Trigonometry **Marks: 200**

ANSWER KEY

Maths

Q.1 C	Q.2 A	Q.3 D	Q.4 D	Q.5 C	Q.6 A	Q.7 B	Q.8 C
Q.9 C	Q.10 B	Q.11 A	Q.12 C	Q.13 C	Q.14 A	Q.15 C	Q.16 B
Q.17 A	Q.18 C	Q.19 C	Q.20 C	Q.21 C	Q.22 A	Q.23 D	Q.24 B
Q.25 D	Q.26 A	Q.27 D	Q.28 C	Q.29 A	Q.30 A	Q.31 A	Q.32 A
Q.33 B	Q.34 B	Q.35 C	Q.36 B	Q.37 D	Q.38 B	Q.39 B	Q.40 C
Q.41 C	Q.42 B	Q.43 B	Q.44 C	Q.45 B	Q.46 A	Q.47 A	Q.48 C
Q.49 D	Q.50 A	Q.51 D	Q.52 B	Q.53 C	Q.54 D	Q.55 B	Q.56 D
Q.57 D	Q.58 B	Q.59 D	Q.60 D	Q.61 D	Q.62 D	Q.63 C	Q.64 A
Q.65 B	Q.66 B	Q.67 C	Q.68 C	Q.69 C	Q.70 C	Q.71 D	Q.72 D
Q.73 D	Q.74 C	Q.75 A	Q.76 C	Q.77 D	Q.78 A	Q.79 B	Q.80 C
Q.81 A	Q.82 C	Q.83 D	Q.84 A	Q.85 B	Q.86 C	Q.87 C	Q.88 C
Q.89 B	Q.90 D	Q.91 B	Q.92 B	Q.93 C	Q.94 A	Q.95 B	Q.96 C
Q.97 C	Q.98 B	Q.99 B	Q.100 A				

Maths

Q.1 The value of $\cos^2 10° - \cos 10° \cdot \cos 50° + \cos^2 50°$ is

Correct option: (C)

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$= \frac{1}{2}(2\cos^2 10° - 2\cos 10° \cos 50° + 2\cos^2 50°)$

$= \frac{1}{2}[1 + \cos 20° - (\cos 60° + \cos 40°) + 1 + \cos 100°]$

$= \frac{1}{2}[2 + (\cos 20° + \cos 100°) - \cos 60° - \cos 40°]$

$= \frac{1}{2}[2 + (2\cos 60° \cos 40°) - \cos 60° - \cos 40°]$

$= \frac{1}{2}\left(2 + \cos 40^\circ - \frac{1}{2} - \cos 40^\circ\right)$

$= \frac{1}{2}\left(\frac{3}{2}\right)$

$= \frac{3}{4}$

Q.2 If a = 16, b = 24, c = 20, then $\cos\frac{B}{2}$ =

Correct option: (A)

$s = \frac{a+b+c}{2} = \frac{16+24+20}{2} = 30$

$\cos\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{ac}}$

$= \sqrt{\frac{30(30-24)}{16 \times 20}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Q.3 $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ =

Correct option: (D)

$\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \cot\left[\cot^{-1}\left(\frac{7}{24}\right)\right] = \frac{7}{24} \ldots$

$\left[\because \cos^{-1}x = \cot^{-1}\frac{x}{\sqrt{1-x^2}}\right]$

Q.4 If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is

Correct option: (D)

Let $\alpha = \cos^{-1}\sqrt{p}$, $\beta = \cos^{-1}\sqrt{1-p}$

and $\gamma = \cos^{-1}\sqrt{1-q}$

$\therefore \cos\alpha = \sqrt{p}$,

$\cos\beta = \sqrt{1-p}$

and $\cos\gamma = \sqrt{1-q}$

$\therefore \sin\alpha = \sqrt{1-p}$, $\sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation can be written as

$\alpha + \beta + \gamma = \frac{3\pi}{4} \Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$

$\Rightarrow \cos(\alpha + \beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$

$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}$

$= -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}} \cdot \sqrt{q}\right)$

$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1 - q = q \Rightarrow q = \frac{1}{2}$

Q.5 If sec θ + tan θ = 4, then sin θ =

Correct option: (C)

$\sec\theta + \tan\theta = 4$

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 4$

$\Rightarrow \frac{1+\sin\theta}{\cos\theta} = 4$

$\Rightarrow \frac{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}} = 4$

$\Rightarrow \cos\frac{\theta}{2} + \sin\frac{\theta}{2} = 4\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$

$\Rightarrow 5\sin\frac{\theta}{2} = 3\cos\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} = \frac{3}{5}$

Now, $\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{2\left(\frac{3}{5}\right)}{1+\left(\frac{3}{5}\right)^2} = \frac{15}{17}$

Q.6 The value of $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $|x| < \frac{1}{2}$, $x \neq 0$

Correct option: (A)

Let $T = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Put $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x^2$

$\therefore\ T = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$

$= \frac{\pi}{4} + \theta$

$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

Q.7 If the angles of a triangle are in the ratio 1: 2: 3, then their corresponding sides are in the ratio

Correct option: (B)

Let the angles of the triangle be x, $2x$ and $3x$.

Then, $x + 2x + 3x = 180° \Rightarrow x = 30°$

$\therefore$ angles of the triangle are 30°, 60° and 90°.

$\therefore$ a : b : c = sin30°: sin60° : sin90°

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$

Q.8 $\sin^4\frac{\pi}{8} + \sin^4\frac{2\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{4\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{6\pi}{8} + \sin^4\frac{7\pi}{8} =$

Correct option: (C)

$\sin^4\frac{\pi}{8} + \sin^4\frac{2\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{4\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{6\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \sin^4\frac{\pi}{8} + \sin^4\frac{\pi}{4} + \sin^4\frac{3\pi}{8} + \sin^4\frac{\pi}{2} + \sin^4\frac{3\pi}{8} + \sin^4\frac{3\pi}{4} + \sin^4\frac{\pi}{8}$ …[∵ sin (π – θ) = sin θ]

$= 2\sin^4\frac{\pi}{8} + 2\sin^4\frac{3\pi}{8} + \sin^4\frac{\pi}{4} + \sin^4\frac{\pi}{2} + \sin^4\frac{3\pi}{4}$

$= \frac{1}{2}\left[\left(2\sin^2\frac{\pi}{8}\right)^2 + \left(2\sin^2\frac{3\pi}{8}\right)^2\right] + \left(\frac{1}{\sqrt{2}}\right)^4 + (1)^4 + \sin^4\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$

$= \frac{1}{2}\left[\left(1-\cos\frac{\pi}{4}\right)^2 + \left(1-\cos\frac{3\pi}{4}\right)^2\right] + \frac{1}{4} + 1 + \cos^4\frac{\pi}{4}$

$= \frac{1}{2}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2 + \left(1+\frac{1}{\sqrt{2}}\right)^2\right] + \frac{5}{4} + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{2}(3) + \frac{5}{4} + \frac{1}{4}$

$= 3$

Q.9 If sin A + cos A = 1, then sin 2A is equal to

Correct option: (C)

Given, sin A + cos A = 1

Squaring on both sides, we get

$(\sin A + \cos A)^2 = 1$

$\Rightarrow 1 + \sin 2A = 1$

$\Rightarrow \sin 2A = 0$

Q.10 If $(a+b)\cos C + (b+c)\cos A + (c+a)\cos B = 72$ and if a$= 18$, b $= 24$, then area of the triangle ABC is

Correct option: (B)

(a + b) cos C + (b + c) cos A + (c + a) cos B = 72

$\therefore$ a cos C + b cos C + b cos A + c cos A + c cos B + a cos B = 72

⇒ a cos C + c cos A + b cos A + a cos B + b cos C + c cos B = 72 … $\begin{bmatrix} \text{By projection} \\ a = b\cos C + c\cos B \\ b = c\cos A + a\cos C \\ c = a\cos B + b\cos A \end{bmatrix}$

⇒ b + c + a = 72

⇒ a + b + c = 72

⇒ 18 + 24 + c = 72 [a = 18, b = 24 …. Given]

⇒ c = 30

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36(36-18)(36-24)(36-30)}$

$= \sqrt{36 \times 18 \times 12 \times 6}$

= 216 sq. units

Q.11 In a ΔABC, if $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in

Correct option: (A)

$\frac{\sin A}{\sin C} = \frac{\sin A\cos B - \cos A\sin B}{\sin B\cos C - \cos B\sin C}$

$\Rightarrow \frac{a}{c} = \frac{a\cos B - b\cos A}{b\cos C - c\cos B}$

⇒ ab cos C – ac cos B = ac cos B – bc cos A

⇒ ab cos C + bc cos A = 2ac cos B

⇒

$\frac{a^2+b^2-c^2}{2} + \frac{b^2+c^2-a^2}{2} = \frac{c^2+a^2-b^2}{1}$

⇒ $b^2 = c^2 + a^2 - b^2$ ⇒ $b^2 = \frac{c^2+a^2}{2}$

⇒ a^2, b^2, c^2 are in A.P.

Q.12 tan 9° – tan 27° – tan 63° + tan 81° =

Correct option: (C) 4

tan 9° – tan 27° – tan 63° + tan 81°

= tan 9° – tan 27° – cot 27° + cot 9° …[∵ tan (90° – θ) = cot θ]

= (tan 9° + cot 9°) – (tan 27° + cot 27°)

$= \frac{1}{sin9^o cos9^o} - \frac{1}{sin27^o cos27^o}$

$= \frac{2}{sin18^o} - \frac{2}{sin54^o}$ …[∵ sin 2θ = 2 sin θ cos θ]

$= 2\left\{\frac{\sin\ 54^o - \sin\ 18^o}{\sin\ 18^o\ \sin\ 54^o}\right\}$

$= 2.\frac{2\ \cos\ 36^o\ \sin\ 18^o}{\sin\ 18^o \sin\ 54^o}$

$= \frac{4\cos 36^\circ}{\cos 36^\circ}$

= 4

Q.13 If θ + ϕ = α and tan θ = k tanϕ (where k > 1), then the value of sin(θ – ϕ) is

Correct option: (C)

Given, tan θ = k tan ϕ

$\Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{k}{1}$

$\Rightarrow \frac{\tan\theta + \tan\phi}{\tan\theta - \tan\phi} = \frac{k+1}{k-1}$ …[Using componendo dividendo]

$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\phi}{\cos\phi}}{\frac{\sin\theta}{\cos\theta} - \frac{\sin\phi}{\cos\phi}} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\sin\theta\cos\phi + \cos\theta\sin\phi}{\sin\theta\cos\phi - \cos\theta\sin\phi} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\sin(\theta+\phi)}{\sin(\theta-\phi)} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\sin\alpha}{\sin(\theta-\phi)} = \frac{k+1}{k-1}$ …[θ + ϕ = α (given)]

⇒ sin(θ – ϕ) = $\frac{k-1}{k+1}$ (sin α)

Q.14 If a cos 2θ + b sin 2θ = c has α and β as its roots, then the value of tan α + tan β is

Correct option: (A)

We have, a cos 2θ + b sin 2θ = c

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$

⇒ a – a $\tan^2$ θ + 2b tan θ = c + c $\tan^2$ θ

⇒ – (a + c) $\tan^2$ θ + 2b tan θ + (a – c) = 0

∴ tan α + tan β = $-\frac{2b}{-(c+a)} = \frac{2b}{c+a}$

Q.15 In a triangle ABC with usual notations if, $\cot\frac{A}{2} = \frac{b+c}{a}$, then the triangle ABC is

Correct option: (C)

$\cot\frac{A}{2} = \frac{b+c}{a}$

$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{k\ (\sin B + \sin C)}{k\ \sin A}$ …[By sine rule]

$$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2} = \frac{\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2} = \cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2} = \frac{B-C}{2} \Rightarrow A = B - C \quad \ldots(i)$$

Now, A + B + C = π

⇒ B – C + B + C = π …[From (i)]

$\Rightarrow 2B = \pi \Rightarrow B = \frac{\pi}{2}$

Q.16 $\frac{\sin 3\theta\cos 4\theta - \sin\theta\cos 2\theta}{\sin 4\theta\sin\theta + \cos 6\theta\cos\theta} =$

Correct option: (B)

$\frac{\sin 3\theta\cos 4\theta - \sin\theta\cos 2\theta}{\sin 4\theta\sin\theta + \cos 6\theta\cos\theta}$

$= \frac{2\sin 3\theta\cos 4\theta - 2\sin\theta\cos 2\theta}{2\sin 4\theta\sin\theta + 2\cos 6\theta\cos\theta}$

$= \frac{\sin 7\theta + \sin(-\theta) - \sin 3\theta - \sin(-\theta)}{\cos 3\theta - \cos 5\theta + \cos 7\theta + \cos 5\theta}$

$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 7\theta}$

$= \frac{2\cos 5\theta\ \sin 2\theta}{2\cos 5\theta\ \cos 2\theta}$

= tan 2θ

Q.17 If α + β + γ = 2π, then

Correct option: (A)

We have, α + β + γ = 2 π

$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\Rightarrow \tan\left(\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}\right) = \tan\pi = 0$

$\Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2} = 0$

$\Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

Q.18 If $\sin 18° = \frac{\sqrt{5}-1}{4}$, then $\cos^2 48° - \sin^2 12°$ has the value

Correct option: (C)

$\cos^2 A - \sin^2 B = \cos(A + B)\cdot\cos(A - B)$

$\therefore\ \cos^2 48° - \sin^2 12° = \cos(60°)\cdot\cos(36°)$

$= \frac{1}{2}\cdot\left(1 - 2\sin^2\frac{36}{2}\right)$

$= \frac{1}{2}(1 - 2\sin^2 18°)$

$= \frac{1}{2}\left[1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2\right]$

$= \frac{\sqrt{5}+1}{8}$

Q.19 Which of the following numbers is/are rational?

Correct option: (C)

$\sin 15° = \sin(45° - 30°) = \frac{\sqrt{3}-1}{2\sqrt{2}}$ = irrational

$\cos 15° = \cos(45° - 30°) = \frac{\sqrt{3}+1}{2\sqrt{2}}$ = irrational

$\sin 15°\cos 15° = \frac{1}{2}(2\sin 15°\cos 15°)$

$= \frac{1}{2}\sin 30° = \frac{1}{2}\cdot\frac{1}{2}$

$= \frac{1}{4}$ = rational

$\sin 15°\cos 75° = \sin 15°\sin 15°$

$= \sin^2 15°$

$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$= \frac{4-2\sqrt{3}}{8}$ = irrational

Q.20 Solve for x $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $x > 0$

Correct option: (C)

$\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \frac{1}{2}\cdot\theta$ ....[Put $x = \tan\theta$]

$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right] = \frac{\theta}{2}$

$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\theta}{2} \Rightarrow \frac{\pi}{4} - \theta = \frac{\theta}{2}$

$\Rightarrow \theta = \frac{\pi}{6} = \Rightarrow \tan^{-1}x = \frac{\pi}{6} \Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Q.21 If $\tan\alpha = \frac{1}{5}$, $\tan\beta = \frac{1}{239}$, then the value of tan(4α – β) is

Correct option: (C)

$\tan 2\alpha = \tan(\alpha + \alpha)$

$= \frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{1}{25}}$

$\therefore \tan 2\alpha = \frac{5}{12}$

$\tan 4\alpha = \tan(2\alpha + 2\alpha)$

$= \frac{\frac{5}{12}+\frac{5}{12}}{1-\frac{25}{144}}$

$= \frac{120}{119}$

$\therefore \tan(4\alpha - \beta) = \frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}$

$= \frac{120\times 239 - 119}{119\times 239 + 120}$

$= \frac{(119+1)\,239-119}{119\times 239+120}$

$= \frac{119\times 239+(239-119)}{119\times 239+120}$

$= 1$

Q.22 The value of

$\cot\left[\sum_{n=1}^{100}\cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right]$ **is**

Correct option: (A)

$\cot\left[\sum_{n=1}^{100}\cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right]$

$=$

$\cot\left[\sum_{n=1}^{100}\cot^{-1}(1+2+4+6+\ldots+2n)\right]$

$= \cot\left\{\sum_{n=1}^{100}\cot^{-1}[1+n(n+1)]\right\}$

$= \cot\left\{\sum_{n=1}^{100}\tan^{-1}\left[\frac{1}{1+n(n+1)}\right]\right\}$

$= \cot\left\{\sum_{n=1}^{100}\tan^{-1}\left[\frac{n+1-n}{1+n(n+1)}\right]\right\}$

$= \cot\left\{\sum_{n=1}^{100}\left[\tan^{-1}(n+1) - \tan^{-1} n\right]\right\}$

$= \cot[(\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + \ldots + (\tan^{-1}101 - \tan^{-1}100)]$

$= \cot(\tan^{-1}101 - \tan^{-1}1)$

$= \cot\left[\tan^{-1}\left(\frac{101-1}{1+101}\right)\right]$

$= \cot\left[\tan^{-1}\left(\frac{100}{102}\right)\right]$

$= \cot\left[\tan^{-1}\left(\frac{50}{51}\right)\right]$

$= \cot\left[\cot^{-1}\left(\frac{51}{50}\right)\right]$

$= \frac{51}{50}$

Q.23 The number of solutions in [0, 2π] of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is

Correct option: (D)

$16^{\sin^2 x} + 16^{\cos^2 x} = 10$

$16^{\sin^2 x} + 16^{1-\sin^2 x} = 10$

$16^{\sin^2 x} + \frac{16}{16^{\sin^2 x}} = 10$

Let $16^{\sin^2 x} = t$

$\therefore\ t + \frac{16}{t} = 10$

$\therefore\ t^2 - 10t + 16 = 0$

$\Rightarrow t = 2$ and $t = 8$

Now, $16^{\sin^2 x} = 2$ and $16^{\sin^2 x} = 8$

$2^{4\sin^2 x} = 2^1$ and $2^{4\sin^2 x} = 2^3$

$\therefore\ 4\sin^2 x = 1$ and $4\sin^2 x = 3$

$\therefore\ \sin^2 x = \frac{1}{4}$ and $\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{1}{2}$ and $\sin x = \pm\frac{\sqrt{3}}{2}$

$\therefore\ x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ and $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ number of solutions = 8.

Q.24 $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ **=**

Correct option: (B)

$\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6} + \frac{\pi}{6} + \frac{2\pi}{3} = \pi$

Q.25 sec 2θ – tan 2θ =

Correct option: (D)

$\sec 2\theta - \tan 2\theta = \frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}$

$= \frac{1-\sin 2\theta}{\cos 2\theta}$

$= \frac{(\cos\theta - \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$

$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$

$= \tan\left(\frac{\pi}{4} - \theta\right)$

Q.26 If $0 \le x \le \pi$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

Then x takes the value

Correct option: (A)

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ …(i)

Let $y = 81^{\sin^2 x}$

$\therefore\ 81^{\cos^2 x} = 81^{(1-\sin^2 x)} = \dfrac{81}{81^{\sin^2 x}} = \dfrac{81}{y}$

$\therefore$ Equation (i) becomes

$y + \dfrac{81}{y} = 30$

$\therefore\ y^2 - 30y + 81 = 0$

$\therefore\ (y - 27)(y - 3) = 0$

$\therefore\ y = 27$ or $y = 3$

$\therefore\ 81^{\sin^2 x} = 27$ or $81^{\sin^2 x} = 3$

$\therefore\ 3^{4\sin^2 x} = 3^3$ or $3^{4\sin^2 x} = 3^1$

$\therefore\ 4\sin^2 x = 3$ or $4\sin^2 x = 1$

$\therefore\ \sin x = \dfrac{\sqrt{3}}{2}, \dfrac{1}{2}$ …[$\because x \in [0, \pi]$]

$\therefore\ x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ …[$\because x \in [0, \pi]$]

Q.27 The value of $2\cot^{-1}\dfrac{1}{2} - \cot^{-1}\dfrac{4}{3}$ is

Correct option: (D)

$2\cot^{-1}\left(\dfrac{1}{2}\right) - \cot^{-1}\left(\dfrac{4}{3}\right)$

$= 2\tan^{-1}(2) - \cot^{-1}\left(\dfrac{4}{3}\right)$

… $\left[\because \cot^{-1}(x) = \tan^{-1}\left(\dfrac{1}{x}\right),\ \text{if } x > 0\right]$

$= \pi + \tan^{-1}\left(\dfrac{4}{-3}\right) - \cot^{-1}\left(\dfrac{4}{3}\right)$ ….[$\because 2\tan^{-1}(x) = \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, if $x > 1$]

$= \pi - \tan^{-1}\left(\dfrac{4}{3}\right) - \cot^{-1}\left(\dfrac{4}{3}\right)$

$= \pi - \left[\tan^{-1}\left(\dfrac{4}{3}\right) + \cot^{-1}\left(\dfrac{4}{3}\right)\right]$

$= \pi - \dfrac{\pi}{2}$

$= \dfrac{\pi}{2}$

Q.28 If A + B + C = 180°, then the value of $\cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2}$ will be

Correct option: (C)

A + B + C = 180°

$\Rightarrow \cot\left(\dfrac{A}{2} + \dfrac{B}{2}\right) = \cot\left(90^o - \dfrac{C}{2}\right)$

$\Rightarrow \dfrac{\cot\frac{A}{2}\,.\,\cot\frac{B}{2} - 1}{\cot\frac{B}{2} + \cot\frac{A}{2}} = \tan\dfrac{C}{2} = \dfrac{1}{\cot\frac{C}{2}}$

$\Rightarrow \left(\cot\dfrac{A}{2}\cot\dfrac{B}{2} - 1\right)\cot\dfrac{C}{2} = \cot\dfrac{B}{2} + \cot\dfrac{A}{2}$

$\Rightarrow \cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2} = \cot\dfrac{C}{2} + \cot\dfrac{B}{2} + \cot\dfrac{A}{2}$

Q.29 $\cos^2\left(\dfrac{\pi}{6} + \theta\right) - \sin^2\left(\dfrac{\pi}{6} - \theta\right) =$

Correct option: (A)

$\cos^2\left(\dfrac{\pi}{6} + \theta\right) - \sin^2\left(\dfrac{\pi}{6} - \theta\right)$

$= \cos\left(\dfrac{\pi}{6} + \theta + \dfrac{\pi}{6} - \theta\right)\cos\left(\dfrac{\pi}{6} + \theta - \dfrac{\pi}{6} + \theta\right)$

…[$\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$]

$= \cos\dfrac{2\pi}{6}\cos 2\theta = \dfrac{1}{2}\cos 2\theta$

Q.30 If $\cos x + \cos y = -\cos\alpha$ and $\sin x + \sin y = -\sin\alpha$, then $\cot\left(\dfrac{x+y}{2}\right) =$

Correct option: (A)

$\cos x + \cos y = -\cos\alpha$ …[Given]

$\Rightarrow 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = -\cos\alpha$ …(i)

$\sin x + \sin y = -\sin\alpha$ …[Given]

$\Rightarrow 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = -\sin\alpha$ …(ii)

Dividing equation (i) by equation (ii), we get

$\cot\left(\dfrac{x+y}{2}\right) = \cot\alpha$

Q.31 cos 38° cos 8° + sin 38° sin 8° is equal to

Correct option: (A)

cos 38° cos 8° + sin 38° sin 8°

= cos (38° – 8°) = cos 30°

Q.32 sin 12° sin 24° sin 48° sin 84° =

Correct option: (A)

sin12° sin 24° sin 48° sin 84°

$= \frac{1}{4}$ (2 sin 12° sin 48°) (2 sin 24° sin 84°)

$= \frac{1}{2}$(cos 36° – cos 60°) (cos 60° – cos 108°)

$= \frac{1}{4}\left(\cos 36^o - \frac{1}{2}\right)\left(\frac{1}{2} + \sin\ 18^o\right)$

$= \frac{1}{4}\left\{\frac{1}{4}(\sqrt{5}+1) - \frac{1}{2}\right\}\left\{\frac{1}{2} + \frac{1}{4}(\sqrt{5}-1)\right\}$

$= \frac{1}{16}$

Consider, cos 20° cos 40° cos 60° cos 80°

$= \frac{1}{2}$[cos (60° – 20°) cos 20° cos (60° + 20°)]

$= \frac{1}{2}\left[\frac{1}{4}\cos\ 3\ (20^o)\right]$ ...

$\left[\because\ \cos\theta\ \cos(60^\circ - \theta)\ \cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta\right]$

$= \frac{1}{8}\cos 60° = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16}$

∴ option [A] is the correct answer.

Q.33 $3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ =$

Correct option: (B)

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Let $\theta = 10^\circ$

$\therefore\ \tan 30^\circ = \dfrac{3\tan 10^\circ - \tan^3 10^\circ}{1 - 3\tan^2 10^\circ}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{3\tan 10^\circ - \tan^3 10^\circ}{1 - 3\tan^2 10^\circ}$

$\Rightarrow \dfrac{1}{\sqrt{3}}\ (1 - 3\tan^2 10^\circ)^2 = 3\tan 10^\circ - \tan^3 10^\circ$

Squaring on both sides, we get

$\frac{1}{3}\ (1 - 3\tan^2 10^\circ)^2 = (3\tan 10^\circ - \tan^3 10^\circ)^2$

$\Rightarrow \frac{1}{3}\ (1 - 6\tan^2 10^\circ + 9\tan^4 10^\circ) = 9\tan^2 10^\circ -$ $6\tan^4 10^\circ + \tan^6 10^\circ$

$\Rightarrow 1 - 6\tan^2 10^\circ + 9\tan^4 10^\circ = 27\tan^2 10^\circ - 18$ $\tan^4 10^\circ + 3\tan^6 10^\circ$

$\Rightarrow 3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1$

Q.34 In a triangle ABC, with usual notations, $\cot\left(\frac{A+B}{2}\right)\cdot\tan\left(\frac{A-B}{2}\right) =$

Correct option: (B)

By Napier's analogy,

$\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\left(\frac{C}{2}\right)$

$\cot\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right)$

$= \left(\frac{a-b}{a+b}\right)\cot\left(\frac{C}{2}\right)\cot\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$= \left(\frac{a-b}{a+b}\right)\cot\left(\frac{C}{2}\right)\tan\left(\frac{C}{2}\right) = \frac{a-b}{a+b}$

Q.35 $\dfrac{\tan\left(\frac{\theta}{2}\right) + \cot\left(\frac{\theta}{2}\right)}{\cot\left(\frac{\theta}{2}\right) - \tan\left(\frac{\theta}{2}\right)} =$

Correct option: (C)

$\dfrac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \dfrac{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$

$= \dfrac{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$

$= \dfrac{1}{\cos\theta} = \sec\theta$

Q.36 The value of sin $\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$ **+ cos** **$(\tan^{-1} 2\sqrt{2})$ =**

Correct option: (B)

$\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos\,[\tan^{-1}(2\sqrt{2})]$

$= \sin\left[\tan^{-1}\frac{2/3}{1 - 1/9}\right] + \cos\,[\tan^{-1}(2\sqrt{2})]$

$= \sin\,[\,\tan^{-1}\frac{3}{4}] + \cos\,[\tan^{-1} 2\sqrt{2}]$

$=$

$\sin\left[\sin^{-1}\frac{\left(\frac{3}{4}\right)}{\sqrt{1+\left(\frac{3}{4}\right)^2}}\right] + \cos\left[\cos^{-1}\frac{1}{\sqrt{1+\left(2\sqrt{2}\right)^2}}\right]$

$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$

Q.37 **If** $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$**,**

then the value of ***x*** **is**

Correct option: (D)

$\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right) \ldots$

$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right) \ldots$

$\left[\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}\right]$

$\Rightarrow x = 3$

Q.38 **With usual notations in ΔABC, a = 3, c = 2 and sin C =** $\frac{2}{3}$**, then ∠A =**

Correct option: (B)

Using sine Rule,

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{3}{\sin A} = \frac{2}{\frac{2}{3}}$

$\Rightarrow \sin A = 1$

$\Rightarrow A = \frac{\pi}{2}$

Q.39 **tan 50° is equal to**

Correct option: (B)

tan 40° = tan (50° – 10°) = $\frac{\tan 50^o - \tan 10^o}{1+\tan 50^o \tan 10^o}$

⇒ tan 40° + tan 40° tan 50° tan 10° = tan 50° – tan 10°

⇒ tan 40° + tan 10° = tan 50° – tan 10° ….[∵ tan 50° = cot 40°]

⇒ 2 tan 10° + tan 40° = tan 50°

Q.40 **The number of integral values of k for which the equestion 7cos** ***x*** **+ 5 sin** ***x*** **= 2k + 1 has a solution is**

Correct option: (C)

$-\sqrt{7^2+5^2} \leq (7\cos x + 5\sin x) \leq \sqrt{7^2+5^2}$

$\Rightarrow -\sqrt{74} \leq (7\cos x + 5\sin x) \leq \sqrt{74}$

$\Rightarrow -8.6 \leq 2k + 1 \leq 8.6$

$\Rightarrow -4.8 \leq 2k \leq 3.8$

Integral values of k are – 4, – 3, – 2, – 1, 0, 1, 2,3

Number of integral values of k = 8

Q.41 **cos 2θ is not equal to**

Correct option: (C)

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

Q.42 **If** $\cot^{-1}(3) + \cot^{-1}(4) + \cot^{-1}(7) = \cot^{-1} x$**, then the value of** ***x*** **is**

Correct option: (B)

$\cot^{-1}(3) + \cot^{-1}(4) + \cot^{-1}(7) = \cot^{-1} x$

⇒

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\ldots\left[\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x\right]$

⇒

$\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{7}{11}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{7}{11}+\frac{1}{7}}{1-\left(\frac{7}{11}\right)\left(\frac{1}{7}\right)}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{60}{70}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow x = \frac{7}{6}$

Q.43 $\frac{sin(B+A)+cos(B-A)}{sin(B-A)+cos(B+A)}$ **=**

Correct option: (B)

$$\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}$$

$$=\frac{\sin(B+A)+\sin\{(90^o-(B-A)\}}{\sin(B-A)+\sin\{(90^o-(A+B)\}}$$

$$=\frac{2\sin(A+45^o)\cos(45^o-B)}{2\sin(45^o-A)\cos(45^o-B)}$$

$$=\frac{\sin(A+45^o)}{\sin(45^o-A)}=\frac{\cos A+\sin A}{\cos A-\sin A}$$

Q.44 In ΔABC, a = 2cm, b = 3cm and c = 4 cm, then angle A is [MP PET 2002]

Correct option: (C)

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{9+16-4}{2\times3\times4}=\frac{7}{8}$$

$$\Rightarrow A=\cos^{-1}\left(\frac{7}{8}\right)$$

Q.45 If 8 cos 2θ + 8 sec 2θ = 65, $0<\theta<\frac{\pi}{2}$, then the value of 4 cos 4θ is equal to

Correct option: (B)

8 cos 2θ + 8 sec 2θ = 65

⇒ $8\cos^2 2\theta+8=65\cos 2\theta$

⇒ $8\cos^2 2\theta-65\cos 2\theta+8=0$

⇒ (cos 2θ – 8) (8 cos 2θ – 1) = 0

Since, cos 2θ ∈[–1, 1]

∴ $\cos 2\theta=\frac{1}{8}$

Now, $4\cos 4\theta=4(2\cos^2 2\theta-1)$

$$=4\left[2\left(\frac{1}{8}\right)^2-1\right]=-\frac{31}{8}$$

Q.46 $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=$

Correct option: (A)

$$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right]$$

$$=\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]=\frac{\pi}{4}-\frac{x}{2}$$

Q.47 $\sin\left(3\sin^{-1}\left(\frac{2}{5}\right)\right)=$

Correct option: (A)

$\sin\left(3\sin^{-1}\left(\frac{2}{5}\right)\right)=\sin 3\theta$,

Where $\theta=\sin^{-1}\left(\frac{2}{5}\right)$ ...

$$\left[\theta=\sin^{-1}\left(\frac{2}{5}\right),\ \sin\theta=\frac{2}{5}\right]$$

$=3\sin\theta-4\sin^3\theta$

$=3\left(\frac{2}{5}\right)-4\left(\frac{2}{5}\right)^3$ ...$\left[\theta=\sin^{-1}\left(\frac{2}{5}\right),\ \sin\theta=\frac{2}{5}\right]$

$$=\frac{6}{5}-\frac{32}{125}=\frac{118}{125}$$

Q.48 If $|\tan A|<1$ and $|A|$ is acute, then $\frac{\sqrt{1+\sin 2A}+\sqrt{1-\sin 2A}}{\sqrt{1+\sin 2A}-\sqrt{1-\sin 2A}}$ is equal to

Correct option: (C)

|tan A| < 1 and |A| is acute.

$\therefore -\frac{\pi}{4}<A<\frac{\pi}{4}\Rightarrow \cos A>\sin A$

$$\therefore \frac{\sqrt{1+\sin 2A}+\sqrt{1-\sin 2A}}{\sqrt{1+\sin 2A}-\sqrt{1-\sin 2A}}$$

$$=\frac{\sqrt{(\cos A+\sin A)^2}+\sqrt{(\cos A-\sin A)^2}}{\sqrt{(\cos A+\sin A)^2}-\sqrt{(\cos A-\sin A)^2}}$$

$$=\frac{|\cos A+\sin A|+|\cos A-\sin A|}{|\cos A+\sin A|-|\cos A-\sin A|}$$

$$=\frac{(\cos A+\sin A)+(\cos A-\sin A)}{(\cos A+\sin A)-(\cos A-\sin A)}=\cot A$$

Q.49 If $\tan\frac{A}{2}=\frac{3}{2}$, then $\frac{1+\cos A}{1-\cos A}=$

Correct option: (D)

Given that, $\tan\frac{A}{2}=\frac{3}{2}$

$$\therefore \frac{1+\cos A}{1-\cos A}=\frac{2\cos^2\frac{A}{2}}{2\sin^2\frac{A}{2}}=\cot^2\frac{A}{2}$$

$$=\left(\frac{2}{3}\right)^2=\frac{4}{9}$$

Q.50 $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ is equal to

Correct option: (A) $\cot\left(7\frac{1}{2}\right)^\circ$

$\cot\frac{A}{2} = \frac{1+\cos A}{\sin A}$

Putting $A = \left(7\frac{1}{2}\right)^{\circ}$, we get

$\cot\left(7\frac{1}{2}\right)^{\circ} = \frac{1+\cos 15^{\circ}}{\sin 15^{\circ}}$

$= \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$

Q.51 The value of

$$\sin^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) \textbf{ is,}$$

Correct option: (D)

$$\sin^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{2}$$

Q.52 In a triangle ABC, with usual notations, 3b = a + c, then $\cot\frac{A}{2}\cdot\cot\frac{C}{2} =$

Correct option: (B)

$$\cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{C}{2}\right)$$

$$= \frac{1}{\tan\left(\frac{A}{2}\right).\tan\left(\frac{C}{2}\right)}$$

$$= \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \times \frac{s(s-c)}{(s-a)(s-b)}} \ldots$$

[By half angle formulae]

$$= \sqrt{\frac{s^2}{(s-b)^2}}$$

$$\therefore \cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{C}{2}\right) = \frac{s}{s-b} \qquad \ldots\text{(i)}$$

2s = a + b + c

⇒ 2s = 3b + b ...[a + c = 3b (given)]

⇒ 2s = 4b

⇒ s = 2b

Substituting s = 2b in equation (i), we get

$$\cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{C}{2}\right) = \frac{2\text{b}}{2\text{b}-b} = \frac{2\text{b}}{b} = 2$$

Q.53 In triangle ABC, A = 30°, b = 8, a = 6, where B = $\sin^{-1}x$, then x =

Correct option: (C)

By sine rule, we have

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\Rightarrow \sin \text{B} = \frac{b\sin A}{a} = \frac{8\sin 30^{o}}{6} = \frac{2}{3}$$

⇒ $\sin(\sin^{-1} x) = \frac{2}{3}$ ...[∵ B = $\sin^{-1}x$ (given)]

⇒ $x = \frac{2}{3}$

Q.54 The area of the triangle ABC is $10\sqrt{3}$ cm^2, angle B is 60° and its perimeter is 20 cm, then l(AC) =

Correct option: (D)

Area of triangle = $\frac{1}{2}$ac sin B

⇒ $10\sqrt{3} = \frac{1}{2}$ac × sin60°

⇒ ac = 40

Using cosine Rule,

$$\cos \text{B} = \frac{c^2+a^2-b^2}{2\text{ac}}$$

$$\Rightarrow \cos 60^{\circ} = \frac{c^2+a^2-b^2}{2\text{ac}}$$

⇒ $\frac{1}{2}$ × 2ac = $(c + a)^2 - 2ac - b^2$

⇒ 40 = $(20 - b)^2 - 80 - b^2$ …

$$\begin{bmatrix} \text{p} = 20 \\ \therefore \text{a} + \text{b} + \text{c} = 20 \\ \text{a} + \text{c} = 20 \ - \ \text{b} \end{bmatrix}$$

⇒ 40 = 400 – 40b + b^2 – 80 – b^2

⇒ 40b = 280

⇒ b = 7

Q.55 The value of

$$2\sin^{-1}\left(\frac{1}{2}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) \textbf{ is}$$

Correct option: (B)

$$2\,\mathbf{sin}^{-1}\left(\frac{1}{2}\right) + \mathbf{cot}^{-1}\left(\frac{1}{\sqrt{3}}\right) = 2 \times \frac{\pi}{6} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Q.56 In Δ ABC, ∠ A = $\frac{\pi}{2}$, then $\cos^2$ B + $\cos^2$ C =

Correct option: (D)

Given, ∠ A = $\frac{\pi}{2}$

In Δ ABC, ∠A + ∠B + ∠C = π

∴ ∠ B + ∠ C = $\frac{\pi}{2}$

⇒ B = $\frac{\pi}{2}$ – C

⇒ $\cos^2 B = \cos^2\left(\frac{\pi}{2} - C\right) = \sin^2 C$

∴ $\cos^2 B + \cos^2 C = \sin^2 C + \cos^2 C = 1$

Q.57 **If $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$, then $\cos\frac{x}{2} =$ _____**

Correct option: (D)

$\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$

∴ $1 + \tan^2 x = \sec^2 x$

∴ $\sec^2 x = \frac{25}{16}$

∴ $\sec x = \frac{-5}{4}$ ...$\left[\because \pi < x < \frac{3\pi}{2}\right]$

∴ $\cos x = \frac{-4}{5}$

∴ $\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$...

$\left[\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}\right]$

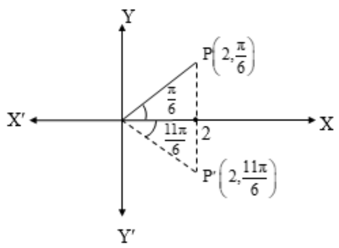
$= -\sqrt{\frac{1}{10}}$

$= \frac{-1}{\sqrt{10}}$

Q.58 **The polar coordinates of P are $\left(2, \frac{\pi}{6}\right)$.**

If Q is the image of P about the X-axis, then the polar coordinates of Q are

Correct option: (B)



Q.59 **$\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} - \cos\frac{6\pi}{7} =$**

Correct option: (D)

$\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} - \cos\frac{6\pi}{7}$

$= \cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{\pi}{7}$ …[∵ cos (π – θ) = – cos θ]

$= 2\left(\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7}\right)$

$= \frac{2\cos\frac{\pi}{14}\left(\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7}\right)}{\cos\frac{\pi}{14}}$

$= \frac{1}{\cos\frac{\pi}{14}}\left(2\cos\frac{\pi}{7}\cos\frac{\pi}{14} - 2\cos\frac{2\pi}{7}\cos\frac{\pi}{14} + 2\cos\frac{3\pi}{7}\cos\frac{\pi}{14}\right)$

$= \frac{1}{\cos\frac{\pi}{14}}\left[\left(\cos\frac{3\pi}{14} + \cos\frac{\pi}{14}\right) - \left(\cos\frac{5\pi}{14} + \cos\frac{3\pi}{14}\right) + \left(\cos\frac{7\pi}{14} + \cos\frac{5\pi}{14}\right)\right]$

$= \frac{1}{\cos\frac{\pi}{14}}\left(\cos\frac{\pi}{14}\right)$

= 1

Q.60 **$\tan\left(\frac{p\pi}{4}\right) = \cot\left(\frac{q\pi}{4}\right)$ if**

Correct option: (D)

$\tan\left(\frac{p\pi}{4}\right) = \cot\left(\frac{q\pi}{4}\right)$

$\Rightarrow \tan\left(\frac{p\pi}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{q\pi}{4}\right)$ ...

$\left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$

$\Rightarrow \frac{p\pi}{4} = n\pi + \frac{\pi}{2} - \frac{q\pi}{4}$ ...[∵ tan θ = tan α ⇒ θ = nπ + α]

$\Rightarrow \frac{p}{4} = n + \frac{1}{2} - \frac{q}{4}$

$\Rightarrow \dfrac{p+q}{4} = \dfrac{2n+1}{2}$

$\Rightarrow$ p + q = 2(2n + 1)

Q.61 If α + β + γ = π, then the expression $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ has the value

Correct option: (D)

$\sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= \sin^2\alpha + \sin(\beta+\gamma)\sin(\beta-\gamma)$

$= \sin^2\alpha + \sin(\pi-\alpha)\sin(\beta-\gamma)$

$= \sin\alpha\,[\sin\alpha + \sin(\beta-\gamma)]$

$= \sin\alpha\,[\sin(\beta+\gamma) + \sin(\beta-\gamma)]$

$= 2\sin\alpha\sin\beta\cos\gamma$

Q.62 The value of $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$ is

Correct option: (D)

$$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right) - \frac{\pi}{4}\right] \quad \ldots$$

$$\left[2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{5}{12}\right) - \frac{\pi}{4}\right]$$

$$= \frac{\tan\left(\tan^{-1}\left(\frac{5}{12}\right)\right) - \tan\frac{\pi}{4}}{1+\tan\left(\tan^{-1}\left(\frac{5}{12}\right)\right)\tan\frac{\pi}{4}}$$

$$= \frac{\frac{5}{12}-1}{1+\frac{5}{12}}$$

$$= \frac{-7}{17}$$

Q.63 $\dfrac{\tan\theta + \cot\theta}{\cot\theta - \tan\theta}$ =

Correct option: (C)

$$\frac{\tan\theta+\cot\theta}{\cot\theta-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\cos 2\theta} = \sec 2\theta$$

Q.64 In a triangle ABC, with usual notations, if $a = 5, \mathrm{b} = 4, \cos(\mathrm{A}-B) = \dfrac{31}{32}$, then $c =$

Correct option: (A)

$$\tan\left(\frac{A-B}{2}\right) = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \sqrt{\frac{1-\left(\frac{31}{32}\right)}{1+\left(\frac{31}{32}\right)}}$$

$$\Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{1}{\sqrt{63}}$$

$$\Rightarrow \frac{1}{9}\cot\frac{C}{2} = \frac{1}{\sqrt{63}}$$

$$\Rightarrow \tan\frac{C}{2} = \frac{\sqrt{7}}{3}$$

Now, $\cos C = \dfrac{1-\tan^2\left(\frac{C}{2}\right)}{1+\tan^2\left(\frac{C}{2}\right)}$

$\Rightarrow \cos C = \dfrac{1-\left(\frac{7}{9}\right)}{1+\left(\frac{7}{9}\right)} = \dfrac{1}{8}$

By using cosine rule, we get

$c^2 = a^2 + b^2 - 2ab\cos C$

$\Rightarrow c^2 = 25 + 16 - 40 \times \dfrac{1}{8} = 36 \Rightarrow c = 6$

Q.65 With usual notation, in triangle ABC, $m\angle A = 30^\circ$ then the value of $\left(1+\dfrac{a}{c}+\dfrac{b}{c}\right)\left(1+\dfrac{c}{b}-\dfrac{a}{b}\right)$ is equal to

Correct option: (B)

$$\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)$$

$$= \left(\frac{\mathrm{c}+\mathrm{a}+\mathrm{b}}{c}\right)\left(\frac{\mathrm{b}+\mathrm{c}-a}{b}\right)$$

$$= \frac{1}{\mathrm{bc}}\left[(\mathrm{b}+\mathrm{c})^2 - a^2\right]$$

$$= \frac{1}{\mathrm{bc}}(b^2 + c^2 + 2bc - a^2)$$

$$= \frac{b^2+c^2-a^2}{\mathrm{bc}} + 2$$

$$= 2\left(\frac{b^2+c^2-a^2}{2\mathrm{bc}}\right) + 2$$

= 2 cos A + 2 …[By cosine rule]

= 2 cos 30° + 2

= $\sqrt{3}$ + 2

Q.66 If $4\sin^{-1}x + 6\cos^{-1}x = 3\pi$, then x =

Correct option: (B)

$4\sin^{-1}x + 6\cos^{-1}x = 3\pi$

$\Rightarrow 4\sin^{-1}x + 4\cos^{-1}x + 2\cos^{-1}x = 3\pi$

$\Rightarrow 4(\sin^{-1}x + \cos^{-1}x) + 2\cos^{-1}x = 3\pi$

$\Rightarrow 4 \times \frac{\pi}{2} + 2\cos^{-1}x = 3\pi$ …[$\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$]

$\Rightarrow 2\cos^{-1}x = \pi$

$\Rightarrow x = \cos\frac{\pi}{2}$

$\Rightarrow x = 0$

Q.67 In ΔABC, with usual notations, if a, b, c are in A.P. then

$$a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right)$$

Correct option: (C)

Since, a, b, c are in A. P.,

$\therefore 2b = a + c$

$$a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right)$$

$$= \frac{a\,(1 + \cos C)}{2} + \frac{c\,(1 + \cos A)}{2}$$

$$= \frac{a + c + a\cos C + c\cos A}{2}$$

$$= \frac{a + c + b}{2}$$

$$= \frac{2b + b}{2}$$

$$= \frac{3b}{2}$$

Q.68 $\frac{sin85^o - sin35^o}{cos65^o} =$

Correct option: (C)

$$\frac{sin85^o - sin35^o}{cos65^o} = \frac{2cos60^o sin25^o}{\sin 25^\circ}$$

$$= 2 \times \frac{1}{2}$$

$$= 1$$

Q.69 If $\cos\theta = -\frac{5}{13}$, where $\frac{\pi}{2} < \theta < \pi$, then $\sin 2\theta =$

Correct option: (C)

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{-5}{13}\right)^2$

$= \frac{144}{169}$

$\therefore \sin\theta = \pm\frac{12}{13}$

$\therefore \sin\theta = \frac{12}{13}$ …[∵ θ lies in the second quadrant]

$\therefore \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$

$\therefore \sin 2\theta = -\frac{120}{169}$

Q.70 If $2\cos^2\theta + 3\cos\theta = 2$, then permissible value of cos θ is

Correct option: (C)

$2\cos^2\theta + 3\cos\theta = 2$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = -2$, which is not possible.

$\therefore \cos\theta = \frac{1}{2}$

Q.71 If $0 < x < 1$, then

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{\frac{1}{2}} =$$

Correct option: (D)

Given,

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{\frac{1}{2}}$$

Let $\cot^{-1}x = \theta$

$\therefore x = \cot\theta$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{1+x^2}}$ …(i)

$\Rightarrow \cos\theta = \frac{x}{\sqrt{1+x^2}}$ …(ii)

$$\therefore \sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\{x\cos\theta + \sin\theta\}^2 - 1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\left\{x \times \frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right\}^2 - 1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2+1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{\frac{1}{2}}$$

$= \sqrt{1+x^2}\left[\left(\sqrt{1+x^2}\right)^2 - 1\right]^{\frac{1}{2}}$

$= \sqrt{1+x^2}\left(1+x^2-1\right)^{\frac{1}{2}}$

$= x\sqrt{1+x^2}$

Q.72 **If sin A = $\frac{4}{5}$ and cos B = $-\frac{12}{13}$, where A and B lie in first and third quadrant respectively, then cos (A + B) =**

Correct option: (D)

cos (A + B) = cos A cos B – sin A sin B

$= \sqrt{1-\frac{16}{25}}\left(-\frac{12}{13}\right) - \frac{4}{5}\sqrt{1-\frac{144}{169}}$

$= \frac{3}{5}\left(-\frac{12}{13}\right) - \frac{4}{5}\left(-\frac{5}{13}\right)$ …[∵ A lies in first quadrant and B lies in third quadrant]

$= -\frac{16}{65}$

Q.73 **If in a triangle ABC, a = 6 cm, b = 8 cm, c = 10 cm, then the value of sin 2A is**

Correct option: (D)

cos A = $\frac{8^2+10^2-6^2}{2.8.10} = \frac{128}{160} = \frac{4}{5}$

∴ sin A = $\frac{3}{5}$

∴ sin 2A = 2 sin A. cos A = $2.\frac{3}{5}.\frac{4}{5} = \frac{24}{25}$

Q.74 $\frac{\tan\ 5A\ -\ \tan\ 3A}{\tan\ 5A\ +\ \tan\ 3A}$ **=**

Correct option: (C)

$\frac{\tan\ 5A\ -\ \tan\ 3A}{\tan\ 5A\ +\ \tan\ 3A} = \frac{\frac{\sin\ 5A}{\cos\ 5A} - \frac{\sin\ 3A}{\cos\ 3A}}{\frac{\sin\ 5A}{\cos\ 5A} + \frac{\sin\ 3A}{\cos\ 3A}}$

$= \frac{\sin 5A.\cos 3A - \cos 5A.\sin 3A}{\sin 5A.\cos 3A + \cos 5A.\sin 3A}$

$= \frac{\sin\ (5A\ -\ 3A)}{\sin\ (5A\ +\ 3A)} = \frac{\sin\ 2A}{\sin\ 8A}$

Q.75 **The general solution of cot θ + tan θ = 2 is**

Correct option: (A)

cot θ + tan θ = 2

∴ $\frac{1}{\tan\theta}$ + tan θ = 2 ⇒ 1 + $\tan^2$ θ = 2 tan θ

∴ $\frac{2\tan\theta}{1+\tan^2\theta}$ = 1 ⇒ sin 2θ = 1

⇒ 2θ = nπ + $(-1)^n \frac{\pi}{2}$

⇒ θ = $\frac{n\pi}{2} + (-1)^n\frac{\pi}{4}$

Q.76 **The general solution of tan 3*x* = 1 is**

Correct option: (C)

tan 3*x* = 1

∴ tan 3*x* = tan$\frac{\pi}{4}$

⇒ 3*x* = nπ + $\frac{\pi}{4}$

∴ $x = n\left(\frac{\pi}{3}\right) + \frac{\pi}{12}$, n ∈ Z

Q.77 $\frac{1-\sin\theta+\cos\theta}{1-\sin\theta-\cos\theta}$ =

Correct option: (D)

We know, sin θ = 2 sin$\frac{\theta}{2}$· cos$\frac{\theta}{2}$

and cos θ = $2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$

$\frac{1-\sin\theta+\cos\theta}{1-\sin\theta-\cos\theta}$ =

$\frac{1-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}+\left(2\cos^2\frac{\theta}{2}-1\right)}{1-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}-\left(1-2\sin^2\frac{\theta}{2}\right)}$

$= \frac{-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}+2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}+2\sin^2\frac{\theta}{2}}$

$= \frac{-2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)}$

$= -\cot\frac{\theta}{2}$

Q.78 **sin 15° + cos 105° =**

Correct option: (A) 0

sin 15° + cos 105°

= sin 15° + cos (90° + 15°)

= sin 15° – sin 15° …[∵ cos(90° + θ) = – sin θ]

= 0

Q.79 $\tan\frac{\pi}{18} + \tan\frac{7\pi}{36} + \tan\frac{\pi}{18}\cdot\tan\frac{7\pi}{36} =$

Correct option: (B)

$\tan\frac{\pi}{4} = 1$

$\therefore \tan\left(\frac{\pi}{18}+\frac{7\pi}{36}\right) = 1$

$\therefore \frac{\tan\frac{\pi}{18}+\tan\frac{7\pi}{36}}{1-\tan\frac{\pi}{18}\tan\frac{7\pi}{36}} = 1$

$\therefore \tan\frac{\pi}{18} + \tan\frac{7\pi}{36} = 1 - \tan\frac{\pi}{18}\tan\frac{7\pi}{36}$

$\therefore \tan\frac{\pi}{18} + \tan\frac{7\pi}{36} + \tan\frac{\pi}{18}\tan\frac{7\pi}{36} = 1$

Q.80 If α, β, γ ∈ [0, π] and α, β, γ are in A.P., then $\frac{\sin\alpha-\sin\gamma}{\cos\gamma-\cos\alpha}$ is equal to

Correct option: (C)

$$\frac{\sin\alpha-\sin\gamma}{\cos\gamma-\cos\alpha} = \frac{2\cos\frac{\alpha+\gamma}{2}\sin\frac{\alpha-\gamma}{2}}{2\sin\frac{\alpha+\gamma}{2}\sin\frac{\alpha-\gamma}{2}}$$

$$= \cot\frac{\alpha+\gamma}{2}$$

But α, β, γ are in A.P. ⇒ $\frac{\alpha+\gamma}{2} = \beta$

∴ $\frac{\sin\ \alpha-\sin\ \gamma}{\cos\ \gamma-\cos\ \alpha}$ = cot β

Q.81 The value of $\operatorname{cosec}^{-1}\left(\sqrt{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is equal to

Correct option: (A)

$$\operatorname{cosec}^{-1}\left(\sqrt{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

Q.82 If cos 7θ = cos θ – sin 4θ, then the general value of θ is

Correct option: (C)

We have, cos 7θ = cos θ – sin 4θ

⇒ sin 4θ = cos θ – cos 7θ

⇒ sin 4θ = 2 sin(4θ) sin(3θ) …

$$\left[\because \cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)\right]$$

⇒ sin 4θ (1 – 2 sin 3θ) = 0

⇒ sin 4θ = 0 or sin 3θ = $\frac{1}{2}$

⇒ 4θ = nπ or sin 3θ = sin $\frac{\pi}{6}$

⇒ θ = $\frac{n\pi}{4}$ or 3θ = nπ + $(-1)^n\frac{\pi}{6}$

$$\Rightarrow \theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$$

Q.83 If $\sin^{-1}(4x) + \sin^{-1}\left(4\sqrt{3}x\right) = -\frac{\pi}{2}$, then the value of x is

Correct option: (D)

Let $\sin^{-1}(4x) = \alpha$ and $\sin^{-1}(4\sqrt{3}x) = \beta$

⇒ sin α = $4x$ and sin β = $4\sqrt{3}x$

α + β = $\frac{-\pi}{2}$

⇒ β = $\frac{-\pi}{2} - \alpha$

⇒ sin β = $\sin\left(\frac{-\pi}{2} - \alpha\right) = -\sin\left(\frac{\pi}{2} + \alpha\right)$

⇒ sin β = – cos α

⇒ cos α = $-4\sqrt{3}x$

Now, cos α = $\sqrt{1-\sin^2\alpha}$

⇒ $-4\sqrt{3}x = \sqrt{1-16x^2}$

Squaring both sides, we get

$48x^2 = 1 - 16x^2$

⇒ $64x^2 = 1$

⇒ $x^2 = \frac{1}{64} \Rightarrow x = \pm\frac{1}{8}$

For $x = \frac{1}{8}$,

$$\sin^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2} \neq \frac{-\pi}{2}$$

For $x = \frac{-1}{8}$,

$$\sin^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{6} - \frac{\pi}{3} = \frac{-\pi}{2}$$

∴ $x = \frac{-1}{8}$ is the required solution.

Q.84 If cos θ = $\frac{8}{17}$ and θ lies in the 1st quadrant, then the value of cos (30° + θ) + cos (45° – θ) + cos (120° – θ) is

Correct option: (A)

cos θ = $\frac{8}{17}$ and $0 < \theta < \frac{\pi}{2}$

⇒ sin θ = $\sqrt{1-\left(\frac{8}{17}\right)^2}$

$= \frac{15}{17}$

cos (30° + θ) + cos (45° – θ) + cos (120° – θ)

= cos 30° cos θ – sin 30° sin θ + cos 45° cos θ + sin45° sin θ + cos 120° cos θ + sin 120° sin θ

$= \cos\theta\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right) - \sin\theta$

$\left(\frac{1}{2}-\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right)$

$= \frac{8}{17}\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right)+\frac{15}{17}$

$\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right)$

$= \frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$

Q.85 $\frac{1-\tan 2^\circ \cot 62^\circ}{\tan 152^\circ - \cot 88^\circ} =$

Correct option: (B)

$\frac{1-\tan 2^\circ \cot 62^\circ}{\tan 152^\circ - \cot 88^\circ} =$

$\frac{1-\tan 2^\circ \cot 62^\circ}{\tan(90^\circ+62^\circ)-\cot(90^\circ-2^\circ)}$

$= \frac{1-\tan 2^\circ \cot 62^\circ}{-\cot 62^\circ - \tan 2^\circ} =$

$\frac{\tan 62^\circ - \tan 2^\circ}{-(1+\tan 2^\circ \tan 62^\circ)}$

= –tan(62° – 2°)

= – tan 60° = $-\sqrt{3}$

Q.86 If cos q+ cos 7q + cos 3q + cos 5q = 0, then q is

Correct option: (C)

(cos θ + cos 7θ) + (cos 3θ + cos 5θ) = 0

⇒ 2 cos 4θ cos 3θ + 2 cos 4θ cos θ = 0

⇒ 2 cos 4θ (cos 3θ + cos θ) = 0

⇒ 4 cos 4θ cos 2θ cos θ = 0

$\Rightarrow 4\frac{\sin 2^3\theta}{2^3 \sin\theta} = 0$ ….

$\left[\because \cos A \cos 2A \cos 2^2 A \cos 2^3 A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}\right]$

⇒ sin 8θ = 0

⇒ 8θ = nπ

$\Rightarrow \theta = \frac{n\pi}{8}$

Q.87 If A > B and tan A – tan B = *x* and cot B – cot A = *y*, then cot(A – B) =

Correct option: (C)

Given, tan A – tan B = *x*

cot B – cot A = *y*

$\Rightarrow \frac{1}{\tan B}-\frac{1}{\tan A}=y$

$\Rightarrow \frac{\tan A-\tan B}{\tan A\cdot\tan B}=y$

$\Rightarrow \tan A\cdot\tan B = \frac{x}{y}$ …(i)

Now, $\cot(A-B) = \frac{1}{\tan(A-B)}$

$= \frac{1+\tan A\cdot\tan B}{\tan A-\tan B}$

$= \frac{1+\frac{x}{y}}{x}$ …[from (i)]

$= \frac{y+x}{xy}$

$= \frac{1}{x}+\frac{1}{y}$

Q.88 If A, B, C, D are the angles of a cyclic quadrilateral taken in order, then cos A + cos B + cos C + cos D =

Correct option: (C)

Since the quadrilateral ABCD is cyclic, we have A + C = 180° and B + D = 180°

∴ cos A = cos (180° – C) = – cos C …(i)

cos B = cos (180° – D) = –cos D …(ii)

∴ cos A + cos B + cos C + cos D = 0 …[From (i) and (ii)]

Q.89 The general solutions of $\sin^2 x \cdot \sec x = \tan x - \sin x + 1$ is

Correct option: (B)

$\sin^2 x \cdot \sec x = \tan x - \sin x + 1$

$\Rightarrow \frac{\sin^2 x}{\cos x} = \frac{\sin x - \sin x\cos x + \cos x}{\cos x}$

$\Rightarrow \sin^2 x = \sin x - \sin x \cdot \cos x + \cos x$

⇒ sin(sin*x* + cos*x*) = sin*x* + cos*x*

⇒ sin*x*(sin*x* + cos*x*) – (sin*x* + cos*x*) = 0

⇒ (sin*x* – 1) (sin*x* + cos*x*) = 0

$\Rightarrow \sin x = 1$ or $\tan x = -1$

$\therefore\ x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = m\pi + \frac{3\pi}{4}$

Q.90 The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x, has real roots. Then p can take any value in the interval

Correct option: (D)

Given equation is

$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = \cos p - 1$, $b = \cos p$, $c = \sin p$

It has real roots.

$\therefore\ b^2 - 4ac \geq 0$

$\Rightarrow \cos^2 p - 4(\cos p - 1)(\sin p) \geq 0$

$\Rightarrow \cos^2 p - 4\sin p \cos p + 4\sin p \geq 0$

$\Rightarrow \cos^2 p - 4\sin p \cos p + 4\sin^2 p + 4\sin p - 4\sin^2 p \geq 0$

$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$

$\therefore$ $(\cos p - 2\sin p)$ is always positive

$\therefore$ $1 - \sin p \geq 0$ for all values of p, $p \in (0, \pi)$

Q.91 If A + B = 45°, then (cot A – 1) (cot B – 1) =

Correct option: (B)

A + B = 45°

$\Rightarrow \tan(A + B) = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \frac{1}{\cot A} + \frac{1}{\cot B} = 1 - \frac{1}{\cot A \cot B}$

$\Rightarrow \cot A + \cot B = \cot A \cot B - 1$

$\Rightarrow \cot A \cot B - \cot A - \cot B = 1$

$\Rightarrow \cot A \cot B - \cot A - \cot B + 1 = 2$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

Q.92 If tan θ = 2 and θ lies in the third quadrant, then the value of sec θ is

Correct option: (B)

$\sec^2\theta = 1 + (2)^2 = 5$

$\therefore\ \sec\theta = -\sqrt{5}$ …[∵ θ lies in IIIrd quadrant]

Q.93 The number of solutions of cos 2θ = sin θ in (0, 2π) is

Correct option: (C)

$\cos 2\theta = \sin\theta \Rightarrow 1 - 2\sin^2\theta = \sin\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\therefore \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

and $\sin\theta = -1 = \sin\frac{3\pi}{2}$

$\Rightarrow \theta = m\pi + (-1)^m \frac{3\pi}{2}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$\therefore$ number of solutions = 3

Q.94 $\sin^2\left(\sin^{-1}\frac{1}{2}\right) + \tan^2(\sec^{-1}2) + \cot^2(\mathrm{cosec}^{-1}4) =$

Correct option: (A)

Let $\mathrm{cosec}^{-1}4 = \theta \Rightarrow \mathrm{cosec}\,\theta = 4$

$\cot^2\theta = \mathrm{cosec}^2\theta - 1 = (4)^2 - 1 = 15$

$\sin^2\left(\sin^{-1}\frac{1}{2}\right) + \tan^2(\sec^{-1}2) + \cot^2(\mathrm{cosec}^{-1}4)$

$= \sin^2 30° + \tan^2 60° + \cot^2\theta$

$= \left(\frac{1}{2}\right)^2 + \left(\sqrt{3}\right)^2 + 15$

$= \frac{73}{4}$

Q.95 The number of solutions of sin x + sin 3x + sin 5x = 0 in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is

Correct option: (B)

$\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow \sin 5x + \sin x + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$ or $2\cos 2x = -1$

$\Rightarrow 3x = n\pi$ or $\cos 2x = \frac{-1}{2}$

$\Rightarrow x = \frac{n\pi}{3}$ or $\cos 2x = -\cos\frac{\pi}{3}$

$\cos 2x = \cos\left(\pi - \frac{\pi}{3}\right)$

$\cos 2x = \cos\frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3} \quad \ldots\left[\because x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$

Q.96 If the sides of a triangle are in the ratio ($\sqrt{3}$ + 1) : $\left(3+\sqrt{3}\right)$: (2 $\sqrt{3}$ + 2), then the largest angle of the triangle will be

Correct option: (C)

Let the common multiple be *x*.

∴ The sides are ($\sqrt{3}$ + 1) *x*, $\left(3+\sqrt{3}\right)x$, $\left(2\sqrt{3}+2\right)x$

∵ $\left(2\sqrt{3}+2\right)x$ is the largest side.

If θ is the angle opposite to side $\left(2\sqrt{3}+2\right)x$, then

$$\cos\theta = \frac{\left[\left(\sqrt{3}+1\right)x\right]^2 + \left[\left(3+\sqrt{3}\right)x\right]^2 - \left[\left(2\sqrt{3}+2\right)x\right]^2}{2\times\left[\left(\sqrt{3}+1\right)x\right]\times\left[\left(3+\sqrt{3}\right)x\right]}$$

= 0

∴ cosθ = 0 ⇒ θ = 90°

Q.97 If tan θ = $\frac{1}{2}$ and tan ϕ = $\frac{1}{3}$, then tan (2θ + ϕ) is equal to

Correct option: (C)

$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = 1$

tan (2θ + ϕ) = tan [θ + (θ + ϕ)]

$= \frac{\tan\theta + \tan(\theta+\phi)}{1 - \tan\theta\tan(\theta+\phi)}$

$= \frac{\frac{1}{2}+1}{1-\frac{1}{2}.1} = 3$

Q.98 1. With usual notations in ΔABC if $a^2 + b^2 - c^2 = ab$, then measurement angle C is

Correct option: (B)

$a^2 + b^2 - c^2 = ab$

$\Rightarrow \frac{a^2+b^2-c^2}{ab} = 1$

$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

$\Rightarrow \cos C = \frac{1}{2}$

$\Rightarrow C = \frac{\pi}{3}$

Q.99 If cosec θ = $\frac{p+q}{p-q}$, then cot $\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$**=**

Correct option: (B)

Given, cosec θ = $\frac{p+q}{p-q} \Rightarrow \frac{1}{\sin\theta} = \frac{p+q}{p-q}$

By componendo – dividendo, we get

$\frac{1+sin\theta}{1-sin\theta} = \frac{p+q+p-q}{p+q-p+q}$

$\Rightarrow\left\{\frac{\cos\frac{\theta}{2}+sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-sin\frac{\theta}{2}}\right\}^2 = \frac{p}{q}$

$\Rightarrow\left\{\frac{1+tan\frac{\theta}{2}}{1-tan\frac{\theta}{2}}\right\}^2 = \frac{p}{q} \Rightarrow$

$\left\{\frac{\tan\frac{\pi}{4}+tan\frac{\theta}{2}}{1-tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right\} = \frac{p}{q}$

$\Rightarrow \tan^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{p}{q} \Rightarrow$

$\cot^2\left(\frac{\pi}{2}+\frac{\theta}{2}\right) = \frac{q}{p}$

$\Rightarrow \cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \sqrt{\frac{q}{p}}$

Q.100 If θ = $\frac{17\pi}{3}$ then tan θ – cot θ =

Correct option: (A)

We have, θ = $\frac{17\pi}{3}$

tan θ – cot θ $= \tan\frac{17\pi}{3} - \cot\frac{17\pi}{3}$

$= \tan\left(6\pi - \frac{\pi}{3}\right) - \cot\left(6\pi - \frac{\pi}{3}\right)$

$= -\tan\frac{\pi}{3} + \cot\frac{\pi}{3}$

$= -\sqrt{3} + \frac{1}{\sqrt{3}}$

$= \frac{-3+1}{\sqrt{3}} = \frac{-2}{\sqrt{3}}$