Differentiation 2 **Marks: 200**

ANSWER KEY

Maths

Q.1 C	Q.2 D	Q.3 C	Q.4 C	Q.5 B	Q.6 D	Q.7 C	Q.8 C
Q.9 D	Q.10 B	Q.11 B	Q.12 C	Q.13 C	Q.14 C	Q.15 C	Q.16 A
Q.17 A	Q.18 D	Q.19 A	Q.20 B	Q.21 B	Q.22 B	Q.23 A	Q.24 D
Q.25 A	Q.26 D	Q.27 B	Q.28 D	Q.29 B	Q.30 C	Q.31 D	Q.32 A
Q.33 A	Q.34 C	Q.35 D	Q.36 C	Q.37 B	Q.38 B	Q.39 B	Q.40 C
Q.41 C	Q.42 D	Q.43 B	Q.44 C	Q.45 C	Q.46 D	Q.47 C	Q.48 C
Q.49 C	Q.50 D	Q.51 A	Q.52 B	Q.53 A	Q.54 D	Q.55 D	Q.56 C
Q.57 B	Q.58 B	Q.59 B	Q.60 B	Q.61 A	Q.62 A	Q.63 C	Q.64 C
Q.65 B	Q.66 A	Q.67 D	Q.68 C	Q.69 D	Q.70 A	Q.71 C	Q.72 A
Q.73 D	Q.74 C	Q.75 A	Q.76 A	Q.77 B	Q.78 B	Q.79 B	Q.80 A
Q.81 C	Q.82 A	Q.83 D	Q.84 A	Q.85 B	Q.86 C	Q.87 B	Q.88 A
Q.89 D	Q.90 D	Q.91 D	Q.92 C	Q.93 D	Q.94 A	Q.95 A	Q.96 A
Q.97 A	Q.98 C	Q.99 B	Q.100 C				

Maths

Q.1 The rate of change of the volume of a sphere with respect to its surface area, when its radius is 2 cm, is

Correct option: (C)

Volume of sphere (V) = $\frac{4}{3}\pi r^3$

Surface area of sphere (A) = $4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ and $\frac{dA}{dr} = 8\pi r$

$\therefore \left(\frac{dV}{dA}\right) = \frac{\left(\frac{dV}{dr}\right)}{\left(\frac{dA}{dr}\right)} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$

$\therefore \left(\frac{dV}{dA}\right)_{r=2} = 1\ \text{cm}^3/\text{cm}^2$

Q.2 A square plate is contracting at the uniform rate 4 cm^2/sec, then the rate at which the perimeter is decreasing, when side of the square is 20 cm, is

Correct option: (D)

Let A, P and X be the area, perimeter and length of side of square respectively at time 't' seconds. Then,

$A = X^2$, $P = 4X$

$\therefore$ $P = 4\sqrt{A}$

Differentiating w.r.t. t, we get

$\frac{dP}{dt} = 4\frac{1}{2\sqrt{A}} \cdot \frac{dA}{dt}$

$= \frac{2}{X} \cdot \frac{dA}{dt}$

$= \frac{2}{20} \times 4 \quad \ldots \left[\begin{array}{l}\text{side} = 20\text{ cm} \\ \frac{dA}{dt} = 4\text{cm}^2/\text{sec}\end{array}\right]$

$= \frac{2}{5}$ cm/sec

Q.3 The approximate value of tan 46° is (given 1° = 0.0175 radians)

Correct option: (C)

Let $f(x) = \tan x$

$\therefore f'(x) = \sec^2 x$

Here, $a = 45° = \left(\frac{\pi}{4}\right)^c$ and $h = 1° = 0.0175^c$

$f(a + h) \approx f(a) + hf'(a)$

$\approx \tan(a) + h\sec^2 a$

$\approx \tan(a) + h\frac{1}{\cos^2 a}$

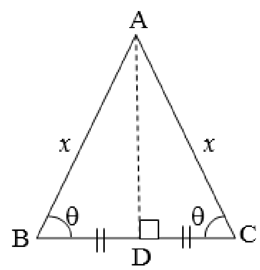
$\approx \tan\left(\frac{\pi}{4}\right) + (0.0175)\frac{1}{\left(1/\sqrt{2}\right)^2}$

$\approx 1 + 0.035$

$\therefore \tan 46^\circ \approx 1.035$

Q.4 A triangular park is enclosed on two sides by a fence and on the third side a straight river bank. The two sides having fence are of same length x. The maximum area (in sq. units) enclosed by the park is

Correct option: (C)



Let ΔABC be on isosceles triangle such that AB = AC = x

$\therefore$ ∠ABC = ∠ACB = θ

Draw seg AD ⊥ side BC at point D.

$\therefore$ ΔABD is a right angled triangle such that AD = x sin θ and BD = x cos θ

Similarly, in ΔACD,

DC = x cos θ

$\therefore$ In ΔABC,

Height = AD = x sin θ

Base = BC = x cos θ + x cos θ = 2x cos θ

$\therefore$ A(ΔABC) = $\frac{1}{2} \times x\sin\theta \times 2x\cos\theta$

$= \frac{x^2}{2}(2\sin\theta\cos\theta)$

$= \dfrac{x^2}{2}\sin 2\theta$

Since, $-1 \le \sin 2\theta \le 1$, for maximum value of $\sin 2\theta$, Maximum area $= \dfrac{x^2}{2}$ sq. units.

Q.5 For $y = \cos(m \sin^{-1}x)$, which of the following is true?

Correct option: (B)

$y = \cos(m \sin^{-1}x)$ ....(i)

$\therefore y_1 = -\sin(m \sin^{-1}x) \cdot \dfrac{m}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\,y_1 = -m \sin(m \sin^{-1}x)$

Differentiating both sides w.r.t. x, we get

$\sqrt{1-x^2}\,y_2 - \dfrac{xy_1}{\sqrt{1-x^2}} = -m\cos(m\sin^{-1}x)$

$\cdot \dfrac{m}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)\,y_2 - xy_1 = -m^2y$ ...[From (i)]

$\Rightarrow (1-x^2)\,y_2 - xy_1 + m^2y = 0$

Q.6 Let f : R → R be a function such that f(x + y) = f(x) + f(y) for all x, y ∈ R. If f(x) is differentiable at x = 0, then which one of the following is incorrect?

Correct option: (D)

$f(x+y) = f(x) + f(y)$ for all $x, y \in R$

Putting $x = 0$ and $y = 0$, we get

$f(0) = f(0) + f(0) \Rightarrow f(0) = 0$

Now, $f'(0) = \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$\Rightarrow f'(0) = \lim_{h\to 0}\dfrac{f(h)}{h}$ ...(i)

$\therefore f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\dfrac{f(x)+f(h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\dfrac{f(h)}{h} = f'(0)$ ...[From (i)]

$\Rightarrow f(x) = xf'(0) + c$

But, $f(0) = 0$

$\therefore c = 0$

Hence, $f(x) = xf'(0)$ for all $x \in R$

Clearly, $f(x)$ is everywhere continuous and differentiable and $f'(x)$ is constant for all $x \in R$.

Hence, option [D] is incorrect.

Q.7 If $y = \cot^{-1}\left(\sqrt{\dfrac{1-\sin x}{1+\sin x}}\right)$, then $\dfrac{dy}{dx}$ =

Correct option: (C)

$y = \cot^{-1}\left(\sqrt{\dfrac{1-\sin x}{1+\sin x}}\right)$

$= \cot^{-1}\sqrt{\dfrac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\frac{\sin x}{2}\right)^2}}$

$= \cot^{-1}\left(\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$

$= \cot^{-1}\left(\dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$= \tan^{-1}\left(\dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$= \tan^{-1}\left(\dfrac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}\right)$

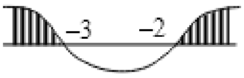
$= \tan^{-1}\left(\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right)$

$\therefore y = \dfrac{\pi}{4}+\dfrac{x}{2}$

$\therefore \dfrac{dy}{dx} = 0 + \dfrac{1}{2} = \dfrac{1}{2}$

Q.8 The function f(x) = $2x^3 + 15x^2 + 36x + 18$ is increasing in the interval

Correct option: (C)



$f(x) = 2x^3 + 15x^2 + 36x + 18$

$\therefore f'(x) = 6x^2 + 30x + 36$

For f(x) to be increasing,

$f'(x) > 0$

$\Rightarrow x^2 + 5x + 6 > 0$

$\Rightarrow (x+3)(x+2) > 0$

$\Rightarrow x \in (-\infty, -3) \cup (-2, \infty)$

Q.9 The position of a point in time t is given by $x = a + bt - ct^2$, $y = at + bt^2$. It's resultant acceleration at time t in seconds is given by

Correct option: (D)

We need to find the resultant acceleration, which is the magnitude of the acceleration given by, Vector $\vec{a} = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right)$

x = a + bt – ct^2

$\therefore \frac{dx}{dt}$ = b – 2ct

$\therefore \frac{d^2x}{dt^2}$ = –2c

y = at + bt^2

$\therefore \frac{dy}{dt}$ = a + 2bt

$\therefore \frac{d^2y}{dt^2}$ = 2b

Acceleration vector $\vec{a}$ is given by,

$\vec{a}$ = (–2c, 2b)

$\Rightarrow |\vec{a}| = \sqrt{(-2c)^2 + (2b)^2}$

$= \sqrt{4c^2 + 4b^2}$

$\Rightarrow |\vec{a}| = 2\sqrt{b^2 + c^2}$ unit/seconds^2

Q.10 The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$ is

Correct option: (B)

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$

$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$\therefore y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

Differentiating w.r.t. θ, we get

$\therefore \frac{dy}{d\theta} = \frac{1}{2}$

Let z = $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta$

$\therefore$ z = $\tan^{-1}\left(\frac{2\sin\theta\sqrt{1-\sin^2\theta}}{1-2\sin^2\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right)$

$= \tan^{-1}\left(\frac{\sin 2\theta}{\cos 2\theta}\right)$

$= \tan^{-1}(\tan 2\theta)$

$\therefore$ z = 2θ

Differentiating w.r.t. θ, we get

$\frac{dz}{d\theta} = 2$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{d\theta}}{\frac{dz}{d\theta}} = \frac{1}{2\cdot 2} = \frac{1}{4}$

Q.11 The radius of a circular plate is increasing at the rate of 0.01 cm/sec, when the radius is 12 cm. Then the rate at which the area increases is

Correct option: (B)

$\frac{dr}{dt}$ = 0.01 cm/sec

Now, Area(A) = πr^2

Differentiating w.r.t. t, we get

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

= 2π(12)(0.01)

= 0.24π

$\therefore$ The rate of increase in area is 0.24π sq. cm/sec.

Q.12 If $y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$, then $\frac{d^2y}{dx^2}$ is

Correct option: (C)

$y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$

$\Rightarrow y = \cos 3x$ ….(i)

$\therefore \frac{dy}{dx} = -3\sin 3x$

$\therefore \frac{d^2y}{dx^2} = -9 \cos 3x$

$\Rightarrow \frac{d^2y}{dx^2} = -9y$ …[From (i)]

Q.13 The law of motion of a body moving along a straight line is $x = \frac{1}{2}$vt, x being its distance from a fixed point on the line at time t and v is its velocity there. Then

Correct option: (C)

$x = \frac{1}{2}vt \Rightarrow 2x = vt \Rightarrow 2\frac{dx}{dt} = v + t\frac{dv}{dt}$

$\Rightarrow 2\frac{d^2x}{dt^2} = \frac{dv}{dt} + t\frac{d^2v}{dt^2} + \frac{dv}{dt}$

But acceleration (f) = $\frac{dv}{dt}$

$\Rightarrow 2f = f + t\frac{df}{dt} + f$

$\Rightarrow \frac{df}{dt} = 0$ ...[$\because t \neq 0$]

$\Rightarrow$ f = constant

Q.14 The maximum and minimum values for the function f(x) = $3x^4 - 4x^3$ on [– 1, 2] are

Correct option: (C)

$f(x) = 3x^4 - 4x^3$

$\therefore f'(x) = 12x^3 - 12x^2$

$\therefore x^2(x-1) = 0 \Rightarrow x = 1, 0$

Now $f''(x) = 36x^2 - 24x$

$f''(1) = 12 > 0$

$f''(0) = 0$

$f(1) = 3 - 4 = -1$

$f(-1) = 3 + 4 = 7$

$f(2) = 48 - 32 = 16$

$\therefore$ Maximum at 2 and Minimum at 1 and Maximum value is 16 and Minimum value is –1.

Q.15 If $x = 2at^2$, $y = at^4$, then $\frac{d^2y}{dx^2}$ at t = 2 is

Correct option: (C)

$x = 2at^2$ and $y = at^4$

$\therefore \frac{dx}{dt} = 4at$ and $\frac{dy}{dt} = 4at^3$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = t^2$

$\therefore \frac{d^2y}{dx^2} = 2t.\frac{dt}{dx} = 2t.\frac{1}{4at} = \frac{1}{2a}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{(t=2)} = \frac{1}{2a}$

Q.16 $\frac{d}{dx}\left[\tan^{-1}\left(\frac{2}{x^{-1}-x}\right)\right] =$

Correct option: (A)

Let $y = \tan^{-1}\left(\frac{2}{x^{-1}-x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$

Q.17 If $f(x) = \sin^{-1}\left(\frac{2\cdot 3^x}{1+9^x}\right)$, then $f'\left(\frac{1}{2}\right)$ equals

Correct option: (A)

$f(x) = \sin^{-1}\left(\frac{2.3^x}{1+9^x}\right) = \sin^{-1}\left(\frac{2.3^x}{1+(3^x)^2}\right)$

Put $3^x = \tan\theta \Rightarrow \theta = \tan^{-1}(3^x)$

$\therefore f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$\therefore f(x) = 2\tan^{-1}(3^x)$

$\therefore f'(x) = 2\cdot\frac{1}{1+(3^x)^2}\cdot 3^x \log 3$

$\therefore f'\left(\frac{1}{2}\right) = 2\cdot\frac{1}{1+\left(3^{\frac{1}{2}}\right)^2}\cdot 3^{\frac{1}{2}} \log 3$

$= \frac{1}{2}\sqrt{3}\log 3 = \sqrt{3}\log\sqrt{3}$

Q.18 If $y = \frac{\tan x + \cot x}{\tan x - \cot x}$, then $\frac{dy}{dx} =$

Correct option: (D) $-2\tan 2x \sec 2x$

$y = \frac{\tan x+\cot x}{\tan x-\cot x} = \frac{\tan x+\frac{1}{\tan x}}{\tan x-\frac{1}{\tan x}}$

$= -\frac{1+\tan^2 x}{1-\tan^2 x} = -\sec 2x$

$\therefore \quad \frac{dy}{dx} = -\sec 2x \tan 2x. \frac{d}{dx}(2x)$
$= -2\sec 2x \tan 2x$

Q.19 If the function f(x) = ax^3 + bx^2 + 11x – 6 satisfies the conditions of Rolle's theorem in [1, 3] and $f'\left(2+\frac{1}{\sqrt{3}}\right)=0$, then a + b =

Correct option: (A)

Since f(x) satisfies the Rolle's theorem,

f(1) = f(3)

∴ a + b + 5 = 27a + 9b + 27

∴ 26a + 8b + 22 = 0

∴ 13a + 4b + 11 = 0 …(i)

Given that $f'\left(2+\frac{1}{\sqrt{3}}\right) = 0$

$f'(x) = 3ax^2 + 2bx + 11$

$\therefore f'\left(2+\frac{1}{\sqrt{3}}\right) =$

$$3a\left(2+\frac{1}{\sqrt{3}}\right)^2 + 2b\left(2+\frac{1}{\sqrt{3}}\right) + 11$$

$$= a\left(2\sqrt{3}+1\right)^2 + 4b + \frac{2b}{\sqrt{3}} + 11$$

$$= (13a + 4b + 11) + 4\sqrt{3}a + \frac{2b}{\sqrt{3}}$$

$$= 0 + \frac{12a+2b}{\sqrt{3}} \quad \text{…[From (i)]}$$

$$\therefore \frac{12a+2b}{\sqrt{3}} = 0$$

∴ 6a + b = 0 …(ii)

Solving (i) and (ii), we get a = 1, b = – 6

∴ a + b = –5

Q.20 The approximate value of square root of 25.2 is

Correct option: (B)

Let f(x) = $\sqrt{x}$

$\therefore f'(x) = \frac{1}{2\sqrt{x}}$

Here, a = 25 and h = 0.2

∴ f (a) = f (25) = $\sqrt{25}$ = 5

and f ′(a) = f ′(25) = $\frac{1}{2\sqrt{25}} = \frac{1}{10}$

∴ f (a + h) ≈ f (a) + h f ′(a)

$\approx 5 + (0.2)\left(\frac{1}{10}\right)$

≈ 5 + 0.02

∴ $\sqrt{25.2}$ ≈ 5.02

Q.21 The combined equation of the tangent and normal to the curve xy = 15 at the point (5, 3) is ________

Correct option: (B)

Given equation of curve is

$xy = 15$

Differentiating w.r.to x, we get

$x\frac{dy}{dx} + y = 1$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

At (5, 3) $\frac{dy}{dx} = \frac{-3}{5}$

Slope of tangent = $\frac{-3}{5}$

Slope of normal = $\frac{5}{3}$

Equation of tangent at (5, 3) is

$y - 3 = \frac{-3}{5}(x-5)$

⇒ 3x + 5y – 30 = 0

Equation of normal at (5, 3) is

$y - 3 = \frac{5}{3}(x-5)$

⇒ 5x – 3y – 16 = 0

∴ Combined equation is

(3x + 5y – 30) (5x – 3y – 16) = 0

⇒ $15x^2 + 16xy - 15y^2 - 198x + 10y + 480 = 0$

Q.22 The function f(x) = $\sin^4 x$ + $\cos^4 x$ is increasing in

Correct option: (B)

f(x) = $\sin^4 x + \cos^4 x$

∴ $f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= 4\sin x \cos x (\sin^2 x - \cos^2 x)$

$= -2\sin 2x \cos 2x$

$= -\sin 4x$

∴ If f(x) is increasing, then f ′(x) > 0

i.e., –sin4x > 0 ⇒ π < 4x < 2π

$\Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$

Q.23 **If f(x) = $\cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$,then f′ (1) is equal to**

Correct option: (A)

Put $x^x = \tan\theta \Rightarrow \theta = \tan^{-1}(x^x)$

$\therefore f(x) = \cot^{-1}\left(\frac{\tan^2\theta - 1}{2\tan\theta}\right)$

$= \cot^{-1}(-\cot 2\theta)$

$= \pi - \cot^{-1}(\cot 2\theta)$

$\therefore f(x) = \pi - 2\theta = \pi - 2\tan^{-1}(x^x)$

$\therefore f'(x) = \frac{-2}{1 + x^{2x}} . x^x(1 + \log x)$

$\therefore f'(1) = \frac{-2}{1 + 1^2} . 1\,(1 + 0) = -1$

Q.24 **If $e^y(x + 1) = 1$, then $\frac{d^2y}{dx^2} =$**

Correct option: (D)

$e^y(x + 1) = 1 \Rightarrow e^y = \frac{1}{x+1}$

$\Rightarrow y = \log\left(\frac{1}{x+1}\right)$

$\Rightarrow y = -\log(x + 1)$

$\therefore \frac{dy}{dx} = -\frac{1}{x+1}$ ...(i)

$\therefore \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(\frac{-1}{x+1}\right)^2$

$= \left(\frac{dy}{dx}\right)^2$ ...[From (i)]

Q.25 **The function f(x) = log x – $\frac{2x}{x+2}$ is increasing for all**

Correct option: (A)

$f(x) = \log x - \frac{2x}{x+2}$

$\Rightarrow f'(x) = \frac{1}{x} - \frac{(x+2)(2) - 2x(1)}{(x+2)^2}$

$= \frac{1}{x} - \frac{4}{(x+2)^2}$

$= \frac{x^2+4}{x(x+2)^2}$

$\Rightarrow f'(x) = \frac{x^2+4}{x(x+2)^2}$

$\therefore$ f ′(x) > 0 for all x > 0

Hence, f(x) is increasing on (0, ∞).

Q.26 **If f(x) = $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, x ∈ (1, ∞), then f ′(x) =**

Correct option: (D)

Given f(x) =

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$,

$\Rightarrow \theta = \tan^{-1}x$

$\therefore$ f(x) =

$\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta)$

$= 2\theta + 2\theta$

$\therefore f(x) = 4\theta$

$\therefore f(x) = 4\tan^{-1}x$

$f'(x) = \frac{4}{1+x^2}$

Q.27 **If f(x) = $\frac{a\sin x + b\cos x}{c\sin x + d\cos x}$ is decreasing for all x, then**

Correct option: (B)

$f(x) = \frac{asinx + bcosx}{csinx + dcosx}$

f(x) will be decreasing, if f ′(x) < 0

$\therefore$

$\frac{1}{(csinx + dcosx)}[(csinx + dcosx)(acosx + bsinx) - (asinx + bcosx)(ccosx + dsinx)] < 0$

$\Rightarrow$

$acsinxcosx - bc\sin^2 x + ad\cos^2 x - bdsinxcosx - ac\sin xcosx + ad\sin^2 x - bc\cos^2 x - bdsinxcosx < 0$

$\Rightarrow$

$ad(\sin^2 x + \cos^2 x) - bc(\sin^2 x + \cos^2 x) < 0$

$\Rightarrow ad - bc < 0$

Q.28 **The approximate value of $\log_{10}998$ is (given that $\log_{10}e = 0.4343$)**

Correct option: (D)

Let $f(x) = \log_{10}x = \dfrac{\log_e x}{\log_e 10} = (\log_{10}e)(\log_e x)$

$= 0.4343(\log_e x)$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{0.4343}{x}$

Let $x = 998 = 1000 - 2 = a + h$

$\therefore$ $a = 1000$, $h = -2$

$f(a) = f(1000)$

$= \log_{10}(1000)$

$= 3\log_{10}10$

$\therefore$ $f(a) = 3$

Also, $f'(a) = f'(1000) = \dfrac{0.4343}{1000}$

$= 0.0004343$

$f(a + h) \approx f(a) + hf'(a)$

$\therefore$ $\log_{10}(998) \approx 3 - 2(0.0004343)$

≈ 2.9991314

Q.29 The function $f(x) = 3x^4 + 16x^3 - 30x^2 + 10$ is increasing for

Correct option: (B)

$f(x) = 3x^4 + 16x^3 - 30x^2 + 10$

$\therefore$ $f'(x) = 12x^3 + 48x^2 - 60x$

For $f(x)$ to be increasing, $f'(x) > 0$

$\Rightarrow 12x(x^2 + 4x - 5) > 0$

$\Rightarrow x(x + 5)(x - 1) > 0$

$\therefore$ $x \in (-5, 0) \cup (1, \infty)$

$\therefore$ $f(x)$ is an increasing function for

$x \in (-5, 0) \cup (1, \infty)$

Q.30 If $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t + \frac{1}{t}\right)$, then $\frac{dy}{dx}$ =

Correct option: (C)

Given, $x = a\left(t - \frac{1}{t}\right)$ ...(i)

and $y = a\left(t + \frac{1}{t}\right)$ ...(ii)

Squaring (i) and (ii), then subtracting we get,

$x^2 - y^2 = a^2(-4) \Rightarrow y^2 - x^2 = 4a^2$

Differentiating w.r.t. x, we get

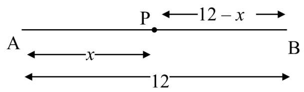
$2y\frac{dy}{dx} - 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y}$

Q.31 If P is a point on the segment AB of length 12 cm, then the position of P for $AP^2 + BP^2$ to be minimum is such that

Correct option: (D)

Let $d(AB) = x$

$\therefore$ $d(BP) = 12 - x$



Let $f(x) = AP^2 + BP^2$

$= x^2 + (12 - x)^2$

$= 2x^2 - 24x + 144$

$\therefore$ $f(x) = x^2 - 12x + 72$

$\therefore$ $f'(x) = 2x - 12$

For maxim, minima,

$f'(x) = 0$

$\Rightarrow 2x - 12 = 0$

$\Rightarrow x = 6$

Also, $f''(x) = 2 > 0$

Hence, $f(x)$ is minimum at $x = 6$.

i.e., P is the midpoint of AB.

Q.32 If $y = \sec^{-1}\left(\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then the value of $\dfrac{dy}{dx}$ =

Correct option: (A)

$y = \cos^{-1}\left(\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

$= \pi/2$

$\therefore \dfrac{dy}{dx} = 0$

Q.33 Let S = {t ∈ R/f(x) = $|x - \pi|(e^{|x|} - 1)\sin|x|$ is not differentiable at t}, then S is

Correct option: (A)

Differentiability at $x = \pi$:

L.h.lim

$= \lim\limits_{h\to 0} \dfrac{|\pi - h - \pi|\left(e^{|\pi-h|} - 1\right)\sin|\pi - h| - 0}{-h}$

$= 0$

R.h.lim

$= \lim\limits_{h\to 0} \dfrac{|\pi + h - \pi|\left(e^{|\pi+h|} - 1\right)\sin|\pi + h| - 0}{h}$

$= 0$

Differentiability at $x = 0$:

L.h.lim =

$\lim\limits_{h\to 0} \dfrac{|-h - \pi|\left(e^{|-h|} - 1\right)\sin|-h| - 0}{-h} = 0$

R.h.lim $= \lim_{h \to 0} \dfrac{|h-\pi|\left(e^{|h|}-1\right)\sin|h| - 0}{h} = 0$

The function f(x) is differentiable at x = 0, π.

⇒ Set S is an empty set.

Q.34 20 is divided into two parts so that the product of the cube of one part and the square of the other part is maximum, then these two parts are

Correct option: (C)

Let $x + y = 20 \Rightarrow y = 20 - x$ …(i)

and $x^3y^2 = z$

$\Rightarrow z = x^3 (20 - x)^2 \Rightarrow z = 400x^3 + x^5 - 40x^4$

$\therefore \dfrac{dz}{dx} = 1200x^2 + 5x^4 - 160x^3$

For maximum or minimum,

$\dfrac{dz}{dx} = 0$

$\Rightarrow 1200x^2 + 5x^4 - 160x^3 = 0$

$\Rightarrow x = 12, 20$

$\dfrac{d^2z}{dx^2} = 2400x + 20x^3 - 480x^2$

$\therefore \left(\dfrac{d^2z}{dx^2}\right)_{x=12} = -5760 < 0$

∴ z is maximum at x = 12.

From (i), $y = 20 - 12 = 8$

∴ The parts of 20 are 12 and 8.

Q.35 The abscissae of the points, where the tangent to curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to X- axis, are

Correct option: (D)

$y = x^3 - 3x^2 - 9x + 5 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 6x - 9$

Since the tangent is parallel to X-axis, $\dfrac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0 \Rightarrow x = -1, 3$

Q.36 The abscissa of the points, where the tangent to the curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to X-axis are

Correct option: (C)

$y = x^3 - 3x^2 - 9x + 5 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 6x - 9$

Since the tangent is parallel to X-axis, $\dfrac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1, 3$

Q.37 If $y = \dfrac{\cos x - \sin x}{\cos x + \sin x}$, then $\dfrac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} =$

Correct option: (B)

$y = \dfrac{\cos x - \sin x}{\cos x + \sin x} = \dfrac{1 - \tan x}{1 + \tan x}$

$\Rightarrow y = \tan\left(\dfrac{\pi}{4} - x\right)$ ...(i)

$\therefore \dfrac{dy}{dx} = -\sec^2\left(\dfrac{\pi}{4} - x\right)$

$\therefore \dfrac{d^2y}{dx^2} = 2\sec^2\left(\dfrac{\pi}{4} - x\right) . \tan\left(\dfrac{\pi}{4} - x\right)$

$\therefore \dfrac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = -2\tan\left(\dfrac{\pi}{4} - x\right) = -2y$ ...[From (i)]

Q.38 If $f(x) = \cos^{-1}\left[\dfrac{1-(\log x)^2}{1+(\log x)^2}\right]$, then f ′(e) =

Correct option: (B)

$f(x) = \cos^{-1}\left[\dfrac{1-(\log x)^2}{1+(\log x)^2}\right]$

Put $\log x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(\log x)$

$\therefore f(x) = \cos^{-1}\left[\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right]$

$\Rightarrow f(x) = \cos^{-1}(\cos 2\theta)$

$\Rightarrow f(x) = 2\theta$

$\Rightarrow f(x) = 2\tan^{-1}(\log x)$

Differentiating w.r.t x, we get

$f'(x) = \dfrac{2}{1+(\log x)^2} \cdot \dfrac{d}{dx}\log x$

$= \dfrac{2}{x\left[1+(\log x)^2\right]}$

$\therefore f'(e) = \dfrac{2}{e\left(1+(\log e)^2\right)} = \dfrac{1}{e}$

Q.39 **If $y = (\tan^{-1} x)^2$ then $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} =$**

Correct option: (B)

$y = (\tan^{-1} x)^2$

$\therefore \frac{dy}{dx} = \frac{2\tan^{-1}x}{1 + x^2}$

$\Rightarrow \frac{dy}{dx}(1 + x^2) = 2\tan^{-1} x$

$\therefore \frac{dy}{dx}(2x) + (1 + x^2)\frac{d^2y}{dx^2} = \frac{2}{1 + x^2}$

$\Rightarrow (x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$

Q.40 **If $y = logx.\, e^{(\tan x + x^2)}$, then $\frac{dy}{dx} =$**

Correct option: (C)

$y = logx.\, e^{(\tan x + x^2)}$

$\therefore \frac{dy}{dx} = e^{(\tan x + x^2)} \cdot \frac{1}{x} + \log x \cdot e^{(\tan x + x^2)} \cdot \frac{d}{dx}(\tan x + x^2)$

$= e^{(\tan x + x^2)} \cdot \frac{1}{x} + \log x \cdot e^{(\tan x + x^2)}(\sec^2 x + 2x)$

$= e^{(\tan x + x^2)}\left[\frac{1}{x} + \left(\sec^2 x + 2x\right)\log x\right]$

Q.41 **If $y = \tan^{-1}\left\{\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right\}$, then $\frac{dy}{dx}$**

Correct option: (C)

$y = \tan^{-1}\left\{\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right\}$

Put a = sin θ and b = cos θ

$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{b}\right)$

$\therefore\ y = \tan^{-1}\left\{\frac{\sin\theta\cos x - \cos\theta\sin x}{\cos\theta\cos x + \sin\theta\sin x}\right\}$

$\Rightarrow y = \tan^{-1}\left\{\frac{\sin(\theta - x)}{\cos(\theta - x)}\right\}$

$\Rightarrow y = \tan^{-1}\{\tan(\theta - x)\}$

$\Rightarrow y = \theta - x$

$\Rightarrow y = \tan^{-1}\left(\frac{a}{b}\right) - x$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = -1$

Q.42 **In the mean value theorem, f(b) – f(a) = (b – a) f ′(c), if a = 4, b = 9 and $f(x) = \sqrt{x}$, then the value of c is**

Correct option: (D)

$f(x) = \sqrt{x}$

$\therefore f(a) = \sqrt{4} = 2,\ f(b) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{3 - 2}{9 - 4} = \frac{1}{5}$

$\therefore \frac{1}{2\sqrt{c}} = \frac{1}{5} \Rightarrow c = \frac{25}{4} = 6.25$

Q.43 **If Mean value theorem holds for the function $f(x) = (x-1)(x-2)(x-3), x \in [0, 4]$ then the values of c as per the theorem are**

Correct option: (B)

$f(x) = (x - 1)(x - 2)(x - 3)$

$f(4) = (4 - 1)(4 - 2)(4 - 3)$

$f(4) = 6$

$f(0) = (0 - 1)(0 - 2)(0 - 3) = -6$

$\therefore$ Using LMVT

$f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{6 - (-6)}{4} = 3$

$\therefore\ f'(c) = 3$ …(i)

Now, $f(x) = (x - 1)(x - 2)(x - 3)$

$= x^3 - 6x^2 + 11x - 6$

$\therefore\ f'(x) = 3x^2 - 12x + 11$

$f'(c) = 3$ …[From (i)]

$\Rightarrow 3c^2 - 12c + 11 = 3$

$\Rightarrow 3c^2 - 12c + 8 = 0$

$\Rightarrow c = \frac{12 \pm \sqrt{48}}{6} = 2 \pm \frac{2\sqrt{3}}{3} = 2 \pm \frac{2}{\sqrt{3}}$

Q.44 **The length of the perpendicular drawn from the origin on the normal to the curve $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is**

Correct option: (C)

$x^2 + 2xy - 3y^2 = 0$

Differentiating w.r.t. x, we get

$$2x + 2y + 2x\frac{dy}{dx} - 3\left(2y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow (x - 3y)\frac{dy}{dx} = -x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x-y}{x-3y}$$

Slope of the normal at (2, 2) is

$$\frac{-1}{\left(\frac{dy}{dx}\right)_{(2,2)}} = -1$$

∴ Equation of the normal at (2, 2) is

$y - 2 = -1(x - 2)$

$\Rightarrow x + y = 4$

Length of perpendicular

$$= \left|\frac{0+0-4}{\sqrt{2}}\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ units}$$

Q.45 If $y = log_5(log_7\ x)$, then $\frac{dy}{dx}$ =

Correct option: (C)

$y = log_5(log_7\ x)$

$$= \frac{\log(\log_7 x)}{\log\ 5}$$

$$= \frac{1}{\log\ 5}[\log(\log x) - \log(\log 7)]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\log\ 5}\left(\frac{1}{x\log x}\right)$$

Q.46 The minimum value of f(a) = $(2a^2 - 3) + 3(3 - a) + 4$ is

Correct option: (D)

$f(a) = 2a^2 - 3a + 10$

$\Rightarrow f'(a) = 4a - 3 \Rightarrow f''(a) = 4 > 0$

For minimum value of f (a),

$f'(a) = 0 \Rightarrow a = \frac{3}{4}$

∴ f(a) is minimum at $a = \frac{3}{4}$.

$$\therefore [f(a)]_{min} = f\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 10 = \frac{71}{8}$$

Q.47 Sides of a square are increasing at the rate 0.5 cm/ sec. When the side is 10 cm long, its area is increasing at the rate of

Correct option: (C)

$A = s^2$

$$\therefore \frac{dA}{dt} = 2s\frac{ds}{dt}$$

$$\therefore \left(\frac{dA}{dt}\right)_{s=10} = 2 \times 10 \times 0.5 = 2 \times 5 = 10 \text{ cm}^2\text{/sec}$$

Q.48 $\frac{d}{dx}\left(3\cos\left(\frac{\pi}{6} + x^\circ\right) - 4\cos^3\left(\frac{\pi}{6} + x^\circ\right)\right)$ = …

Correct option: (C)

$\frac{d}{dx}\left(3\cos\left(\frac{\pi}{6} + x^\circ\right) - 4\cos^3\left(\frac{\pi}{6} + x^\circ\right)\right)$

$= \frac{d}{dx}\left[-\left(4\cos^3\left(\frac{\pi}{6} + x^\circ\right) - 3\cos\left(\frac{\pi}{6} + x^\circ\right)\right)\right]$

$= \frac{d}{dx}\left(-\cos 3\left(\frac{\pi}{6} + x^\circ\right)\right)$ …[∵ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$]

$= \frac{d}{dx}\left(-\cos\left(\frac{\pi}{2} + 3x^\circ\right)\right)$

$= \frac{d}{dx}\left(\sin 3x \times \frac{\pi}{180}\right)$

$= \frac{3\pi}{180}\left(\cos 3x \times \frac{\pi}{180}\right)$

$= \frac{\pi}{60}\cos 3x^\circ$

Q.49 Suppose the cubic $x^3 - px + q$ has three real roots where p > 0 and q > 0. Then which one of the following holds?

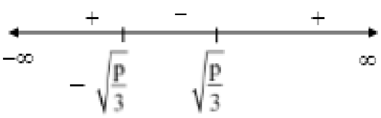
Correct option: (C)

Let $f(x) = x^3 - px + q$. Then,

$f'(x) = 3x^2 - p$

$$= 3\left(x - \sqrt{\frac{p}{3}}\right)\left(x + \sqrt{\frac{p}{3}}\right)$$

The signs of $f'(x)$ for different values of x are as shown below:



Since $f'(x)$ changes its sign form positive to negative in the neighbourhood of $-\sqrt{\frac{p}{3}}$.

So, $-\sqrt{\frac{p}{3}}$ is a point of local maximum. Similarly,

$x = \sqrt{\frac{p}{3}}$ is a point of local minimum.

Q.50 The approximate value of $5^{2.01}$ is____, where ($\log_e 5 = 1.6095$)

Correct option: (D)

Let $f(x) = 5^x$

$\therefore f'(x) = 5^x \log 5$

Here, a = 2 and h = 0.01

$\therefore f(a + h) \approx f(a) + h\, f'(a)$

$\approx f(2) + 0.01\, f'(2)$
$\approx 5^2 + 0.01\,(5^2 \times \log 5)$
$\approx 25 + 0.01\,(25 \times 1.6095)$
≈ 25.4024

Q.51 If θ denotes the acute angle between the curves $y = 10 - x^2$ and $y = 2 + x^2$, at a point of the intersection, then |tan θ| is equal to

Correct option: (A)

$y = 10 - x^2$ …(i)

$y = 2 + x^2$ …(ii)

Solving (i) and (ii), we get
$y = 6$

From (i), $6 = 10 - x^2 \Rightarrow x = \pm 2$

Differentiating (i) w.r.t. x, we get
$$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$$

Differentiating (ii) w.r.t. x, we get
$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

∴
$$\tan\theta = \left|\frac{4-(-4)}{1+4(-4)}\right| = \left|\frac{8}{-15}\right| \Rightarrow |\tan\theta| = \frac{8}{15}$$

Q.52 Derivative of log (sec θ + tan θ) with respect to sec θ at $\theta = \frac{\pi}{4}$ is

Correct option: (B)

Let $y = \log(\sec\theta + \tan\theta)$ and $z = \sec\theta$

$$\therefore \frac{dy}{d\theta} = \frac{1}{\sec\theta + \tan\theta} \cdot (\sec\theta\tan\theta + \sec^2\theta) = \sec\theta$$

and $\frac{dz}{d\theta} = \sec\theta\tan\theta$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{d\theta}}{\frac{dz}{d\theta}} = \frac{\sec\theta}{\sec\theta\tan\theta} = \frac{1}{\tan\theta} = \cot\theta$$

$$\therefore \left(\frac{dy}{dz}\right)_{\theta=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$$

Q.53 If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then the value of $\frac{dy}{dx}$ at $x = \sqrt{3}$ is

Correct option: (A)

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\therefore y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$= \sin^{-1}(\sin 2\theta) + \sec^{-1}(\sec 2\theta)$
$= 2\theta + 2\theta$
$= 4\theta$
$= 4\tan^{-1}x$

$$\therefore \frac{dy}{dx} = \frac{4}{1+x^2}$$

At, $x = \sqrt{3}$

$$\frac{dy}{dx} = \frac{4}{1+\left(\sqrt{3}\right)^2} = \frac{4}{1+3} = 1$$

Q.54 If $f(x) = \cot^{-1}\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)$, $0 < x < \frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is

Correct option: (D)

Since, $1 - \sin\theta = \left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right)^2$ and $1 + \sin\theta = \left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)^2$

$\therefore f(x) = \cot^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$
$= \cot^{-1}\left(\frac{\cot\frac{x}{2} - 1}{1 + \cot\frac{x}{2}}\right)$
$= \cot^{-1}\left[\cot\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$

$\therefore f(x) = \frac{\pi}{4} + \frac{x}{2}$
$\therefore f'(x) = \frac{1}{2}$
$\therefore f'\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Q.55 If $a(4 + x^2) = x$ and $y - x^3 = a^2$ then $\frac{dy}{dx}$ at $x = 1$ is ________

Correct option: (D)

Let $a(4 + x^2) = x$
$\Rightarrow a = \frac{x}{4+x^2}$

$\therefore \quad \frac{da}{dx} = \frac{(4+x^2) - x(0+2x)}{(4+x^2)^2}$

$\therefore \quad \frac{da}{dx} = \frac{4-x^2}{(4+x^2)^2}$

$y - x^3 = a^2$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} - 3x^2 = 2a.\ \frac{da}{dx}$

$\Rightarrow \frac{dy}{dx} - 3x^2 = 2\left(\frac{x}{4+x^2}\right) \cdot \left(\frac{4-x^2}{(4+x^2)^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{2x(4-x^2)}{(4+x^2)^3} + 3x^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x=1)} = \frac{2\times 3}{(5)^3} + 3 = \frac{381}{125}$

Q.56 Let $y = t^{12} + 3$ and $x = t^{10} + 7.5$, then $\frac{d^2y}{dx^2}$ is

Correct option: (C)

$\frac{dy}{dt} = 12t^{11}$ and $\frac{dx}{dt} = 10t^9$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t^2}{5}$

$\therefore \frac{d^2y}{dx^2} = \frac{6}{5}\cdot 2t \cdot \frac{dt}{dx} = \frac{12t}{5} \cdot \frac{1}{10t^9} = \frac{6}{25t^8}$

Q.57 If $f(x) = x \cdot e^{x(1-x)}$, then $f(x)$ is

Correct option: (B)

$f(x) = x\, e^{x(1-x)}$

$\therefore \quad f'(x) = x\, e^{x(1-x)}[x(-1) + (1-x)] + e^{x(1-x)}(1)$

$= e^{x(1-x)}(x - 2x^2 + 1)$

For f(x) to be increasing, $f'(x) \geq 0$

$\Rightarrow e^{x(1-x)}(x - 2x^2 + 1) \geq 0$

$\Rightarrow x - 2x^2 + 1 \geq 0$

$\Rightarrow 2x^2 - x - 1 \leq 0$

$\Rightarrow (2x+1)(x-1) \leq 0$

$\Rightarrow x \in \left[-\frac{1}{2}, 1\right]$

For $f(x)$ to be decreasing, $f'(x) \leq 0$

$\Rightarrow (2x+1)(x-1) \geq 0$

$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$

Q.58 The radius of circular plate is increasing at the rate of 3 cm/sec. The rate of change of the circumference is

Correct option: (B)

$\frac{dr}{dt} = 3$

Now, circumference (C) = $2\pi r$

Differentiating w.r.t. 't', we get $\frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi$

$(3) = 6\pi$

$\therefore$ The rate of increase in circumference

$= 6\pi$ cm/sec

Q.59 The approximate value of $(1.002)^{300}$ using differentiation is

Correct option: (B)

Let $f(x) = x^{300}$

$\therefore \quad f'(x) = = 300x^{299}$

Here, a = 1 and h = 0.002

$f(a) = f(1) = (1)^{300} = 1$ and

$f'(a) = f'(1) = 300 \times (1)^{299} = 300$

$\therefore \quad f(a+h) \approx f(a) + hf'(a)$

$\therefore \quad (1.002)^{300} = 1 + (0.002)(300)$

$= 1 + 0.6$

$\therefore \quad (1.002)^{300} = 1.6$

Q.60 If $f(x) = 3x^3 + 2x^2 f'(1) + xf''(2) + f'''(3)$ then f(x) =_______.

Correct option: (B)

$f(x) = 3x^3 + 2x^2 f'(1) + xf''(2) + f'''(3)$

$\therefore \quad f'(x) = 9x^2 + 4x f'(1) + f''(2) \quad \ldots(i)$

$f''(x) = 18x + 4f'(1) \quad \ldots(ii)$

$f'''(x) = 18 \quad \ldots(iii)$

$\therefore \quad f''(3) = 18$

Substituting $x = 1$ in equation (i), we get

$f'(1) = 9 + 4f'(1) + f''(2)$

$\therefore \quad 3f'(1) + f''(2) = -9 \quad \ldots(iv)$

Substituting $x = 2$ in (ii), we get

$f''(2) = 36 + 4f'(1)$

$\Rightarrow 7f'(1) = -45 \quad \ldots$[From (iv)]

From (iii) and (iv), we get

$\Rightarrow f'(1) = \frac{-45}{7}$, $f''(2) = \frac{72}{7}$

Substituting values of $f'(1)$, $f''(2)$ and $f'''(3)$ in given equation, we get

$f(x) = 3x^3 + 2x^2\left(\frac{-45}{7}\right) + x\left(\frac{72}{7}\right) + 18$

$= \frac{1}{7}(21x^3 - 90x^2 + 72x + 126)$

Q.61 **If** $\log_{10}\left(\frac{x^2-y^2}{x^2+y^2}\right) = 2$**, then** $\frac{dy}{dx} =$

Correct option: (A)

$\log_{10}\left(\frac{x^2-y^2}{x^2+y^2}\right) = 2$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = 10^2$

$\Rightarrow x^2 - y^2 = 100x^2 + 100y^2$

$\Rightarrow 99x^2 + 101y^2 = 0$

Differentiating w.r.t. x, we get

$99(2x) + 101\left(2y\frac{dy}{dx}\right) = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{99x}{101y}$

Q.62 **If** $f(x) = \cos^{-1}x,\ g(x) = e^x$ **and**

$h(x) = g(f(x))$, **then** $\frac{h'(x)}{h(x)} =$

Correct option: (A)

$f(x) = \cos^{-1} x$

$g(x) = e^x$

$h(x) = g(f(x))$

$= e^{\cos^{-1}x}$

$h'(x) = e^{\cos^{-1}x} \cdot \frac{-1}{\sqrt{1-x^2}}$

$\therefore \frac{h'(x)}{h(x)} = \frac{e^{\cos^{-1}x} \times \frac{-1}{\sqrt{1-x^2}}}{e^{\cos^{-1}x}} = \frac{-1}{\sqrt{1-x^2}}$

Q.63 **Let g(x) be the inverse of the function f(x) and f ′(x) =** $\frac{1}{1+x^3}$**, then g′(x) is equal to**

Correct option: (C)

Since g(x) is the inverse of f(x).

$\therefore fog(x) = x$

$\Rightarrow \frac{d}{dx}[fog(x)] = \frac{d}{dx}(x)$

$\Rightarrow f'[g(x)] \cdot g'(x) = 1$

$\Rightarrow \frac{1}{1+[g(x)]^3} \cdot g'(x) = 1 \ldots$

$\left[\because f'(x) = \frac{1}{1+x^3}(given)\right]$

$\Rightarrow g'(x) = 1 + [g(x)]^3$

Q.64 **If x = a (1 – cos θ), y = a(θ – sin θ), then** $\frac{d^2y}{dx^2} =$

Correct option: (C)

$x = a(1 - \cos\theta)$ and $y = a(\theta - \sin\theta)$

$\therefore \frac{dx}{d\theta} = a\sin\theta$ and $\frac{dy}{d\theta} = a(1-\cos\theta)$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1-\cos\theta)}{a\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\therefore \frac{dy}{dx} = \tan\frac{\theta}{2}$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = \frac{1}{2}\sec^2\frac{\theta}{2} \times \frac{d\theta}{dx}$

$= \frac{1}{2}\sec^2\frac{\theta}{2} \times \frac{1}{a\sin\theta}$

$= \frac{\text{cosec}\theta}{2a\cos^2\left(\frac{\theta}{2}\right)}$

Q.65 **If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of** $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ **is $\sqrt{x}\cdot$ g(x), then g(x) equals**

Correct option: (B)

Let $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left[\frac{6x^{\frac{3}{2}}}{1-\left(3x^{\frac{3}{2}}\right)^2}\right]$

$= \tan^{-1}\left[\frac{2 \times 3x^{\frac{3}{2}}}{1-\left(3x^{\frac{3}{2}}\right)^2}\right]$

$= 2\tan^{-1} 3x^{\frac{3}{2}}$

$\therefore \frac{dy}{dx} = \frac{2}{1+\left(3x^{\frac{3}{2}}\right)^2} \cdot 3 \times \frac{3}{2} \times x^{\frac{1}{2}} = \frac{9}{1+9x^3}\sqrt{x}$

Comparing with $\sqrt{x}g(x)$, we get

$g(x) = \frac{9}{1+9x^3}$

Q.66 **If x = sec θ – cos θ and y = sec^n θ – cos^n θ, then**

Correct option: (A)

$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$

and

$\frac{dy}{d\theta} = n\sec^{n-1}\theta . \sec\theta\tan\theta - n\cos^{n-1}\theta . (-\sin\theta)$

$= n\sec^n\theta\tan\theta + n\cos^{n-1}\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{n\sec^n\theta\tan\theta + n\cos^{n-1}\theta\sin\theta}{\sec\theta\tan\theta + \sin\theta}$

$\frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{\sec\theta + \cos\theta}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2(\sec^n\theta + \cos^n\theta)^2}{(\sec\theta + \cos\theta)^2}$

$= \frac{n^2[(\sec^n\theta - \cos^n\theta)^2 + 4\sec^n\theta\cos^n\theta]}{(\sec\theta - \cos\theta)^2 + 4\sec\theta.\cos\theta}$

$= \frac{n^2(y^2+4)}{x^2+4}$

$\therefore (x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2 (y^2 + 4)$

Q.67 If the curves $y^2 = 6x$ and $9x^2$ + by $^2 = 16$ intersect each other at right angles, then the value of b is

Correct option: (D)

Let the given curves intersect each other at P(x_1, y_1).

$y^2 = 6x$

Differentiating w.r.t. x, we get

$2y\frac{dy}{dx} = 6 \Rightarrow \left(\frac{dy}{dx}\right)_P = \frac{3}{y_1}$

$9x^2 + by^2 = 16$

Differentiating w.r.t. x, we get

$18x + 2by\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_P = -\frac{9x_1}{by_1}$

Since the given curves intersect each other at right angles, $\left(\frac{3}{y_1}\right)\left(\frac{-9x_1}{by_1}\right) = -1$

$\Rightarrow \frac{27x_1}{by_1^2} = 1$

$\Rightarrow b = \frac{9}{2}$ $\quad \ldots[y_1^2 = 6x_1]$

Q.68 The displacement of a particle at time 't' is s = $t^3 - 4t^2 - 5t$, then the velocity of the particle at t = 2 sec. is

Correct option: (C)

$s = t^3 - 4t^2 - 5t$

Differentiating w.r.t. t on both sides, we get $\frac{ds}{dt} = 3t^2 - 8t - 5$

$\left.\frac{ds}{dt}\right|_{(t=2)} = 3 \times (2)^2 - 8 \times 2 - 5$

$= -9$

$\therefore$ velocity = –9 units/sec

Q.69 The diameter of a circle is increasing at the rate of 1 cm/sec. When its radius is π cm, then rate of increase of its area, is

Correct option: (D)

$\frac{dD}{dt} = 1$ cm/sec and r = π cm

$A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2$

$\frac{dA}{dt} = \frac{\pi D}{2}\frac{dD}{dt} = \frac{\pi \times (2\pi)}{2} \times 1 = \pi^2$

$\therefore$ The rate of increase in area = π^2 cm^2/sec

Q.70 If $x = at^2$, $y = 2at$, then $\frac{d^2x}{dy^2}$ =

Correct option: (A)

$x = at^2$

$\therefore \frac{dx}{dt} = 2at$

$y = 2at$

$\therefore \frac{dy}{dt} = 2a$ $\quad \ldots$(i)

$\therefore \frac{dx}{dy} = \frac{2at}{2a} \Rightarrow \frac{dx}{dy} = t$

$\therefore \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}(t)$

$\therefore \frac{d^2x}{dy^2} = \frac{dt}{dy}$

$\therefore \frac{d^2x}{dy^2} = \frac{1}{2a}$ $\quad \ldots$[From (i)]

Q.71 The area of a rectangle will be maximum for the given perimeter, when rectangle is a

Correct option: (C)

Let x and y be the lengths of two adjacent sides of the rectangle.

Then, its perimeter is P = 2(x + y) ….(i)

$\Rightarrow y = \frac{P-2x}{2}$

Area of rectangle, A = xy

$= x\left(\frac{P-2x}{2}\right) = \frac{Px - 2x^2}{2}$

$\therefore \frac{dA}{dx} = \frac{P-4x}{2}$ and $\frac{d^2A}{dx^2} = -2$

For maximum or minimum,

$\frac{dA}{dx} = 0$

$\Rightarrow \dfrac{P-4x}{2}=0$

⇒ P = 4x

⇒ 2x + 2y = 4x ….[From (i)]

⇒ $x = y$

$\therefore \left(\dfrac{d^2A}{dx^2}\right)_{x=y} = -2 < 0$

Hence, the area of a rectangle will be maximum when rectangle is a square.

Q.72 If x = a cos θ, y = b sin θ, then $\dfrac{dy}{dx}$ =

Correct option: (A)

x = a cos θ and y = b sin θ

$\therefore \dfrac{dx}{d\theta} = -$ a sin θ and $\dfrac{dy}{d\theta}$ = b cos θ

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{b\cos\theta}{-a\sin\theta} = \left(-\dfrac{b}{a}\right)\cot\theta$

Q.73 If $y = \log\left(x+\sqrt{x^2+a^2}\right)$, then $\dfrac{d^2y}{dx^2}$ is equal to

Correct option: (D)

$y = \log\left(x+\sqrt{x^2+a^2}\right)$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{x+\sqrt{x^2+a^2}}.\left(1+\dfrac{1}{2\sqrt{x^2+a^2}}.2x\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x+\sqrt{x^2+a^2}} \times \dfrac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2+a^2}}$

$\therefore \dfrac{d^2y}{dx^2} = \dfrac{-1}{2}\left(x^2+a^2\right)^{-\frac{3}{2}}.2x = \dfrac{-x}{(x^2+a^2)^{\frac{3}{2}}}$

Q.74 The derivative of $\cot^{-1} x$ w.r.t $\log(1+x^2)$ is

Correct option: (C)

Let u = $\cot^{-1} x$ and v = $\log(1+x^2)$

u = $\cot^{-1} x$

$\dfrac{du}{dx} = \dfrac{-1}{1+x^2}$

v = $\log(1+x^2)$

$\dfrac{dv}{dx} = \dfrac{1}{1+x^2} \times \dfrac{d}{dx}(1+x^2)$

$\therefore \dfrac{dv}{dx} = \dfrac{2x}{1+x^2}$

Now, $\dfrac{du}{dv} = \dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \dfrac{\frac{-1}{1+x^2}}{\frac{2x}{1+x^2}} = \dfrac{-1}{2x}$

Q.75 For every value of x, the function f(x) = $\dfrac{1}{5^x}$ is

Correct option: (A)

f(x) = 5^{-x}

$\therefore f'(x) = -5^{-x}\log_e 5 = -\dfrac{\log_e 5}{5^x}$

⇒ $f'(x) < 0$ for all x

i.e., f(x) is decreasing for all x.

Q.76 If $y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, then $\dfrac{dy}{dx}$ =

Correct option: (A)

Put x = tan θ ⇒ θ = $\tan^{-1}x$

$\therefore y = \sin^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\cos 2\theta)$

$= \sin^{-1}\left(\sin\left(\dfrac{\pi}{2}-2\theta\right)\right)$

$= \dfrac{\pi}{2} - 2\theta = \dfrac{\pi}{2} - 2\tan^{-1}x$

$\therefore \dfrac{dy}{dx} = -\dfrac{2}{1+x^2}$

Q.77 If

$x = \log_e\left(\dfrac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}\right), \tan\dfrac{y}{2} = \sqrt{\dfrac{1-t}{1+t}}$

. Then $(y_1)_{t=\frac{1}{2}}$ has the value

Correct option: (B)

$x = \log_e\left(\dfrac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}\right)$

$\Rightarrow e^x = \dfrac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}$

$\Rightarrow e^x = \tan\left(\dfrac{\pi}{4}-\dfrac{y}{2}\right)$ …(i) $\left[\because \tan\dfrac{\pi}{4}=1\right]$

Differentiating w.r.t. x, we get

$e^x = \sec^2\left(\dfrac{\pi}{4}-\dfrac{y}{2}\right)\cdot\left(\dfrac{-1}{2}\right)\dfrac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = -2e^x\cos^2\left(\frac{\pi}{4} - \frac{y}{2}\right)$

When t = $\frac{1}{2}$,

$\tan\frac{y}{2} = \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$

$\Rightarrow \tan\frac{y}{2} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{y}{2} = \frac{\pi}{6}$

Substituting $\frac{y}{2} = \frac{\pi}{6}$ in (i), we get

$e^x = \tan\frac{\pi}{12} = 2 - \sqrt{3}$

$\therefore \left(\frac{dy}{dx}\right)_{t=\frac{1}{2}} = -2\left(2-\sqrt{3}\right)\cos^2\frac{\pi}{12}$

$= -2\left(2-\sqrt{3}\right)\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2$

$= \frac{-1}{2}\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right) = -\frac{1}{2}$

Q.78 A bullet is shot horizontally and its distance S cm at time t second is given by S = 1200 t – 15 t^2, then the distance covered by the bullet when it comes to the rest, is

Correct option: (B)

s = 1200t – 15t^2

Velocity (v) = $\frac{ds}{dt}$ = 1200 – 30t

When bullet stopped, $\frac{ds}{dt} = 0$

$\Rightarrow$ t = 40

Hence, required distance

= 1200(40) – 15 × (40)^2 = 24000 cm

Q.79 If $x^2 + y^2 = t + \frac{2}{t}$ and $x^4 + y^4 = t^2 + \frac{4}{t^2}$, then $x^3y\,\frac{dy}{dx}$ =

Correct option: (B)

$x^2 + y^2 = t + \frac{2}{t}$

Squaring on both sides, we get

$x^4 + y^4 + 2x^2y^2 = t^2 + \frac{4}{t^2} + 4$

$\Rightarrow \left(t^2 + \frac{4}{t^2}\right) + 2x^2y^2 = t^2 + \frac{4}{t^2} + 4$

$\Rightarrow x^2y^2 = 2$ ...(i)

Differentiating both sides w.r.t. x, we get

$x^2.2y\,\frac{dy}{dx} + y^2.2x = 0$

$\Rightarrow x^2\,y\,\frac{dy}{dx} = -xy^2$

$\Rightarrow x^3\,y\,\frac{dy}{dx} = -x^2\,y^2$

$\Rightarrow x^3y\,\frac{dy}{dx} = -2$ ...[From (i)]

Q.80 If tan x = $\frac{2t}{1-t^2}$ and sin y = $\frac{2t}{1+t^2}$, then the value of $\frac{dy}{dx}$ is

Correct option: (A)

Given: tan x = $\frac{2t}{1-t^2}$, sin y = $\frac{2t}{1+t^2}$

Put t = tan θ, θ = tan^{-1} t

$\therefore$ tan x = $\frac{2tan\theta}{1-tan^2\theta}$ and sin y = $\frac{2tan\theta}{1+tan^2\theta}$

tan x = tan 2θ and sin y = sin 2θ

$\Rightarrow x = 2\theta$ and $y = 2\theta$

$\therefore y = x$

$\therefore \frac{dy}{dx} = 1$

Q.81 A particle moves so that the space described in time t is square root of a quadratic function of t, then

Correct option: (C)

$s = \sqrt{at^2+bt+c}$

$\therefore v = \frac{ds}{dt} = \frac{1}{2}\frac{2at+b}{\sqrt{at^2+bt+c}}$

$= \frac{2at+b}{2s}$

acceleration = $\frac{d^2s}{dt^2} = \frac{dv}{dt}$

$= \frac{2s(2a) - (2at+b)\cdot 2\frac{ds}{dt}}{4s^2}$

$= \frac{4a\,s - 2(2a\,t+b)\frac{(2a\,t+b)}{2s}}{4s^2}$

$= \frac{4as^2 - (2at+b)^2}{4s^3}$

$= \frac{4a(at^2+bt+c) - (4a^2t^2+4abt+b^2)}{4s^3}$

$= \dfrac{4ac - b^2}{4s^3}$

$\therefore$ acceleration varies as $\dfrac{1}{s^3}$

Q.82 If $y = (x \log x)^{\log(\log x)}$, then $\frac{dy}{dx} =$

Correct option: (A)

$(x\log x)^{\log(\log x)}\left\{\frac{1}{x\log x}[\log x + \log(\log x)] + \log(\log x)\left(\frac{1}{x} + \frac{1}{x\log x}\right)\right\}$

$y = (x \log x)^{\log(\log x)}$

Taking logarithm on both sides, we get

$\log y = \log(\log x)[\log x + \log(\log x)]$

Differentiating w.r.t. x, we get

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x\log x}[\log x + \log(\log x)] + \log(\log x)\left(\frac{1}{x} + \frac{1}{x\log x}\right)$

$\Rightarrow \frac{dy}{dx}$

$= (x\log x)^{\log(\log x)}\left\{\frac{1}{x\log x}[\log x + \log(\log x)] + \log(\log x)\left(\frac{1}{x} + \frac{1}{x\log x}\right)\right\}$

Q.83 The point on the curve $y^2 = 2(x - 3)$ at which the normal is parallel to the line $y - 2x + 1 = 0$ is

Correct option: (D)

$y^2 = 2(x - 3)$ …(i)

Differentiating w.r.t. x on both sides, we get

$2y\dfrac{dy}{dx} = 2$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{y}$

Slope of the normal $= \dfrac{-1}{\frac{dy}{dx}} = -y$.

Slope of the given line is 2

Since the normal is parallel to the given line

$\therefore y = -2$

From (i), $(-2)^2 = 2(x - 3)$

$\Rightarrow 2x = 10$

$\Rightarrow x = 5$

$\therefore$ Co-ordinates of the point are (5, –2).

Q.84 The minimum value of $f(x) = \sin x(1 + \cos x)$ is

Correct option: (A)

$f(x) = \sin x(1 + \cos x)$

$= \sin x + \sin x \cos x$

$\therefore f(x) = \sin x + \frac{1}{2}\sin 2x$

$\therefore f'(x) = \cos x + \cos 2x = 2\cos\frac{x}{2}\cos\frac{3x}{2}$

$\therefore f'(x) = 0 \Rightarrow \cos\frac{x}{2} = 0$ or $\cos\frac{3x}{2} = 0$

$\therefore \frac{x}{2} = \frac{\pi}{2}$ or $\frac{3x}{2} = \frac{\pi}{2}$

$\therefore x = \pi$ or $x = \frac{\pi}{3}$

$f''(x) = -\sin x - 2\sin 2x < 0$, only when $x = \frac{\pi}{3}$

$\therefore$ The maximum value of function is at $\frac{\pi}{3}$

$\therefore f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right) = \frac{\sqrt{3}}{2} \times \frac{3}{2} = \frac{3\sqrt{3}}{4}$

Q.85 If $(a + bx)e^{\frac{y}{x}} = x$, then $x^3\dfrac{d^2y}{dx^2}$ is equal to

Correct option: (B)

$(a + bx).\ e^{\frac{y}{x}} = x$

$\Rightarrow e^{\frac{y}{x}} = \dfrac{x}{a + bx}$

$\Rightarrow \dfrac{y}{x} = \log\left(\dfrac{x}{a + bx}\right)$

$\Rightarrow \dfrac{y}{x} = \log x - \log(a + bx)$

Differentiating w.r.t. x, we get

$\dfrac{x\frac{dy}{dx} - y.1}{x^2} = \dfrac{1}{x} - \dfrac{b}{a + bx}$

$\Rightarrow x\dfrac{dy}{dx} - y = x^2\left(\dfrac{1}{x} - \dfrac{b}{a + bx}\right)$

$\Rightarrow x\dfrac{dy}{dx} - y = \dfrac{ax}{a + bx}$ …(i)

Again, differentiating w.r.t. x, we get

$x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - \dfrac{dy}{dx} = \dfrac{(a + bx)a - ax(b)}{(a + bx)^2}$

$\Rightarrow x\dfrac{d^2y}{dx^2} = \dfrac{a^2}{(a + bx)^2}$

$\Rightarrow x^3\dfrac{d^2y}{dx^2} = \dfrac{x^2a^2}{(a + bx)^2}$

$\Rightarrow x^3\dfrac{d^2y}{dx^2} = \left(\dfrac{ax}{a + bx}\right)^2$

$\Rightarrow x^3\dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$ …[From (i)]

Q.86 If $y = \log\left|\dfrac{x + \sqrt{x^2 + 25}}{\sqrt{x^2 + 25} - x}\right|$, then $\dfrac{dy}{dx} =$

Correct option: (C)

$$y = \log\left|\frac{x+\sqrt{x^2+25}}{\sqrt{x^2+25}-x} \times \frac{\sqrt{x^2+25}+x}{\sqrt{x^2+25}+x}\right|$$

$$\Rightarrow y = \log\left|\frac{\left(x+\sqrt{x^2+25}\right)^2}{25}\right|$$

$$\Rightarrow y = 2\log\left|x+\sqrt{x^2+25}\right| - \log 25$$

$\Rightarrow$

$$\frac{dy}{dx} = \frac{2}{\left(x+\sqrt{x^2+25}\right)}\left[1+\frac{1}{2\sqrt{x^2+25}}.(2x)\right]-0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\left(x+\sqrt{x^2+25}\right)}{\left(x+\sqrt{x^2+25}\right)\sqrt{x^2+25}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{x^2+25}}$$

Q.87 If $y = \tan^{-1}\left(\sqrt{x^2+y^2}\right) + \cot^{-1}\left(\sqrt{x^2+y^2}\right)$, then $\frac{dy}{dx}$ =

Correct option: (B)

Given: $y = \tan^{-1}\left(\sqrt{x^2+y^2}\right) + \cot^{-1}\left(\sqrt{x^2+y^2}\right)$

$y = \frac{\pi}{2} \quad \ldots\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \frac{dy}{dx} = 0$

Q.88 If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(1)$ =

Correct option: (A)

Given, $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$, then $\theta = \tan^{-1}x$

$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

$\therefore y' = \frac{1}{2(1+x^2)}$

$\therefore y'(1) = \frac{1}{4}$

Q.89 A stone is dropped into a quiet lake and waves move in circles at speed of 10 cm/sec. At the instant when the radius of the circular wave is 15 cm. how fast is the enclosed area increasing?

Correct option: (D)

Let r be the radius.

Then, $\frac{dr}{dt}$ = 10 cm/sec

Area = A = πr^2

$\therefore \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi(15)(10) \quad \ldots[\because r = 15 \text{ cm}]$

$= 300\pi$ cm^2/sec

Q.90 If the line $9x - y - 14 = 0$ is tangent to the curve $y = ax^3 + bx + 2$ at point (2, 4), then

Correct option: (D)

Point (2, 4) lies on the curve $y = ax^3 + bx + 2$

$\therefore 4 = 8a + 2b + 2$

$\therefore 4a + b = 1 \quad \ldots.(i)$

Also, slope of tangent at (2, 4) is equal to slope of line $9x - y - 14 = 0$

$\Rightarrow \frac{dy}{dx} = 9$

$\Rightarrow 3ax^2 + b = 9$

At $x = 2$,

$3a \times 4 + b = 9$

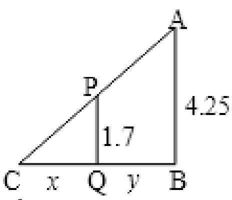
$\Rightarrow 12a + b = 9 \quad \ldots.(ii)$

Solving (i) and (ii), we get

a = 1, b = –3

Q.91 A man of height 1.7 metre is moving away from a lamp post at the rate of 2.7 m/sec. If the height of the lamp post be 4.25 metre, then the rate at which the shadow of the man is lengthening is

Correct option: (D)



$\frac{dy}{dt} = 2.7$

From the figure,

$\frac{x}{1.7} = \frac{x+y}{4.25}$

$x = \frac{2}{3}y \Rightarrow \frac{dx}{dt} = \frac{2}{3}.\frac{dy}{dt}$

$\therefore$ Required rate of length of shadow

$= \frac{dx}{dt} = 1.8$ m/s

Q.92 If the radius of a circle is increasing at a uniform rate of 2cm./sec. The rate of increase of area of circle at the instant when the radius is 20 cm, is

Correct option: (C)

Given, $\frac{dr}{dt}$ = 2 cm/sec, where r be the radius of circle and t be the time.

Now, area of circle is given by A = πr^2

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi \,.\, 20 \,.2$

$\therefore \frac{dA}{dt} = 80\,\pi$ cm^2/sec

∴ The rate of change of area of circle with respect to time is 80 π cm^2/sec.

Q.93 If $y = \tan^{-1}\left(\frac{1-\cos 3x}{\sin 3x}\right)$, then $\frac{dy}{dx}$ =

Correct option: (D)

Given : $y = \tan^{-1}\left(\frac{1-\cos 3x}{\sin 3x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{2\sin^2\frac{3x}{2}}{2\sin\left(\frac{3x}{2}\right).\cos\left(\frac{3x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{3x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{3x}{2}\right)\right)$

$\Rightarrow y = \frac{3x}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2}$

Q.94 The minimum value of f(x) = $a^2\cos^2 x + b^2\sin^2 x$ if $a^2 > b^2$, is

Correct option: (A)

$f(x) = a^2\cos^2 x + b^2\sin^2 x$

$f'(x) = 2a^2\cos x(-\sin x) + 2b^2\sin x \cdot \cos x$

$= -a^2\sin 2x + b^2\sin 2x$

$= \sin 2x(b^2 - a^2)$

For maximum minimum,

$f'(x) = 0$

$\Rightarrow \sin 2x\,(b^2 - a^2) = 0$ …[since $b^2 - a^2 \neq 0$, $a^2 > b^2$ Given]

$\Rightarrow \sin 2x = 0$

$\Rightarrow 2x = 0$ or $2x = \pi$

$\Rightarrow x = 0$ or $x = \frac{\pi}{2}$

Now, $f''(x) = (b^2 - a^2)\, 2\cos 2x$

$= 2\cos 2x\,(b^2 - a^2)$

$f''(0) = 2 \times 1 \times (b^2 - a^2)$

$= 2(b^2 - a^2)$

i.e., $f''(0) < 0$

∴ f(x) is maximum at x = 0

Minimum value of f(x) is

$f\left(\frac{\pi}{2}\right) = a^2\left(\cos\frac{\pi}{2}\right)^2 + b^2\left(\sin\frac{\pi}{2}\right)^2$

$= b^2$

Q.95 The sum of two natural numbers is 16. Their product is maximum if the numbers are

Correct option: (A)

Let x and y be two natural numbers such that $x + y = 16$ and the product is xy.

$xy = x(16 - x) = 16x - x^2 = f(x)$

$\therefore f'(x) = 16 - 2x$

$\therefore f''(x) = -2$

Roots of $f'(x) = 0$,

i.e., $16 - 2x = 0$, i.e., $x = 8$

$f'(8) = 16 - 16 = 0$

∴ f is maximum when $x = 8$, $y = 8$

∴ The product is maximum if $x = 8$, $y = 8$

Q.96 If $y = e^{\frac{x^2}{1+x^2}}$, then $\frac{dy}{dx}$ =

Correct option: (A)

$y = e^{\frac{x^2}{1+x^2}}$

$\therefore \quad \frac{dy}{dx} = e^{\frac{x^2}{1+x^2}} \frac{d}{dx}\left(\frac{x^2}{1+x^2}\right)$

$= e^{\frac{x^2}{1+x^2}}.\left[\frac{(1+x^2).(2x) - x^2.(0+2x)}{(1+x^2)^2}\right]$

$= \frac{2x\,e^{\frac{x^2}{1+x^2}}}{(1+x^2)^2}$

Q.97 Let C be a curve given by $y(x) = 1 + \sqrt{4x-3}$, $x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is

Correct option: (A)

$y(x) = 1 + \sqrt{4x-3}$

$\therefore \frac{dy}{dx} = \frac{4}{2\sqrt{4x-3}} = \frac{2}{\sqrt{4x-3}}$

$\Rightarrow \frac{2}{\sqrt{4x-3}} = \frac{2}{3}$

$\Rightarrow x = 3$

$\therefore y = 4$

$\therefore$ Equation of normal is

$y - 4 = \frac{-3}{2}(x-3)$

$\Rightarrow 2y - 8 = -3x + 9$

$\Rightarrow 3x + 2y - 17 = 0$

$\therefore$ Option (A) i.e., (1, 7) satisfies above equation.

Q.98 If $f(x) = x^3 - 10x^2 + 200x - 10$, then

Correct option: (C)

$f(x) = x^3 - 10x^2 + 200x - 10$

$\Rightarrow f'(x) = 3x^2 - 20x + 200$

For $f(x)$ to be increasing $f'(x) > 0$

$\Rightarrow 3x^2 - 20x + 200 > 0$

$$\Rightarrow 3\left(x^2 - \frac{20}{3}x + \frac{200}{3} + \frac{100}{9} - \frac{100}{9}\right) > 0$$

$$\Rightarrow 3\left[\left(x - \frac{10}{3}\right)^2 + \frac{500}{9}\right] > 0$$

$$\Rightarrow 3\left(x - \frac{10}{3}\right)^2 + \frac{500}{3} > 0$$

Always increasing throughout real line.

Q.99 Differentiation of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right)$ is

Correct option: (B)

Let $u = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

and $v = \cos^{-1}\left[\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right]$

Put $x = \tan\theta$, then $\theta = \tan^{-1} x$

$$\therefore u = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\therefore u = \frac{\tan^{-1} x}{2} \quad \ldots\text{(i)}$$

$$v = \cos^{-1}\left[\sqrt{\frac{1+\sqrt{1+\tan^2\theta}}{2\sqrt{1+\tan^2\theta}}}\right]$$

$$= \cos^{-1}\left[\sqrt{\frac{1+\sec\theta}{2\sec\theta}}\right]$$

$$= \cos^{-1}\left[\sqrt{\frac{1+\frac{1}{\cos\theta}}{\frac{2}{\cos\theta}}}\right]$$

$$= \cos^{-1}\left[\sqrt{\frac{1+\cos\theta}{2}}\right]$$

$$= \cos^{-1}\left(\sqrt{\frac{2\cos^2\left(\frac{\theta}{2}\right)}{2}}\right)$$

$$= \cos^{-1}\left(\cos\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\therefore v = \frac{\tan^{-1} x}{2} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get

$u = v$

$\therefore \quad \frac{du}{dv} = 1$

Q.100 The function f defined by $f(x) = (x+2)e^{-x}$ is

Correct option: (C)

$f(x) = (x + 2)e^{-x}$

$\therefore \quad f'(x) = e^{-x} - e^{-x}(x + 2) = -e^{-x}(x + 1)$

For $f(x)$ to be increasing,

$-e^{-x}(x + 1) > 0 \Rightarrow e^{-x}(x + 1) < 0$

$\Rightarrow (x + 1) < 0$

$\Rightarrow x < -1$

$\therefore \quad x \in (-\infty, -1)$

$\therefore$ The function is increasing in $(-\infty, -1)$.

$\therefore$ For $f(x)$ to be decreasing, $-e^{-x}(x + 1) < 0$

$\Rightarrow e^{-x}(x + 1) > 0$

$\Rightarrow x + 1 > 0$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

$\therefore$ The function is decreasing in $(-1, \infty)$.