



Error and Vector

Marks: 50

ANSWER KEY

Physics

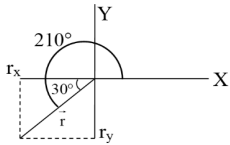
Q.1 D	Q.2 B	Q.3 C	Q.4 B	Q.5 B	Q.6 A	Q.7 C	Q.8 A
Q.9 B	Q.10 D	Q.11 C	Q.12 B	Q.13 C	Q.14 A	Q.15 A	Q.16 D
Q.17 D	Q.18 D	Q.19 B	Q.20 D	Q.21 D	Q.22 C	Q.23 B	Q.24 B
Q.25 D	Q.26 C	Q.27 B	Q.28 C	Q.29 A	Q.30 D	Q.31 D	Q.32 C
Q.33 A	Q.34 A	Q.35 A	Q.36 A	Q.37 C	Q.38 B	Q.39 C	Q.40 C
Q.41 D	Q.42 D	Q.43 D	Q.44 B	Q.45 B	Q.46 A	Q.47 C	Q.48 A
Q.49 D	Q.50 D						

## Physics

**Q.1** A displacement vector  $\vec{r}$  has magnitude of 25 m and makes an angle of  $210^\circ$  with the x-axis.

Then its y-component is -

**Correct option: (D)**



$$|\vec{r}| = r = 25$$

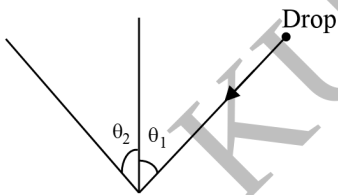
$$|\vec{r}_y| = r \sin 30^\circ = 25 \times \frac{1}{2} = 12.5 \text{ m}$$

$$\vec{r}_y = -12.5 \text{ m}$$

**Q.2** A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2 m/s. Find the angle  $\alpha$  with the vertical at which the wind screen should be placed so that the rain drops falling vertically downwards with velocity 6 m/s strikes the wind screen perpendicularly.

**Correct option: (B)**

Let vertical direction limit vector be  $\hat{j}$ , horizontal  $\hat{i}$



$$\theta_1 + \theta_2 = 90^\circ$$

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$$\vec{V}_{\text{car}} = 2\hat{i} \text{ and } \vec{V}_{\text{drops}} = 6\hat{j}$$

$$\vec{V}_{\text{drops w.r.t. car}} = \vec{V}_{\text{drops}} - \vec{V}_{\text{car}} = -2\hat{i} + 6\hat{j}$$

$$\cos \theta_1 = \frac{\hat{j} \cdot (-2\hat{i} + 6\hat{j})}{|\hat{j}| \cdot |-2\hat{i} + 6\hat{j}|} \Rightarrow \cos \theta_1 = \frac{6}{\sqrt{2^2 + 6^2} \cdot 1} = \frac{6}{\sqrt{40}}$$

$$\cos \theta_1 = \frac{3}{\sqrt{10}}$$

$$\sin \theta_2 = \frac{1}{\sqrt{10}}$$

$$\tan \theta_2 = 3 \Rightarrow \theta_2 = \tan^{-1} 3$$

Hence, the wind stream is inclined at  $\tan^{-1} 3$  with the vertical.

**Q.3** Consider a particle moving with a speed of 5 m/s towards east. After 10 sec velocity of the particle is 5 m/s towards north. Find the average acceleration and its direction.

**Correct option: (C)**

Average acceleration is the change in velocity divided by the total time taken.

$$\vec{v}_i = 5\hat{i}; \vec{v}_f = 5\hat{j}$$

$$\therefore \vec{v}_f - \vec{v}_i = 5\hat{j} - 5\hat{i}$$

Time interval = 10 sec

We know that,

$$\text{Average acceleration} = \frac{\vec{v}_f - \vec{v}_i}{\text{Time interval}}$$

$$\text{acc}_{\text{avg}} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{1}{2}(\hat{j} - \hat{i}) \text{ m/s}^2$$

$$|\text{acc}_{\text{avg}}| = \left| \frac{1}{2}(\hat{j} - \hat{i}) \right| = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

Unit vector along that direction is:

$$\frac{1}{\sqrt{2}}(\hat{j} - \hat{i}) \quad [45^\circ \text{ due west of north}]$$

**Q.4** The following observations were taken for determining surface tension T of water by capillary method:

diameter of capillary,  $D = 1.25 \times 10^{-2}$  m rise of water,  $h = 1.45 \times 10^{-2}$  m.

Using  $g = 9.80 \text{ m/s}^2$  and the simplified relation

$$T = \frac{r h g}{2} \times 10^3 \text{ N/m, the possible error in}$$

surface tension is closest to:

**Correct option: (B)**

$$D = 1.25 \times 10^{-2} \text{ m; } h = 1.45 \times 10^{-2} \text{ m}$$

The maximum permissible error in D

$$= \Delta D = 0.01 \times 10^{-2} \text{ m}$$

The maximum permissible error in h

$$= \Delta h = 0.01 \times 10^{-2} \text{ m}$$

g is given as a constant and is errorless.

$$T = \frac{rhg}{2} \times 10^3 \text{ N/m} = \frac{dhg}{4} \times 10^3 \text{ N/m}$$

$$\therefore \% \text{ error } \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h}$$

$$\therefore \frac{\Delta T}{T} \times 100 = \frac{\Delta d}{d} \times 100 + \frac{\Delta h}{h} \times 100$$

$$= \left( \frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}} \right) \times 100$$

$$= \frac{100}{125} + \frac{100}{145}$$

$$\therefore \frac{\Delta T}{T} = 0.8 \% + 0.7 \% = 1.5 \%$$

**Q.5** If  $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$  then the value

of  $|\vec{A} + \vec{B}|$  is

$$(\tan 60^\circ = \sqrt{3}, \cos 60^\circ = 0.5)$$

**Correct option: (B)**

Given: For two vectors  $\vec{A}$  and  $\vec{B}$ ,

$$|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B}).$$

$$AB \sin \theta = \sqrt{3} AB \cos \theta$$

$$\therefore \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

$\therefore$  The magnitude of  $\vec{A} + \vec{B}$  is:

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos 60^\circ}$$

Substituting  $\cos 60^\circ = 0.5$ :

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + AB}$$

**Q.6** The formula for percentage error is

**Correct option: (A)**

**Q.7** A body is acted upon by two forces of magnitudes  $F_1 = \sqrt{2} \text{ N}$  and  $F_2 = 3 \text{ N}$  which are inclined at  $45^\circ$  to each other. The magnitude of resultant force acting on the body is

**Correct option: (C)**

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{(\sqrt{2})^2 + (3)^2 + 2(\sqrt{2})(3)\cos 45^\circ}$$

$$F = \sqrt{2 + 9 + 6} = \sqrt{17} \text{ N}$$

**Q.8** If length and breadth of a plate are  $(40 \pm 0.2) \text{ cm}$  and  $(30 \pm 0.1) \text{ cm}$ , the absolute error in measurement of area is

**Correct option: (A)**

$$A = l \times b \Rightarrow \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} \Rightarrow$$

$$\Delta A = A \left[ \frac{\Delta l}{l} + \frac{\Delta b}{b} \right]$$

$$\Delta A = b\Delta l + l\Delta b = 10 \text{ cm}^2$$

**Q.9** The resultant of two vectors  $2\vec{A}$  and  $3\vec{A}$  is  $\vec{R}$ . If the second vector is doubled, the resultant vector also gets doubled. The angle ' $\theta$ ' between the vectors  $2\vec{A}$  and  $3\vec{A}$  is

**Correct option: (B)**

$$R^2 = (2A)^2 + (3A)^2 + 2 \times 2A \times 3A \times \cos \theta$$

$$R^2 = 4A^2 + 9A^2 + 12A^2 \cos \theta$$

$$R^2 = 13A^2 + 12A^2 \cos \theta \dots (i)$$

$$(2R)^2 = (2A)^2 + (6A)^2 + 2 \times 2A \times 6A \times \cos \theta$$

$$4R^2 = 4A^2 + 24A^2 \cos \theta$$

$$R^2 = 10A^2 + 6A^2 \cos \theta \dots (ii)$$

From (i) and (ii)

$$13A^2 + 12A^2 \cos \theta = 10A^2 + 6A^2 \cos \theta$$

$$3A^2 = -6A^2 \cos \theta$$

$$\therefore \cos \theta = -\frac{1}{2} = -0.5$$

$$\Rightarrow \theta = \cos^{-1}(-0.5)$$

**Q.10** The error in the measurement of length (L) of the simple pendulum is 0.1% and the error in time period (T) is 3%. The maximum possible error in the measurement of  $\frac{L}{T^2}$  is

**Correct option: (D)**

Maximum possible error in measurement of

$$\frac{L}{T^2} = \left( \frac{\Delta L}{L} + 2 \frac{\Delta T}{T} \right) \% = (0.1 + 2 \times 3) \% = 6.1\%$$

**Q.11** If the error in measuring the radius of the sphere is 2.5% and in mass is 1.5%, then the error in measurement of the density of material of the sphere is

**Correct option: (C)**

Given: Error in radius = 2.5 %

Error in mass = 1.5 %

Density of a sphere is given by,

$$\rho = \frac{m}{v}$$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\therefore \rho \propto \frac{m}{r^3}$$

$$\frac{\Delta\rho}{\rho} \times 100 = \left[ \frac{\Delta m}{m} + 3 \cdot \frac{\Delta r}{r} \right] \times 100$$

$$\therefore \frac{\Delta\rho}{\rho} = 0.015 + 3 \times 0.025$$

$$= 0.015 + 0.075$$

$$= 0.09$$

$$\therefore \text{Error \%} = 9 \%$$

**Q.12** The length, breadth and height of a rectangular block of wood were measured to be

$l = 13.12 \pm 0.02$  cm,  $b = 7.18 \pm 0.01$  cm,

$h = 4.16 \pm 0.02$  cm.

The percentage error in the volume of the block will be

**Correct option: (B)**

$$\frac{\Delta V}{V} \times 100 = \left( \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h} \right) \times 100\%$$

$$\therefore \frac{\Delta V}{V} \times 100 = \left( \frac{0.02}{13.12} + \frac{0.01}{7.18} + \frac{0.02}{4.16} \right) \times$$

100%

$$= 0.77\%$$

**Q.13** A physical quantity A can be determined by measuring parameters B, C, D and E using the relation  $A = \frac{B^\alpha C^\beta}{D^\gamma E^\delta}$ . If the

maximum errors in the measurement are b %, c %, d % and e % then maximum error in the value of A is

**Correct option: (C)**

$$\text{Given: } A = \frac{B^\alpha C^\beta}{D^\gamma E^\delta}$$

$$\text{Error contributed by B} = \alpha \times \left( \frac{\Delta B}{B} \times 100 \right) = \alpha$$

$\times b\%$

$$\text{Error contributed by C} = \beta \times \left( \frac{\Delta C}{C} \times 100 \right) = \beta \times$$

$c\%$

$$\text{Error contributed by D} = \gamma \times \left( \frac{\Delta D}{D} \times 100 \right) = \gamma \times$$

$d\%$

$$\text{Error contributed by E} = \delta \times \left( \frac{\Delta E}{E} \times 100 \right) = \delta \times$$

$e\%$

$\therefore$  Percentage error in A is given as,

$$\frac{\Delta A}{A} \times 100 = (\% \text{ error contributed by B}) + (\% \text{ error contributed by C}) + (\% \text{ error contributed by D}) + (\% \text{ error contributed by E})$$

$$= (\alpha b + \beta c + \gamma d + \delta e) \%$$

**Q.14** Two perpendicular components are

**Correct option: (A)**

**Q.15** If  $\vec{A} + \vec{B} + \vec{C} = \vec{0}$  and

$$|\vec{A}| = |\vec{B}| = |\vec{C}| = 1, \text{ Find}$$

$$\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C} + \vec{C} \cdot \vec{A}.$$

**Correct option: (A)**

$$\vec{A} + \vec{B} + \vec{C} = \vec{0}$$

$$\vec{A} + \vec{B} = -\vec{C}$$

$$|\vec{A} + \vec{B}| = |-\vec{C}|$$

$$\sqrt{A^2 + B^2 + 2\vec{A} \cdot \vec{B}} = |-\vec{C}|$$

$$A^2 + B^2 + 2\vec{A} \cdot \vec{B} = |-\vec{C}|^2$$

$$\therefore (|A| = |B| = |C| = 1)$$

$$\vec{A} \cdot \vec{B} = \frac{1 - 1 - 1}{2} = -\frac{1}{2}$$

By same logic

$$\vec{B} \cdot \vec{C} = \frac{-1}{2}$$

$$\vec{C} \cdot \vec{A} = \frac{-1}{2}$$

So,  $\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C} + \vec{C} \cdot \vec{A}$

$$\left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right) = \frac{-3}{2}$$

**Q.16** If vector  $\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ , its magnitude is

**Correct option: (D)**

$$\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$$

$$|\vec{A}| = \sqrt{(3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$$

**Q.17** Of the following the dimensionless error is

**Correct option: (D)**

**Q.18** The pressure on a square plate is measured by measuring the force acting on the plate and length of the sides of the plate. The maximum error in the measurement of force and length are respectively 4% and 2%, the percentage error in the measurement of pressure is

**Correct option: (D)**

$$P = \frac{F}{A} = \frac{F}{l^2}$$

so maximum error in pressure (P)

$$\left(\frac{\Delta P}{P} \times 100\right)_{\max} = \frac{\Delta F}{F} \times 100 + 2 \frac{\Delta l}{l} \times 100$$

$$= 4\% + 2 \times 2\% = 8\%$$

**Q.19** Displacement is a  
**Correct option: (B)**

**Q.20** The measure of accuracy is  
**Correct option: (D)**

**Q.21** If  $\vec{P} \cdot \vec{Q} = \frac{PQ}{2}$ , then the angle between

$\vec{P}$  and  $\vec{Q}$  is:

**Correct option: (D)**

$$\vec{P} \cdot \vec{Q} = \frac{PQ}{2}$$

$$\Rightarrow PQ \cos \theta = \frac{PQ}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

**Q.22** Find the torque  $\tau$  exerted by force

$\hat{i} + \hat{j} + 2\hat{k}$  at point P (2, 3, 4) w.r.t

origin. Given,  $\tau = \rightarrow r \times \rightarrow F$ ,  $\rightarrow r$  is the position vector of point at which force is acting w.r.t to the given point (in this case origin).

**Correct option: (C)**

The force  $\rightarrow F$  is given in Cartesian coordinates.

Express the position vector of point P in Cartesian coordinates and find the cross product

$$\tau = \rightarrow r \times \rightarrow F.$$

Here  $\rightarrow r = \rightarrow OP = \rightarrow P$  [ $\rightarrow P$  : Position vector of point P]

$$\Rightarrow \rightarrow r = 2\hat{i} + 3\hat{j} + 4\hat{k} \quad \text{and}$$

$$\rightarrow F = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \tau = \rightarrow r \times \rightarrow F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(2) - \hat{j}(0) + \hat{k}(-1) = 2\hat{i} - \hat{k}$$

**Q.23** If the resultant vector forms an angle of  $45^\circ$ , then the two components are

**Correct option: (B)**

**Q.24** Estimate the mean absolute error from the following data.

20.17, 21.23, 20.79, 22.07, 21.78

**Correct option: (B)**

$$a_m = \frac{20.17 + 21.23 + 20.79 + 22.07 + 21.78}{5}$$

$$a_m = 21.21$$

$$|\Delta a_1| = |21.21 - 20.17| = 1.04$$

$$|\Delta a_2| = |21.21 - 21.23| = 0.02$$

$$|\Delta a_3| = 0.42$$

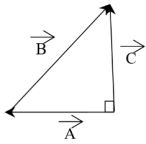
$$|\Delta a_4| = 0.86$$

$$|\Delta a_5| = 0.57$$

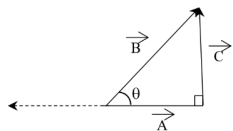
$$|\Delta a_m| = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$

$$= \frac{1.04 + 0.02 + 0.42 + 0.86 + 0.57}{5} = 0.58$$

**Q.25** In the adjoining vector diagram, what is the angle between  $\vec{A}$  and  $\vec{B}$ ? (Given:  $C = B/2$ ).



**Correct option: (D)**



$$\sin \theta = \frac{c}{b} = \frac{\frac{B}{2}}{B} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\text{Angle b/w } \vec{A} \text{ and } \vec{B} = 180^\circ - 30^\circ = 150^\circ$$

**Q.26** The components of a vector along the x and y directions are  $(n + 1)$  and 1 respectively. If the coordinate system is rotated by an angle  $60^\circ$  then the components change to n and 3. Find the value of n.

**Correct option: (C)**

Length of the vector does not change on rotation.

$$\therefore \sqrt{(n + 1)^2 + 1^2} = \sqrt{n^2 + 3^2}$$

$$\therefore (n + 1)^2 + 1 = n^2 + 9$$

$$\therefore 2n = 7 \Rightarrow n = 3.5$$

**Q.27** A constant force  $\vec{F} = 3\hat{i} - 2\hat{j} - \hat{k}$

newton has a displacement

$\vec{r} = 2\hat{i} - 3\hat{j} - 3\hat{k}$  metre in 2 second. The

work done and the power are respectively

**Correct option: (B)**

$$\text{Work done} = \vec{F} \cdot \vec{r}$$

$\therefore$  Work done =

$$(3\hat{i} - 2\hat{j} - \hat{k}) \cdot (2\hat{i} - 3\hat{j} - 3\hat{k})$$

$$\therefore \text{Work done} = (6 + 6 + 3)$$

$$\therefore \text{Work done} = 15 \text{ joule}$$

$$\text{Power} = \frac{\text{workdone}}{\text{time}}$$

$$\therefore \text{Work done} = \frac{15}{2}$$

$$\therefore \text{Work done} = 7.5 \text{ watt.}$$

**Q.28** Error in the measurement of radius of the sphere is 2%. The error in the calculated value of its volume is

**Correct option: (C)**

$$\frac{\Delta r}{r} \times 100 = 2\% \text{ and } V = \frac{4}{3}\pi r^3$$

Percentage error in volume

$$= \frac{\Delta V}{V} \% = 3 \left( \frac{\Delta r}{r} \times 100 \right) = 6\%$$

**Q.29** Given  $\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k})$ ,

$$\vec{B} = (3\hat{i} + \hat{j} - 2\hat{k}) \text{ and}$$

$$\vec{C} = (3\hat{i} + 2\hat{j} + \hat{k}) \cdot (\vec{A} + \vec{B}) \cdot \vec{C} \text{ will be}$$

**Correct option: (A)**

$$\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k})$$

$$\vec{B} = (3\hat{i} + \hat{j} - 2\hat{k})$$

$$\vec{C} = (3\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{A} + \vec{B} = (2 + 3)\hat{i} + (-3 + 1)\hat{j} + (1 - 2)\hat{k}$$

$$= 5\hat{i} - 2\hat{j} - \hat{k}$$

$$(\vec{A} + \vec{B}) \cdot \vec{C} = (5\hat{i} - 2\hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} + \hat{k})$$

$$= 15 - 4 - 1$$

$$= 10$$

**Q.30** Choose the INCORRECT statement out of the following.

**Correct option: (D)**

**Q.31** Two equal vectors have a resultant equal to either of them. The angle between them is

**Correct option: (D)**

$$\text{Since, } R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$A = B = R$$

$$\therefore A^2 = 2A^2 + 2A^2 \cos \theta$$

$$\cos \theta = -\frac{1}{2} = \cos 120^\circ$$

$$\therefore \theta = 120^\circ$$

**Q.32** A wire has a mass  $0.3 \pm 0.003$  gram, radius  $0.5 \pm 0.005$  mm and length  $6 \pm 0.06$  cm. The maximum percentage error in the measurement of its density is

**Correct option: (C)**

Here,

$$\frac{\Delta m}{m} = \frac{.003}{0.3}, \frac{\Delta r}{r} = \frac{0.005}{0.5}, \frac{\Delta L}{L} = \frac{0.06}{6}$$

$$\text{As } \rho = \frac{m}{(\pi r^2)L},$$

$$\therefore \left( \frac{\Delta \rho}{\rho} \right) \times 100 =$$

$$\begin{aligned} & \left( \frac{\Delta m}{m} + \frac{\Delta r}{r} + \frac{\Delta L}{L} \right) \times 100 \\ & = \left( \frac{0.003}{0.3} + \frac{2 \times 0.005}{0.5} + \frac{0.06}{6} \right) \times 100 \\ & = 1 + 2 + 1 = 4\% \end{aligned}$$

**Q.33** An experiment, the values of refractive indices of glass were found to be 1.54, 1.53, 1.44, 1.54, 1.56 and 1.45 in successive measurements

**i) Relative error ii) Percentage error**

**Correct option: (A)**

Mean value = e ( $x_m$ )

$$= \frac{1.54 + 1.53 + 1.44 + 1.54 + 1.56 + 1.45}{6}$$

$$= 1.51$$

Absolute errors in measurement are

$$\Delta x_1 = |1.51 - 1.54| = 0.03,$$

$$\Delta x_2 = |1.51 - 1.53| = 0.02,$$

$$\Delta x_3 = |1.51 - 1.44| = 0.07,$$

$$\Delta x_4 = |1.51 - 1.54| = 0.03,$$

$$\Delta x_5 = |1.51 - 1.56| = 0.05,$$

$$\Delta x_6 = |1.51 - 1.45| = 0.06,$$

Mean absolute error ( $\overline{\Delta x_m}$ )

$$= \frac{0.03 + 0.02 + 0.07 + 0.03 + 0.05 + 0.06}{6}$$

$$= 0.04$$

(rounded off to two places of decimal)

Relative error

$$= \pm \frac{\overline{\Delta x_m}}{x} = \pm \frac{0.04}{1.51} = \pm 0.02649 \approx \pm 0.03$$

(rounded off to two places of decimal)

Percentage error = Relative Error  $\times$  100

$$= \pm 0.03 \times 100$$

$$= \pm 3\%$$

**Q.34** If  $\vec{A} = \hat{i} + \hat{j} + 3\hat{k}$ ,  $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$ ,

and  $\vec{C} = 3\hat{i} - 3\hat{j} - 12\hat{k}$ , then find the

angle between the vectors  $(\vec{A} + \vec{B} + \vec{C})$

and  $(\vec{A} \times \vec{B})$  in degrees.

**Correct option: (A)**

Let  $\vec{P} = \vec{A} + \vec{B} + \vec{C} = 3\hat{i} - 5\hat{k}$  and

$$\vec{Q} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -1 & 1 & 4 \end{vmatrix} = 5\hat{i} - 7\hat{j} + 3\hat{k}$$

Angle between  $\vec{P}$  &  $\vec{Q}$  is given by

$$\cos \theta = \frac{\vec{P} \cdot \vec{Q}}{PQ} = \frac{15 - 15}{PQ} = 0 \Rightarrow \theta = 90^\circ$$

**Q.35** The length and breadth of a rectangular object are 25.2cm and 16.8cm respectively and have been measured to an accuracy of 0.1cm. Relative error and percentage error in the area of the object are.

**Correct option: (A)**

$$A = l \times b; \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{A} \times 100 = \left( \frac{\Delta l}{l} + \frac{\Delta b}{b} \right) \times 100 =$$

$$\left( \frac{0.1}{25.2} + \frac{0.1}{16.8} \right) \times 100$$

$$= (0.0039 + 0.0059) \times 100 = 0.0099 \times 100$$

$$= 0.99 \approx 1\%$$

$\therefore$  Percentage error = Relative error  $\times$  100

$$= 0.01$$

**Q.36** The magnitude of the difference between mean value and each individual value is called \_\_\_\_\_.

**Correct option: (A)**

**Q.37** Two resistors of resistances  $R_1 = 200 \pm 5 \Omega$  and  $R_2 = 200 \pm 10 \Omega$  are connected (a) in series, (b) in parallel. the equivalent resistance in series and parallel combination respectively is

**Correct option: (C)**

(a) The equivalent resistance of series combination is,  $R = R_1 + R_2$

$$\therefore R = (200 \pm 5) + (200 \pm 10) = 400 \pm 15 \Omega$$

(b) The equivalent resistance of parallel combination

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \dots(i)$$

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200 \times 200}{200 + 200} = 100 \Omega$$

Differentiating equation (i),

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = (R'^2) \frac{\Delta R_1}{R_1^2} + (R'^2) \frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{100}{200}\right)^2 \times 5 + \left(\frac{100}{200}\right)^2 \times 10$$

$$= 3.75 \Omega$$

$$\therefore R' = 100 \pm 3.75 \Omega$$

**Q.38** A body of mass 10 kg is placed on a smooth inclined plane making an angle of  $30^\circ$  with the horizontal, the component of the force of gravity trying to move the body down the inclined plane is [ $g = 9.8 \text{ m/s}^2$ ]

**Correct option: (B)**

Component of force of gravity =  $F_y = F \sin \theta$

$$F_y = mg \sin 30^\circ = 10 \times 9.8 \times \frac{1}{2} = 49 \text{ N}$$

**Q.39** A physical quantity  $P = \frac{A^3 \sqrt{B}}{C^{-4} D^{3/2}}$ . Out of

the quantities A, B, C and D, the quantity which brings maximum percentage error in P is

**Correct option: (C)**

In error analysis, the percentage error in a physical quantity depends on the power/exponent of each variable. Greater the power, more impact its error has on the overall expression.

Given:

$$P = \frac{A^3 \sqrt{B}}{C^4 D^{3/2}}$$

Take percentage error:

$$\frac{\Delta P}{P} \times 100 = \left(3 \frac{\Delta A}{A} + \frac{1}{2} \frac{\Delta B}{B} + 4 \frac{\Delta C}{C} + \frac{3}{2} \frac{\Delta D}{D}\right) \times 100$$

So, C has the highest power (4)  $\rightarrow$  its error contributes most significantly to the percentage error in P.

**Q.40** The resistance  $R = \frac{V}{I}$  where  $V = (100 \pm 5) \text{ V}$

and  $I = (10 \pm 0.2) \text{ A}$ . The percentage error in R is

**Correct option: (C)**

$$R = \frac{V}{I}$$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{5}{100} + \frac{0.2}{10} = 0.07$$

$$\therefore \text{Percentage error: } \frac{\Delta R}{R} \times 100 = 7\%$$

**Q.41** The area of a triangle formed by the sides of vector  $\vec{A}$  and  $\vec{B}$  is

**Correct option: (D)**

**Q.42** The vector sum of two forces  $\vec{A}$  and  $\vec{B}$  is perpendicular to their vector difference. Hence forces  $\vec{A}$  and  $\vec{B}$  are

**Correct option: (D)**

The sum of the two forces,

$$\vec{F}_1 = \vec{A} + \vec{B} \dots(i)$$

The difference of the two forces,

$$\vec{F}_2 = \vec{A} - \vec{B} \dots(ii)$$

Since sum of the two forces is perpendicular to their difference,

$$\vec{F}_1 \cdot \vec{F}_2 = 0$$

$$\Rightarrow (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$\Rightarrow A^2 - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - B^2 = 0$$

$$\therefore A^2 = B^2 \Rightarrow |A| = |B|$$

Thus, the forces are equal to each other in magnitude.

**Q.43** A particle moves in x - y plane under the action of a force  $\vec{F}$  such that the x and y components of linear momentum  $\vec{p}$  at any time t are  $2 \cos t$  and  $2 \sin t$ . Find the angle between  $\vec{F}$  and  $\vec{p}$  at a given time.

**Correct option: (D)**

$$\vec{p} = 2 \cos t \hat{i} + 2 \sin t \hat{j}$$

$$\therefore |\vec{p}| = 2\sqrt{2} \text{ units}$$

$$\text{Now, } \vec{F} = \frac{d\vec{p}}{dt} = -2 \sin t \hat{i} + 2 \cos t \hat{j}$$

Taking dot product,

$$\vec{F} \cdot \vec{p} = (2 \cos t \hat{i} + 2 \sin t \hat{j}) \cdot (-2 \sin t \hat{i} + 2 \cos t \hat{j})$$

$$= (2)(-2) + (2)(2)$$

$$= 0$$

$$\text{As } \vec{F} \cdot \vec{p} = 0, \theta = 90^\circ$$

**Q.44** Identify the vector quantity among the following

**Correct option: (B)**

Angular momentum has both magnitude and direction, therefore is a vector quantity

**Q.45** The errors which are estimated by statistical methods are

**Correct option: (B)**

**Q.46** Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

**Correct option: (A)**

$$R = \frac{V}{I} \Rightarrow \pm \frac{\Delta R}{R} = \pm \frac{\Delta V}{V} \pm \frac{\Delta I}{I}$$

$$= 3 + 3 = 6\%$$

**Q.47** When a copper sphere is heated, maximum percentage change will be observed in

**Correct option: (C)**

The percentage change in volume ( $3\alpha$ ) is the largest because it depends on three dimensions (length, width, and height), while the area ( $2\alpha$ ) depends on two dimensions, and the radius ( $\alpha$ ) depends on one dimension.

**Q.48** In the determination of Young's modulus ( $Y = \frac{4MLg}{\pi ld^2}$ ) by using Searle's method, a

wire of length  $L = 2 \text{ m}$  and diameter  $d = 0.5 \text{ mm}$  is used. For a load  $M = 2.5 \text{ Kg}$ , an extension

$l = 0.25 \text{ mm}$  in the length of the wire is observed. Quantities  $d$  and  $l$  are measured using a screw gauge and a micrometer, respectively. They have the same pitch of  $0.5 \text{ mm}$ . The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the  $Y$  measurement are

**Correct option: (A)**

Least count of both instrument

$$\Delta d = \Delta l = \frac{0.5}{100} \text{ mm} = 5 \times 10^{-3} \text{ mm}$$

$$Y = \frac{4MLg}{\pi ld^2}$$

$$\left( \frac{\Delta Y}{Y} \right)_{\max} = \frac{\Delta l}{l} + 2 \frac{\Delta d}{d}$$

Error in measurement of  $l$ ,

$$\frac{\Delta l}{l} = \frac{0.5/100 \text{ mm}}{0.25 \text{ mm}} = 2\%$$

Error in measurement of  $d$ ,

$$2 \frac{\Delta d}{d} = \frac{2 \times \frac{0.5}{100}}{0.5 \text{ mm}} = \frac{0.5/100}{0.25} = 2\%$$

**Q.49** The maximum and minimum magnitude of the resultant of two given vectors are 17 units and 7 units respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is

**Correct option: (D)**

$$R_{\max} = A + B = 17 \text{ when } \theta = 0^\circ$$

$$R_{\min} = A - B = 7 \text{ when } \theta = 180^\circ$$

by solving,  $A = 12$  and  $B = 5$

When  $\theta = 90^\circ$

$$\text{then } R = \sqrt{A^2 + B^2}$$

$$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

**Q.50** A wire has mass  $(0.2 \pm 0.002)$  gram, radius  $(0.4 \pm 0.004)$ mm and length  $(6 \pm 0.06)$ cm . The percentage error in the measurement of its density is  
**Correct option: (D)**

$$\text{Here, } \frac{\Delta m}{m} = \frac{0.002}{0.2}, \frac{\Delta r}{r} = \frac{0.004}{0.4}, \frac{\Delta L}{L} = \frac{0.06}{6}$$

$$\text{As } \rho = \frac{m}{(\pi r^2)L},$$

$$\therefore \left( \frac{\Delta \rho}{\rho} \right) \times 100 =$$

$$\left( \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta L}{L} \right) \times 100$$

$$= \left( \frac{0.002}{0.2} + \frac{2 \times 0.004}{0.4} + \frac{0.06}{6} \right) \times 100$$

$$= 1 + 2 + 1 = 4\%$$

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