7} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of a is

Correct option: (A)

Hyperbola is $\frac{x^2}{144} - \frac{y^2}{18} = \frac{1}{25}$

Here, a = $\sqrt{\frac{144}{25}}$, b = $\sqrt{\frac{81}{25}}$

$\Rightarrow e_1 = \sqrt{1+\frac{81}{144}} = \sqrt{\frac{225}{144}} = \frac{15}{12} = \frac{5}{4}$

$\Rightarrow$ foci = $(ae_1, 0) = \left(\frac{12}{5}\times\frac{5}{4}, 0\right) = (3, 0)$

Focus of ellipse = (ae, 0)

$\Rightarrow$ ae = 3 …(i)

Using

$b^2 = a^2 - a^2e^2$

$7 = a^2 - (3)^2$

$\Rightarrow a^2 = 7 + 9$

$\Rightarrow a^2 = 16$

$\Rightarrow a = 4$

Q.40 The equation of the circle concentric with the circle $2x^2 + 2y^2 + 10x + 12y - 16 = 0$ and having an area equal to 25π square units is

Correct option: (C)

Given equation of circle is

$2x^2 + 2y^2 + 10x + 12y - 16 = 0$

$\Rightarrow x^2 + y^2 + 5x + 6y - 8 = 0$

$\therefore$ Centre ≡ $\left(\frac{-5}{2}, -3\right)$

Centre of required circle ≡ $\left(\frac{-5}{2}, -3\right)$

Area = 25π

$\Rightarrow \pi r^2 = 25\pi$

$\Rightarrow r = 5$

$\therefore$ Equation of required circle is

$\left(x + \frac{5}{2}\right)^2 + (y + 3)^2 = 5^2$

$\Rightarrow x^2 + y^2 + 5x + 6y + \frac{25}{4} + 9 = 25$

$\Rightarrow 4x^2 + 4y^2 + 20x + 24y - 39 = 0$

Q.41 The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are roots of the equation $y^2 + 2py - q^2 = 0$. Then the equation of the circle with AB as diameter is given by

Correct option: (D)

Let $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$.

According to the given condition,

$x_1 + x_2 = -2a$, $x_1x_2 = -b^2$

$y_1 + y_2 = -2p$, $y_1y_2 = -q^2$

The equation of the circle with A (x_1, y_1) and B (x_2, y_2) as the end points of diameter is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow x^2 + y^2 - x(x_1 + x_2) - y(y_1 + y_2) + x_1x_2 + y_1y_2 = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Q.42 Line $y = x + a\sqrt{2}$ is a tangent to the circle $x^2 + y^2 = a^2$ at

Correct option: (D)

Suppose that the point be (h, k).

Tangent at (h, k) is

$hx + ky = a^2$ and $x - y = -\sqrt{2}a$

or $\frac{h}{1} = \frac{k}{-1} = \frac{a^2}{-\sqrt{2a}}$ or $h = -\frac{a}{\sqrt{2}}$, $k = \frac{a}{\sqrt{2}}$

Therefore, point of contact is $\left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$.

Q.43 If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are tangents of a circle, then radius of that circle is

Correct option: (C)

The equation of the tangents are

$5x - 12y + 10 = 0$ and $5x - 12y - 16 = 0$

Hence, they are parallel to each other. The perpendicular distance between these two lines is the diameter of the circle

$$2r = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

$c_1 = 10$; $c_2 = -16$; $a = 5$; $b = -12$

$$\therefore 2r = \left|\frac{10 - (-16)}{\sqrt{5^2 + 12^2}}\right| = \left|\frac{26}{13}\right| = 2$$

$\Rightarrow r = 1$

Q.44 If the line $y = 2x + c$ be a tangent to the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then c =

Correct option: (B)

$c = \pm\sqrt{b^2 + a^2m^2}$

$= \pm\sqrt{4 + 8(4)}$

$= \pm 6$

Q.45 The equation of a hyperbola with foci at (6, 5) and (– 4, 5) and eccentricity = $\frac{5}{4}$ is

Correct option: (A)

Centre of the hyperbola is midpoint of foci.

Hence, its centre is (1, 5).

Also, distance between foci is 2ae = 10

$\Rightarrow a = 4 \qquad \ldots\left[\because e = \frac{5}{4}\right]$

$\Rightarrow a^2 = 16$

Now, $b^2 = a^2(e^2 - 1)$

$= a^2e^2 - a^2 = 25 - 16 \Rightarrow b^2 = 9$

Hence, equation of hyperbola is

$$\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$$

Q.46 If the line $y = 7x - 25$ meets the circle $x^2 + y^2 = 25$ at the points A, B, then the distance between A and B is

Correct option: (C)

$y = 7x - 25$ ....(i)

and $x^2 + y^2 = 25$

$\therefore x^2 + (7x - 25)^2 = 25$

$\Rightarrow x^2 + 49x^2 + 625 - 350x = 25$

$\Rightarrow 50x^2 - 350x + 600 = 0$

$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x = 3, 4$

Substituting $x = 3,4$ in (i), we get

$y = 21 - 25 \Rightarrow y = -4$, $y = 28 - 25 \Rightarrow y = 3$

Let A ≡ (3, –4), B ≡ (4, 3)

Using distance formula, we get

$$AB = \sqrt{(3-4)^2 + (-4-3)^2}$$

$$= \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$$

Q.47 The length of tangent from the point (2, – 3) to the circle $2x^2 + 2y^2 = 1$ is

Correct option: (C)

Equation of the circle is $2x^2 + 2y^2 - 1 = 0$

$\Rightarrow x^2 + y^2 - \frac{1}{2} = 0$

Length of the tangent from the point (2, – 3) is

$$\sqrt{2^2 + (-3)^2 - \frac{1}{2}} = \sqrt{13 - \frac{1}{2}} = \frac{5}{\sqrt{2}}$$

Q.48 The auxiliary equation of circle of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is

Correct option: (A)

Q.49 An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4 cm, the necessary length of the string and the distance between the pins respectively in cm, are

Correct option: (D)

Given, 2a = 6, 2b = 4

i.e., a = 3, b = 2

Also, $e^2 = 1 - \frac{b^2}{a^2} = \frac{5}{9}$

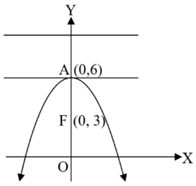
$\Rightarrow e = \frac{\sqrt{5}}{3}$

$\therefore$ Distance between the pins = 2ae = $2\sqrt{5}$cm

and length of string = 2a + 2ae = $6 + 2\sqrt{5}$cm

Q.50 If (0, 6) and (0, 3) are respectively the vertex and focus of a parabola, then its equation is

Correct option: (D)



Here vertex ≡ (0, 6) and focus ≡ (0, 3)

⇒ Y axis is the axis of the parabola and parabola is opening downwards.

Also a = 3

⇒ $x^2 = -4(3)(y - 6)$

⇒ $x^2 + 12y = 72$

Q.51 The equation of the circle which touches both the axes and whose radius is a, is

Correct option: (A)

Required equation is $(x - a)^2 + (y - a)^2 = a^2$

⇒ $x^2 + y^2 - 2ax - 2ay + a^2 = 0$

Q.52 Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then the equation of the hyperbola is

Correct option: (B)

The equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$.

Let e be its eccentricity.

Then, e = $\sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

The foci of the ellipse are S$\left(\sqrt{3}, 0\right)$ and S′ $\left(-\sqrt{3}, 0\right)$.

Eccentricity of the hyperbola = $\frac{1}{e} = \frac{2}{\sqrt{3}}$

∴ $b^2 = a^2\left(\frac{4}{3} - 1\right) = \frac{a^2}{3}$ ….(i)

The hyperbola passes through S$\left(\sqrt{3}, 0\right)$.

∴ $\frac{3}{a^2} - 0 = 1 \Rightarrow a^2 = 3$

Putting $a^2 = 3$ in (i), we get

$b^2 = 1$

Hence, the equation of the hyperbola is

$\frac{x^2}{3} - \frac{y^2}{1} = 1$ i.e., $x^2 - 3y^2 = 3$.

Q.53 The locus of a point of intersection of two lines $x\sqrt{3} - y = k\sqrt{3}$ and $\sqrt{3}kx + ky = \sqrt{3}$, k ∈ R, describes

Correct option: (B)

Given equations of line

$x\sqrt{3} - y = k\sqrt{3}$ …(i)

and $\sqrt{3}kx + ky = \sqrt{3}$

⇒ $\sqrt{3}x + y = \frac{\sqrt{3}}{k}$ …(ii)

Multiplying equations (i) and (ii), we get

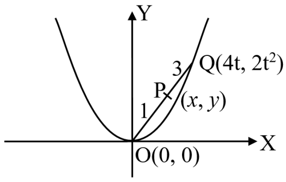
$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x + y\right) = \left(k\sqrt{3}\right)\left(\frac{\sqrt{3}}{k}\right)$

⇒ $3x^2 - y^2 = 3$

⇒ $\frac{x^2}{1} - \frac{y^2}{3} = 1$, which is a hyperbola

Q.54 Let O be the vertex and Q be any point on the parabola $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is

Correct option: (D)



$x^2 = 8y$

⇒ a = 2

By internal division formula,

P(x, y) = $\left(t, \frac{t^2}{2}\right)$

∴ $x = t,\ y = \frac{t^2}{2}$

⇒ $x^2 = 2y$

Q.55 The locus of the point of intersection of the lines $ax \sec\theta + by \tan\theta = a$ and $ax \tan\theta + by \sec\theta = b$, where θ is the parameter, is

Correct option: (D)

Squaring and subtracting, we get

$a^2x^2 - b^2y^2 = a^2 - b^2$, which is the equation of hyperbola.

Q.56 Number of common tangents to the circles $x^2 + y^2 - 6x - 14y + 48 = 0$ and $x^2 + y^2 - 6x = 0$ are

Correct option: (C)

$x^2 + y^2 - 6x - 14y + 48 = 0$

$\therefore \ C_1(3,7), r_1 = \sqrt{10}$

Again $x^2 + y^2 - 6x = 0$

$\therefore \ C_2(3,0), r_2 = 3$

Now $l(C_1C_2)$= distance between centres

$\therefore \ l(C_1C_2) = \sqrt{0^2+7^2} = 7$ and

$r_1 + r_2 = \sqrt{10} + 3 < l(C_1C_2)$

⇒ The given circles are disjoint.

⇒ Number of common tangents is 4.

Q.57 The equation of the circle with centre (2, 2) which passes through (4, 5) is

Correct option: (B)

Centre (2, 2) and

r = $\sqrt{(4-2)^2+(5-2)^2}$

= $\sqrt{13}$

Hence, required equation is

$(x-2)^2 + (y-2)^2 = \left(\sqrt{13}\right)^2$

⇒ $x^2 + y^2 - 4x - 4y - 5 = 0$

Q.58 If the line $y = 2x + \lambda$ be a tangent to the hyperbola $36x^2 - 25y^2 = 3600$, then λ =

Correct option: (C)

If $y = 2x + \lambda$ is tangent to given hyperbola, then

$\lambda = \pm\sqrt{a^2m^2 - b^2}$

$= \pm\sqrt{(100)(4) - 144}$

$= \pm\sqrt{256} = \pm 16$

Q.59 The equation of locus of a point, the tangents from which to the circle $2x^2 + 2y^2 = 32$ are at right angle, is

Correct option: (B)

The locus of the point of intersection of two perpendicular tangents is the director circle.

The equation of the director circle of the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.

Equation of circle is $x^2 + y^2 = 16$

Equation of director circle is $x^2 + y^2 = 2(16)$

i.e., $x^2 + y^2 = 32$

Q.60 The equations of the tangents to the circle $x^2 + y^2 = 16$ which are parallel to $2x + y - 1 = 0$ are

Correct option: (C)

The perpendicular distance from the centre of the given circle to the tangent line is the radius of the given circle.

The centre is (0, 0) and its radius is 4.

The equation of the tangent line, which is parallel to the given line is $2x + y + k = 0$

$\therefore \frac{k}{\sqrt{4+1}} = \pm 4$

$\Rightarrow k = \pm 4\sqrt{5}$

∴The equations of the tangents are $2x + y \pm 4\sqrt{5} = 0$

Q.61 If e and e′ are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola $5x^2 - 4y^2 = 45$, then ee′ =

Correct option: (D)

$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9}$

$\Rightarrow e = \frac{2}{3}$

and $e'^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{\frac{45}{4}}{9} = \frac{9}{4}$

$\Rightarrow e' = \frac{3}{2}$

$\therefore \quad ee' = 1$

Q.62 The equations of the directrices of the ellipse $16x^2 + 25y^2 = 400$ are

Correct option: (D)

$16x^2 + 25y^2 = 400$

$\Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$

$\Rightarrow$ e = $\sqrt{1-\frac{16}{25}} = \frac{3}{5}$

Therefore, directrices are $x = \pm \frac{5}{\frac{3}{5}}$ or $3x = \pm 25$

Q.63 A point on the parabola whose focus is S(1, –1) and whose vertex is A(1, 1) is

Correct option: (A)

Equation of parabola having vertex (p, q) and focus (p, b + q) is given by

$(x - p)^2 = 4b(y - q)$

Given, vertex A = (1, 1) and focus S = (1, –1)

$\therefore$ p = 1, q = 1, b = –2

$\therefore$ Equation of parabola is

$(x - 1)^2 = 4(-2)(y - 1)$

i.e. $x^2 - 2x + 8y - 7 = 0$

Only $\left(3, \frac{1}{2}\right)$ satisfies the above equation of parabola.

Q.64 If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - y + 4 = 0$, then the value of θ is

Correct option: (C)

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

Here, $a^2 = 4$, $b^2 = 9$

Equation of the tangent at (2 sec θ, 3 tan θ) is

$$\frac{x(2\sec\theta)}{4} - \frac{y(3\tan\theta)}{9} = 1$$

$$\Rightarrow \frac{x\sec\theta}{2} - \frac{y\tan\theta}{3} = 1$$

$\Rightarrow 3x\sec\theta - 2y\tan\theta = 6$

Slope of this tangent = $\frac{-3\sec\theta}{-2\tan\theta} = \frac{3}{2\sin\theta}$

Slope of line $3x - y + 4 = 0$ is 3.

Since this line is parallel to the tangent, their slopes are equal.

$\therefore \frac{3}{2\sin\theta} = 3$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

Q.65 The condition for which the straight line $y = mx + c$ touches the parabola $y^2 = 4ax$ is

Correct option: (B)

Solve $y = mx + c$ and $y^2 = 4ax$, to get $m^2x^2 + (2mc - 4a)x + c^2 = 0$ …(i)

For line to be tangent, roots of (i) must be equal

$\Rightarrow$ discriminant = 0

$\Leftrightarrow 4(mc - 2a)^2 - 4m^2c^2 = 0$

$\Rightarrow$ a = mc

Q.66 If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is

Correct option: (C)

According to the condition, $\frac{2a}{e} = 3(2ae)$

$\Rightarrow e = \frac{1}{\sqrt{3}}$

Q.67 The equation of circle with centre at (2, –3) and the circumference 10π units is

Correct option: (A)

Given, circumference = 10π

$\therefore 2\pi r = 10\pi$

$\Rightarrow r = 5$

$\therefore$ the equation of the circle is

$(x-2)^2 + (y+3)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 4x + 6y - 12 = 0$

Q.68 Four distinct points (2a, 3a), (1, 0), (0, 1) and (0, 0) lie on the circle for

Correct option: (A)

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since the circle passes through (0, 0), (1, 0) and (0, 1), we get

c = 0

1 + 2g + c = 0

$\Rightarrow g = \frac{-1}{2}$

1 + 2f + c = 0

$\Rightarrow f = \frac{-1}{2}$

$\therefore$ The equation of circle is $x^2 + y^2 - x - y = 0$

Since, (2a, 3a) lies on this circle.

$\therefore (2a)^2 + (3a)^2 - 2a - 3a = 0$

$\Rightarrow 13a^2 = 5a$

$\Rightarrow a = \frac{5}{13}$

Q.69 The equation of the tangent to the hyperbola $2x^2 - 3y^2 = 6$ which is parallel to the line $y = 3x + 4$, is

Correct option: (C)

Let tangent be $y = 3x + c$

$c = \pm \sqrt{a^2m^2 - b^2} = \pm \sqrt{3(9) - 2} = \pm 5$

$\Rightarrow y = 3x \pm 5$

Q.70 The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ is

Correct option: (A)

$16x^2 - 3y^2 - 32x - 12y - 44 = 0$

$\Rightarrow 16(x^2 - 2x) - 3(y^2 + 4y) - 44 = 0$

$\Rightarrow 16(x - 1)^2 - 3(y + 2)^2 = 48$

$\Rightarrow \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$

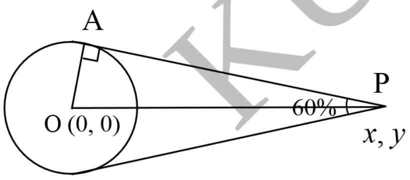
Comparing with standard form, we get

$a^2 = 3, b^2 = 16$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{3}} = \sqrt{\frac{19}{3}}$

Q.71 The locus of point of intersection of the tangents to the circle $x^2 + y^2 = 16$, such that the angle between them is 60°, is

Correct option: (B)



Given equation of circle is

$x^2 + y^2 = 16$

$C \equiv (0, 0)$

r = 4 units

In ΔAPO,

$\angle P = 30°$

$\therefore \sin 30° = \frac{OA}{OP}$

$\frac{1}{2} = \frac{4}{OP}$

$\Rightarrow OP = 8$ units

$\Rightarrow \sqrt{(x-0) + (y-0)^2} = 8$

$\Rightarrow x^2 + y^2 = 8^2$

$\Rightarrow x^2 + y^2 = 64$

Q.72 The equation of the circle which touches X-axis and whose centre is (1, 2), is

Correct option: (B)

Since the circle touches X-axis,

radius = 2.

$\therefore$ the equation of the circle is

$(x - 1)^2 + (y - 2)^2 = 2^2$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$

Q.73 The eccentricity of the ellipse $y^2 + 4x^2 - 12x + 6y + 14 = 0$ is

Correct option: (A)

$y^2 + 4x^2 - 12x + 6y + 14 = 0$

$\Rightarrow 4(x^2 - 3x) + y^2 + 6y = -14$

$\Rightarrow 4\left(\mathbf{x} - \frac{3}{2}\right)^2 + (\mathbf{y} + 3)^2 = 4$

$\Rightarrow \left(\mathbf{x} - \frac{3}{2}\right)^2 + \frac{(\mathbf{y}+3)^2}{4} = 1$

$\therefore \quad a^2 = 1, b^2 = 4$

$e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{3}}{2}$

Q.74 The equation of the tangents to the conic $3x^2 - y^2 = 3$ perpendicular to the line $x + 3y = 2$ is

Correct option: (A)

Slope of $x + 3y - 2 = 0$ is $-\frac{1}{3}$.

$\therefore$ Slope of required tangent = 3

Tangent to $\frac{x^2}{1} - \frac{y^2}{3} = 1$ and perpendicular to

$x + 3y - 2 = 0$ is given by

$y = 3x \pm \sqrt{9 - 3} = 3x \pm \sqrt{6}$

Q.75 The centre and radius of a circle $x = 4a\left(\frac{1-t^2}{1+t^2}\right), y = \frac{8at}{1+t^2}$, are respectively

Correct option: (B)

$$x = 4a\left(\frac{1-t^2}{1+t^2}\right)$$

$$y = 4a\left(\frac{2t}{1+t^2}\right)$$

Squaring and adding, we get

$$x^2 + y^2 = 16a^2\frac{\left(1-t^2\right)^2}{\left(1+t^2\right)^2} + \frac{64a^2t^2}{\left(1+t^2\right)^2}$$

$$= \frac{16a^2}{\left(1+t^2\right)^2}\left[\left(1-t^2\right)^2 + 4t^2\right]$$

$$= \frac{16a^2}{\left(1+t^2\right)^2}\left(1+t^2\right)^2$$

$\therefore$ $x^2 + y^2 = 16a^2$

$\therefore$ Centre is (0, 0) and radius = 4a units