Differentiation **Marks: 120**

ANSWER KEY

Maths

Q.1 D	Q.2 A	Q.3 A	Q.4 C	Q.5 A	Q.6 A	Q.7 D	Q.8 B
Q.9 C	Q.10 B	Q.11 A	Q.12 B	Q.13 B	Q.14 B	Q.15 C	Q.16 B
Q.17 A	Q.18 A	Q.19 B	Q.20 D	Q.21 B	Q.22 D	Q.23 D	Q.24 B
Q.25 A	Q.26 B	Q.27 D	Q.28 C	Q.29 D	Q.30 A	Q.31 B	Q.32 C
Q.33 B	Q.34 B	Q.35 C	Q.36 D	Q.37 C	Q.38 C	Q.39 B	Q.40 B
Q.41 D	Q.42 C	Q.43 C	Q.44 C	Q.45 B	Q.46 A	Q.47 A	Q.48 A
Q.49 C	Q.50 D	Q.51 B	Q.52 D	Q.53 D	Q.54 D	Q.55 D	Q.56 B
Q.57 B	Q.58 C	Q.59 B	Q.60 C				

KUNAL ACADEMY

SOLUTION

Maths

Q.1 **If u = log $\left(\sqrt{x-1}-\sqrt{x+1}\right)$ and $v=\sqrt{x+1}+\sqrt{x-1}$ then $\frac{\mathrm{du}}{\mathrm{dv}}=\ldots.$**

Correct option: (D)

u = log $\left(\sqrt{x-1}-\sqrt{x+1}\right)$

$\therefore$

$\frac{du}{dx}=\frac{1}{\sqrt{x-1}-\sqrt{x+1}}\cdot\frac{d}{dx}\left(\sqrt{x-1}-\sqrt{x+1}\right)$

$=\frac{1}{\sqrt{x-1}-\sqrt{x+1}}\left(\frac{1}{2\sqrt{x-1}}-\frac{1}{2\sqrt{x+1}}\right)$

$=\frac{1}{\sqrt{x-1}-\sqrt{x+1}}\left(\frac{\sqrt{x+1}-\sqrt{x-1}}{2\sqrt{x^2-1}}\right)$

$=\frac{-1}{2\sqrt{x^2-1}}$

v = $\sqrt{x+1}+\sqrt{x-1}$

$\therefore\quad\frac{dv}{dx}=\frac{1}{2\sqrt{x+1}}+\frac{1}{2\sqrt{x-1}}$

$=\frac{\sqrt{x-1}+\sqrt{x+1}}{2\sqrt{x^2-1}}$

$\therefore\quad\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{-1}{\sqrt{x-1}+\sqrt{x+1}}=\frac{-1}{v}$

Q.2 **If $y=(x\log x)^{\log(\log x)}$, then $\frac{dy}{dx}=$**

Correct option: (A)

$(x\log x)^{\log(\log x)}\left\{\frac{1}{x\log x}[\log x+\log(\log x)]+\log(\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)\right\}$

$y=(x\log x)^{\log(\log x)}$

Taking logarithm on both sides, we get

$\log y=\log(\log x)[\log x+\log(\log x)]$

Differentiating w.r.t. x, we get

$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{x\log x}[\log x+\log(\log x)]+\log(\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)$

$\Rightarrow\frac{dy}{dx}$

$=(x\log x)^{\log(\log x)}\left\{\frac{1}{x\log x}[\log x+\log(\log x)]+\log(\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)\right\}$

Q.3 **If $y=\left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx}=$**

Correct option: (A)

$y=\left(1+\frac{1}{x}\right)^x$

Taking logarithm on both sides, we get

$\log y=x\log\left(1+\frac{1}{x}\right)$

Differentiating w.r.t. x, we get

$\frac{1}{y}\cdot\frac{dy}{dx}=\log\left(1+\frac{1}{x}\right)+x\cdot\frac{1}{1+\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)$

$\Rightarrow\frac{1}{y}\cdot\frac{dy}{dx}=\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}$

$\Rightarrow\frac{dy}{dx}=\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$

Q.4 **If f(1) = 3, f ′(1) = 2, then $\frac{d}{dx}$ {log f(e^x + 2x)} at x = 0 is**

Correct option: (C)

Let $y=\frac{d}{dx}[\log f(e^x+2x)]$

$=\frac{1}{f(e^x+2x)}\cdot\frac{d}{dx}[f(e^x+2x)]$

$=\frac{1}{f(e^x+2x)}\cdot f'(e^x+2x).\frac{d}{dx}(e^x+2x)$

$=\frac{f'(e^x+2x)(e^x+2)}{f(e^x+2x)}$

$\therefore(y)_{(x=0)}=\frac{f'(1).3}{f(1)}=\frac{2.3}{3}=2$

Q.5 **If $f(x)=\sin^{-1}\left(\frac{2\cdot3^x}{1+9^x}\right)$, then $f'\left(\frac{1}{2}\right)$ equals**

Correct option: (A)

f(x) = $\sin^{-1}\left(\frac{2.3^x}{1+9^x}\right)=\sin^{-1}\left(\frac{2.3^x}{1+(3^x)^2}\right)$

KUNAL ACADEMY

Put $3^x = \tan\theta \Rightarrow \theta = \tan^{-1}(3^x)$

$\therefore$ $f(x) = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$\therefore$ $f(x) = 2\tan^{-1}(3^x)$

$\therefore$ $f'(x) = 2\cdot\dfrac{1}{1+(3^x)^2}\cdot 3^x \log 3$

$\therefore$ $f'\left(\dfrac{1}{2}\right) = 2\cdot\dfrac{1}{1+\left(3^{\frac{1}{2}}\right)^2}\cdot 3^{\frac{1}{2}}\log 3$

$= \dfrac{1}{2}\sqrt{3}\log 3 = \sqrt{3}\log\sqrt{3}$

Q.6 The slope of tangent to the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point M (2, –1) is

Correct option: (A)

$x = 2$ and $x = t^2 + 3t - 8$

$\therefore t^2 + 3t - 8 = 2$

$\therefore t^2 + 3t - 10 = 0$

$\Rightarrow t = 2$ or -5

$y = -1$ and $y = 2t^2 - 2t - 5$

$\therefore 2t^2 - 2t - 5 = -1$

$\therefore 2t^2 - 2t - 4 = 0$

$\Rightarrow t = 2$ or -1

$\therefore$ t = 2 satisfies both equations corresponding to co-ordinates of point M.

Hence, t = 2

Now, $\frac{dy}{dt} = 4t - 2$

$\frac{dx}{dt} = 2t + 3$

$\therefore \frac{dy}{dt} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t-2}{2t+3}$

At t = 2,

$\frac{dy}{dx} = \frac{4\times2-2}{2\times2+3} = \frac{6}{7}$

Q.7 If $e^{-y}\cdot y = x$, then $\dfrac{dy}{dx}$ is

Correct option: (D)

$e^{-y}\cdot y = x$

$\Rightarrow e^{-y} = \dfrac{x}{y}$

$\Rightarrow e^{y} = \dfrac{y}{x}$

Taking log on both sides, we get

$y = \log y - \log x$

Differentiating w.r.t. 'x', we get

$\dfrac{dy}{dx} = \dfrac{1}{y}\dfrac{dy}{dx} - \dfrac{1}{x}$

$\dfrac{(y-1)}{y}\dfrac{dy}{dx} = -\dfrac{1}{x}$

$\dfrac{dy}{dx} = \dfrac{y}{x(1-y)}$

Q.8 The minimum value of function $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on [0, 3] is equal to

Correct option: (B)

$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$

$\therefore f'(x) = 12x^3 - 24x^2 + 24x - 48$

$= 12(x^3 - 2x^2 + 2x - 4) = 12[(x-2)(x^2+2)]$

For maximum or minimum of $f(x)$, $f'(x) = 0$

$\Rightarrow 12[(x-2)(x^2+2)] = 0$

$\Rightarrow x = 2$.

Now, $f''(x) = 12(3x^2 - 4x + 2)$

$\therefore f''(2) = 12(12 - 8 + 2) = 72 > 0$

$\therefore$ f has minimum at $x = 2$ and the minimum value is $f(2) = 48 - 64 + 48 - 96 + 25 = -39$

Q.9 If $f(x) = |x - 2|$, where $x \in [0, 4]$, then the Rolle's theorem cannot be applied to the function because

Correct option: (C)

$f(x) = |x-2| = \begin{cases} x-2 & , \ x \geq 2 \\ -(x-2) & , \ x < 2 \end{cases}$

$f(x)$ is continuous at every point in [0, 4]

$f(0) = 2 - 0 = 2$ and $f(4) = 4 - 2 = 2$

$\therefore$ $f(0) = f(4)$

$f'(x) = 1, \quad x \geq 2$

$f'(x) = -1, \ x < 2$

$\therefore$ $f(x)$ is not differentiable at $x = 2$.

Hence, Rolle's theorem cannot be applied.

Q.10 The equation of tangent to the curve $y = \cos(x + y)$ where $-2\pi \leq x \leq 2\pi$ and which is parallel to the line $x + 2y = 0$, is

Correct option: (B)

$y = \cos(x + y)$

$\therefore$ $\dfrac{dy}{dx} = -\sin(x+y)\cdot\left(1 + \dfrac{dy}{dx}\right)$

$\therefore$ $\dfrac{dy}{dx} = -\sin(x+y) - \sin(x+y)\dfrac{dy}{dx}$

$\therefore$ $(1 + \sin(x+y))\dfrac{dy}{dx} = -\sin(x+y)$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{-\sin(x+y)}{1+\sin(x+y)}$

Slope of line $x + 2y = 0$ is $\dfrac{-1}{2}$.

KUNAL ACADEMY

Since the tangent is parallel to $x + 2y = 0$.

$\therefore \quad \frac{-1}{2} = \frac{-\sin(x+y)}{1+\sin(x+y)}$

$\Rightarrow 1 + \sin(x + y) = 2\sin(x + y)$

$\Rightarrow \sin(x + y) = 1$

$\Rightarrow x + y = \frac{\pi}{2}$

$\Rightarrow y = \cos(x + y) = \cos\left(\frac{\pi}{2}\right) = 0$

$\Rightarrow y = 0$

$\therefore$ point P $= \left(\frac{\pi}{2}, 0\right)$

Equation of tangent at $\left(\frac{\pi}{2}, 0\right)$ is

$y - 0 = \frac{-1}{2}\left(x - \frac{\pi}{2}\right)$

$\Rightarrow 2y = -x + \frac{\pi}{2}$

$\Rightarrow 4y = -2x + \pi$

$\Rightarrow 2x + 4y - \pi = 0$

Q.11 A manufacturer can sell x items at the price of ₹ $(330 - x)$ each. The cost of producing x items is ₹ $x^2 + 10x - 12$. How many items must be sold so that his profit is maximum?

Correct option: (A)

Given:

The selling price of one item = ₹ $(330 - x)$

$\therefore$ The selling price of x items = ₹ $(330 - x)\,x$

The cost price of x items = ₹ $(x^2 + 10x - 12)$

We need to maximize profit

$\therefore$ Profit = Selling Price – Cost Price

$= (330 - x)\,x - (x^2 + 10x - 12)$

$= 330x - x^2 - x^2 - 10x + 12$

$\therefore$ Profit $= 12 + 320x - 2x^2$

Let $f(x) = 12 + 320x - 2x^2$

$\Rightarrow f'(x) = 320 - 4x$

$\therefore$ For maxima, $f'(x) = 0$

$\Rightarrow 320 - 4x = 0$

$\Rightarrow x = 80$

$\therefore \quad f''(x) = -4$

$f''(80) = -4 < 0$

$\Rightarrow$ f is maximum at $x = 80$.

Q.12 The derivative of

$$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$$ **is**

Correct option: (B)

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$\therefore \quad y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1 - \tan\theta}{1 + \tan\theta}\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] = \frac{\pi}{4} - \theta$

$\therefore \quad y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Differentiating w.r.t. x, we get

$\Rightarrow \frac{dy}{dx} = \frac{-1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$

Q.13 The two parts of 100 for which the sum of double of first and square of second part is minimum, are

Correct option: (B)

Let one number be $(100 - x)$ and then another is x.

Therefore $f(x) = 2(100 - x) + x^2 = x^2 - 2x + 200$

$\therefore f'(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$

Here $f''(x) = 2 > 0$

Therefore function is minimum at $x = 1$.

So the numbers are 99 and 1.

Q.14 The equation of the normal to the curve $x = \theta + \sin\theta, y = 1 + \cos\theta$ **at** $\theta = \frac{\pi}{2}$ **is**

Correct option: (B)

$x = \theta + \sin\theta$, $y = 1 + \cos\theta$

$\frac{dx}{d\theta} = 1 + \cos\theta$, $\frac{dy}{d\theta} = -\sin\theta$

$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta}{1 + \cos\theta}$

KUNAL ACADEMY

$\therefore \quad \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-\sin\left(\frac{\pi}{2}\right)}{1+\cos\left(\frac{\pi}{2}\right)} = \frac{-1}{1+0} = -1$

At $\theta = \frac{\pi}{2}$,

$x = \frac{\pi}{2} + \sin\frac{\pi}{2} = 1 + \frac{\pi}{2}$, $y = 1 + \cos\frac{\pi}{2} = 1$

$\therefore$ Equation of the normal is

$$(y-1) = \frac{-1}{\left(\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}\right)}\left(x - 1 - \frac{\pi}{2}\right)$$

$\therefore \quad (y-1) = 1\left(x - 1 - \frac{\pi}{2}\right)$

$\therefore \quad 2x - 2y - \pi = 0$

Q.15 If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is _____

Correct option: (C)

$y = f(x^2 + 2)$

$\therefore \frac{dy}{dx} = f'(x^2 + 2).(2x)$

$\therefore \left(\frac{dy}{dx}\right)_{x=1} = f'(1^2 + 2).(2 \times 1)$

$= f'(3).2 = 5.2 = 10$

Q.16 If x is real, then the difference between the greatest and least values of $\frac{x^2 - x + 1}{x^2 + x + 1}$ is

Correct option: (B)

Let $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow \frac{dy}{dx} =$

$$\frac{(x^2 + x + 1)(2x - 1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2}$$

$\Rightarrow \frac{dy}{dx} = \frac{2x^2 - 2}{(x^2 + x + 1)^2}$

For maximum or minimum,

$\frac{dy}{dx} = 0$

$\Rightarrow 2x^2 - 2 = 0$

$\Rightarrow x = -1, 1$

$\frac{d^2y}{dx^2} = \frac{4(-x^3 + 3x + 1)}{x^2 + x + 1}$

At $x = -1$, $\frac{d^2y}{dx^2} < 0$, the function will occupy maximum value.

$\therefore \quad f(-1) = 3$

At $x = 1$, $\frac{d^2y}{dx^2} > 0$, the function will occupy minimum value.

$\therefore \quad f(1) = \frac{1}{3}$

$$Max.\,value - Min.\,value = 3 - \frac{1}{3} = \frac{8}{3}$$

Q.17 If $x^2 + y^2 = 1$, then $\frac{d^2x}{dy^2} =$

Correct option: (A)

$x^2 + y^2 = 1$

Differentiating w.r.t. x, we get

$2x + 2y\frac{dy}{dx} = 0$

$\therefore \quad \frac{dy}{dx} = -\frac{x}{y}$ …(i)

$\Rightarrow \frac{dx}{dy} = -\frac{y}{x}$ …(ii)

Differentiating w.r.t. y, we get

$$\frac{d^2x}{dy^2} = -1\left[\frac{x(1) - y\frac{dx}{dy}}{x^2}\right]$$

$$= -1\left[\frac{x - y\frac{dx}{dy}}{x^2}\right]$$

$$= -1\left[\frac{x - y \times \left(-\frac{y}{x}\right)}{x^2}\right] \quad \text{…[From (i)]}$$

$$= -1\left[\frac{x + \frac{y^2}{x}}{x^2}\right]$$

$$= -\left[\frac{x^2 + y^2}{x^3}\right]$$

$= -\frac{1}{x^3}$ …[$\because x^2 + y^2 = 1$ (given)]

Q.18 The angle between the curves $xy = 6$ and $x^2y = 12$ is

Correct option: (A)

KUNAL ACADEMY

Solving $xy = 6$ and $x^2y = 12$, we get

$x = 2, y = 3$

$\therefore$ Consider, $xy = 6$

$\Rightarrow y = \frac{6}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{-6}{x^2}$

$m_1 = \left(\frac{dy}{dx}\right)_{\text{at } x=2} = \frac{-6}{4} = \frac{-3}{2}$

Now, $x^2y = 12$

$\Rightarrow y = \frac{12}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{12(-2)}{x^3} = \frac{-24}{x^3}$

$m_2 = \frac{dy}{dx}_{\text{at } x=2} = \frac{-24}{8} = -3$

Using, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$

$\Rightarrow \theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$

$= \tan^{-1}\left|\frac{\frac{-3}{2} + 3}{1 + \left(\frac{-3}{2}\right)(-3)}\right|$

$= \tan^{-1}\left(\frac{3}{11}\right)$

Q.19 If Mean value theorem holds for the function

$f(x) = (x-1)(x-2)(x-3), x \in [0, 4]$

then the values of c as per the theorem are

Correct option: (B)

$f(x) = (x-1)(x-2)(x-3)$

$f(4) = (4-1)(4-2)(4-3)$

$f(4) = 6$

$f(0) = (0-1)(0-2)(0-3) = -6$

$\therefore$ Using LMVT

$f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{6 - (-6)}{4} = 3$

$\therefore$ $f'(c) = 3$ …(i)

Now, $f(x) = (x-1)(x-2)(x-3)$

$= x^3 - 6x^2 + 11x - 6$

$\therefore$ $f'(x) = 3x^2 - 12x + 11$

$f'(c) = 3$ …[From (i)]

$\Rightarrow 3c^2 - 12c + 11 = 3$

$\Rightarrow 3c^2 - 12c + 8 = 0$

$\Rightarrow c = \frac{12 \pm \sqrt{48}}{6} = 2 \pm \frac{2\sqrt{3}}{3} = 2 \pm \frac{2}{\sqrt{3}}$

Q.20 If $f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$ is strictly increasing for all real values of x, then

Correct option: (D)

$f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$

Differentiating w.r.t.x, we get

$f'(x) = \frac{(\sin x + \cos x) \times (k\cos x - 2\sin x) - (k\sin x + 2\cos x) \times (\cos x - \sin x)}{(\sin x + \cos x)^2}$

$= \frac{(k\sin x\cos x - 2\sin^2 x + k\cos^2 x - 2\sin x\cos x) - (k\sin x\cos x - k\sin^2 x + 2\cos^2 x - 2\sin x\cos x)}{(\sin x + \cos x)^2}$

$= \frac{(k-2)\sin^2 x + (k-2)\cos^2 x}{(\sin x + \cos x)^2}$

$= \frac{(k-2)(\sin^2 x + \cos^2 x)}{(\sin x + \cos x)^2}$

$f'(x) = \frac{k-2}{(\sin x + \cos x)^2}$

function $f(x)$ is strictly increasing ...[Given]

$\therefore$ $f'(x) > 0$

$\therefore$ $\frac{k-2}{(\sin x - \cos x)^2} > 0$

$\Rightarrow k - 2 > 0$

$\Rightarrow k > 2$

Q.21 If $\frac{x}{\sqrt{1+x}} + \frac{y}{\sqrt{1+y}} = 0$, $x \neq y$, then

$(1+x)^2\frac{dy}{dx} =$

Correct option: (B)

$\frac{x}{\sqrt{1+x}} + \frac{y}{\sqrt{1+y}} = 0$

$\Rightarrow x\left(\sqrt{1+y}\right) + y\left(\sqrt{1+x}\right) = 0$

$\Rightarrow x\left(\sqrt{1+y}\right) = -y\left(\sqrt{1+x}\right)$

KUNAL ACADEMY

Squaring on both sides, we get

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + xy^2$

$\Rightarrow x^2 - y^2 = xy^2 - x^2y$

$\Rightarrow x^2 - y^2 = -xy(x-y)$

$\Rightarrow (x+y) = -xy$

$\Rightarrow y = \dfrac{-x}{1+x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-(1+x)(1)+x}{(1+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\Rightarrow \frac{dy}{dx}(1+x)^2 = -1$$

Q.22 **If $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$, then $f'(1)$ =**

Correct option: (D)

$y = (1+x)(1+x^2)(1+x^4)(1+x^8)$ …(i)

Taking 'log' on both sides, we get

$\log y = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}$$

…(ii)

At $x = 1$, (i) $\Rightarrow y = 16$

$\therefore$ (ii) $\Rightarrow \left(\dfrac{dy}{dx}\right)_{x=1} = 16\left(\dfrac{1}{2} + \dfrac{2}{2} + \dfrac{4}{2} + \dfrac{8}{2}\right) =$ 120

Q.23 **Let $\phi(x)$ be the inverse of the function $f(x)$ and $f'(x) = \dfrac{1}{1+x^5}$, then $\dfrac{d}{dx}\phi(x)$ is**

Correct option: (D)

Given

$f^{-1}(x) = \phi(x)$

$\therefore \quad f[f^{-1}(x)] = f[\phi(x)]$

$\Rightarrow x = f[\phi(x)]$

Differentiating w.r.t. x, we get

$\Rightarrow 1 = f'[\phi(x)] \cdot \phi'(x)$

$\Rightarrow \phi'(x) = \dfrac{1}{f'[\phi(x)]}$

$\Rightarrow \phi'(x) = \dfrac{1}{\frac{1}{1+(\phi(x))^5}} \quad \ldots\left[f'(x) = \dfrac{1}{1+x^5}\right]$

$\therefore \quad \phi'(x) = 1 + (\phi(x))^5$

Q.24

The derivative of every even function

Correct option: (B)

Let f(x) be an even function and $f'(x)$ be its derivative.

As the function is even hence $f(x) = f(-x)$

Differentiating on both sides w.r.t. x we get,

$f'(x) = -f'(-x)$ which satisfies the definition of odd function.

Q.25 **If $x = a\cos^4\theta$, $y = a\sin^4\theta$, then $\dfrac{dy}{dx}$ at $\theta = \dfrac{3\pi}{4}$ is**

Correct option: (A)

$x = a\cos^4\theta$ and $y = a\sin^4\theta$

$\therefore \dfrac{dx}{d\theta} = -4a\cos^3\theta\sin\theta$

and $\dfrac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{-\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$

$\therefore$

$$\left(\frac{dy}{dx}\right)_{\left(\theta=\frac{3\pi}{4}\right)} = -\tan^2\left(\frac{3\pi}{4}\right) = -(-1)^2 = -1$$

Q.26 **If $f(x) = \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then**

$\lim\limits_{x\to 0}\dfrac{f'(x)}{x}$ **is equal to**

Correct option: (B)

$f(x) = \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$

KUNAL ACADEMY

$$\therefore\ f'(x)=\begin{vmatrix}1 & \sin x & \cos x\\ 2x & \tan x & -x^3\\ 2 & \sin 2x & 5x\end{vmatrix}$$

$$\begin{vmatrix}x & \cos x & \cos x\\ x^2 & \sec^2 x & -x^3\\ 2x & 2\cos 2x & 5x\end{vmatrix}\begin{vmatrix}x & \sin x & -\sin x\\ x^2 & \tan x & -3x^2\\ 2x & \sin 2x & 5\end{vmatrix}$$

$$\therefore\ \begin{vmatrix}1 & \sin x & \cos x\\ 2 & \frac{\tan x}{x} & -x^2\\ 2 & \sin 2x & 5x\end{vmatrix}\begin{vmatrix}1 & \cos x & \cos x\\ x & \sec^2 x & -x^3\\ 2 & 2\cos 2x & 5x\end{vmatrix}$$

$$\begin{vmatrix}1 & \sin x & -\sin x\\ x & \tan x & -3x^2\\ 2 & \sin 2x & 5\end{vmatrix}$$

$$\therefore\ \lim_{x\to 0}\frac{f'(x)}{x}\begin{vmatrix}1 & 0 & 1\\ 2 & 1 & 0\\ 2 & 0 & 0\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ 2 & 2 & 0\end{vmatrix}\begin{vmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 2 & 0 & 5\end{vmatrix}$$

$= -2 - 2 + 0 = -4.$

Q.27 The value of a for which the function f(x) = a sin x + $\frac{1}{3}$sin 3x has an extremum at x = $\frac{\pi}{3}$ is

Correct option: (D)

f(x) = a sin x + $\frac{1}{3}$sin 3x

$\Rightarrow$ f ′(x) = a cos x + $\frac{1}{3}$3 cos 3x

$\Rightarrow$ f ′(x) = a cos x + cos 3x

$\Rightarrow$ f ′ $\left(\frac{\pi}{3}\right)$ = a cos $\frac{\pi}{3}$ + cosπ = $\frac{a}{2}$ − 1

$\Rightarrow \frac{a}{2} - 1 = 0$

$\Rightarrow$ a = 2

Q.28 The equation of tangent to the curves x = a secθ, y = a tanθ at θ = $\frac{\pi}{6}$ is

Correct option: (C)

At θ = $\frac{\pi}{6}$,

x = a sec $\frac{\pi}{6} = \frac{2a}{\sqrt{3}}$ and y = a tan $\frac{\pi}{6} = \frac{a}{\sqrt{3}}$

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sec^2\theta}{a\sec\theta\tan\theta}=\frac{1}{\sin\theta}=\operatorname{cosec}\theta$$

$$\therefore\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{6}} = \operatorname{cosec}\frac{\pi}{6} = 2$$

∴ Equation of the tangent at $\left(\frac{2a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$ is

$$y-\frac{a}{\sqrt{3}}=2\left(x-\frac{2a}{\sqrt{3}}\right)$$

$\Rightarrow 2x - y = \sqrt{3}\,a$

Q.29 The equation of the horizontal tangent to the curve $y = e^x + e^{-x}$ is

Correct option: (D)

$y = e^x + e^{-x}$ ...(i)

$$\therefore\ \frac{dy}{dx} = e^x - e^{-x}$$

The slope of the horizontal tangent is $\frac{dy}{dx} = 0$

∴ $0 = e^x - e^{-x}$

$\Rightarrow e^x = e^{-x}$

$\Rightarrow e^{2x} = 1$

$\Rightarrow x = 0$

Substituting x = 0 in (i), we get

$y = e^0 + e^0 = 2$

Q.30 If $y = e^{\frac{x^2}{1+x^2}}$, then $\frac{dy}{dx}$ =

Correct option: (A)

$y = e^{\frac{x^2}{1+x^2}}$

$$\therefore\ \frac{dy}{dx} = e^{\frac{x^2}{1+x^2}}\ \frac{d}{dx}\left(\frac{x^2}{1+x^2}\right)$$

$$= e^{\frac{x^2}{1+x^2}}.\left[\frac{(1+x^2).(2x)-x^2.(0+2x)}{(1+x^2)^2}\right]$$

$$= \frac{2x\,e^{\frac{x^2}{1+x^2}}}{(1+x^2)^2}$$

Q.31 Let P(x) be a polynomial of degree 2, with P(2) = –1, P′(2) = 0, P″ (2) = 2, then P(1.001) is

Correct option: (B)

Let P(x) = ax^2 + bx + c

$\Rightarrow P'(x)$ = 2ax + b

$\Rightarrow$ P ″(x) = 2a

P ″(2) = 2a

$\Rightarrow$ 2 = 2a

$\Rightarrow$ a = 1

$P'(2)$ = 2a(2) + b

KUNAL ACADEMY

$\Rightarrow 0 = 4a + b$

$\Rightarrow 0 = 4(1) + b$

$\Rightarrow b = -4$

$P(2) = a(2)^2 + b(2) + c$

$\Rightarrow -1 = 4a + 2b + c$

$\Rightarrow -1 = 4(1) + 2(-4) + c$

$\Rightarrow c = 3$

$\therefore \quad P(x) = x^2 - 4x + 3$

$\Rightarrow P'(x) = 2x - 4$

$x = 1.001 = 1 + 0.001 = a + h$

Here, $a = 1, h = 0.001$

$P(a) = P(1) = 1 - 4 + 3 = 0$

$P'(a) = P'(1) = 2 - 4 = -2$

$\therefore \quad P(1.001) \approx 0 + (0.001)(-2) \approx -0.002$

Q.32 If $y = \tan^{-1}(\sec x + \tan x)$, then $\frac{dy}{dx} =$

Correct option: (C)

$y = \tan^{-1}(\sec x + \tan x)$

$= \tan^{-1}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$

$= \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$

$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right]$

$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$

$= \tan^{-1}\left[\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right]$

$= \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$

$\therefore \quad y = \frac{\pi}{4} + \frac{x}{2}$

$\therefore \quad \frac{dy}{dx} = \frac{1}{2}$

Q.33 The sum of two natural numbers is 20. Their product is maximum if the numbers are

Correct option: (B)

Let x and y be two natural numbers such that $x + y = 20$ and the product is xy.

$f(x) = xy = x(20 - x) = 20x - x^2$

$\therefore f'(x) = 20 - 2x$

$\therefore f''(x) = -2 < 0$

$f'(x) = 0$

$\Rightarrow 20 - 2x = 0$, i.e., $x = 10$

$\therefore$ f is maximum when $x = 10, y = 10$

$\therefore$ The product is maximum if $x = 10, y = 10$

Q.34 The edge of a cube is increasing at the rate of 5cm/sec. How fast is the volume of the cube increasing when the edge is 12cm long?

Correct option: (B) 2160 cm^3/sec

$\frac{da}{dt} = 5$ ....(i)

But if a is edge of a cube, then $V = a^3$

$\therefore \frac{dV}{dt} = 3a^2\frac{da}{dt} = 3a^2 . 5$

$= 15a^2 = 15 \times (12)^2$ ...[$\because$ edge a = 12 cm]

$= 2160 \text{ cm}^3/\text{sec}$

Q.35 Let $f(x) = x^2 + 2x + 2$, $g(x) = -x^2 + 2x - 1$ and a, b be the extreme values of $f(x)$, $g(x)$ respectively. If c is the extreme value of $\frac{f}{g}(x)$ (for $x \neq 1$), then $a + 2b + 5c + 4 =$

Correct option: (C)

$f(x) = x^2 + 2x + 2$

$\therefore f'(x) = 2x + 2$

Consider $f'(x) = 0$, we get

$2x + 2 = 0$

i.e., $x = -1$

$f(x)$ has extreme value at $x = -1$.

$\therefore$ Extreme value of $f(x) = f(-1) = 1 = a$

$g(x) = -x^2 + 2x - 1$

$\therefore g'(x) = -2x + 2$

Consider $g'(x) = 0$, we get

$-2x + 2 = 0$

$\therefore x = 1$

$\therefore g(x)$ has extreme value at $x = 1$.

$\therefore$ Extreme value of $g(x) = g(1) = 0 = b$

Now,

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 2x + 2}{-x^2 + 2x - 1} = \frac{x^2 + 2x + 2}{-(x - 1)^2}$$

KUNAL ACADEMY

∴

$$\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} = \frac{-2(x-1)^2(x+1)+2(x-1)(x^2+2x+2)}{(x-1)^4}$$

Consider, $\frac{d\left(\frac{f}{g}(x)\right)}{dx} = 0$

i.e., $-x^2 + 1 + x^2 + 2x + 2 = 0$

$\therefore x = \frac{-3}{2}$

$\therefore \frac{f}{g}(x)$ has extreme value at $x = \frac{-3}{2}$.

$\therefore$ Extreme value of $\frac{f}{g}(x) = \frac{f}{g}\left(\frac{-3}{2}\right) = -\frac{1}{5} = c$

$\therefore a + 2b + 5c + 4 = 4$

Q.36 **For $x \in \left(0, \frac{5\pi}{2}\right)$, define f($x$) = $\int_0^x \sqrt{t}\sin t\,dt$. Then, f has [AIEEE 2011]**

Correct option: (D)

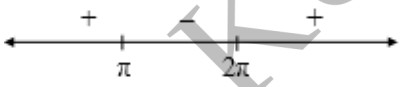
$f(x) = \int_0^x \sqrt{t}\sin t\,dt \Rightarrow f'(x) = \sqrt{x}\sin x$

For local maximum or minimum of f(x),

$f'(x) = 0 \Rightarrow \sqrt{x}\sin x = 0$

$\Rightarrow x = \pi, 2\pi \ldots \left[\because x \in \left(0, \frac{5\pi}{2}\right)\right]$

The changes in signs of f ′(x) in the neighbourhoods of π and 2π are as shown below:



Clearly, f ′(x) changes its sign from positive to negative in the neighborhood of $x = \pi$ and negative to positive in the neighborhood of $x = 2\pi$. Thus, f(x) has a local maximum at $x = \pi$ and a local minimum at $x = 2\pi$.

Q.37 **If $\tan y = \frac{x\sin\alpha}{1 - x\cos\alpha}$ and $\frac{dy}{dx} = \frac{m}{x^2 + 2nx + 1}$, then $m^2 + n^2$ is**

Correct option: (C)

$$\tan y = \frac{x\sin\alpha}{1 - x\cos\alpha}$$

$$\therefore \quad y = \tan^{-1}\left(\frac{x\sin\alpha}{1 - x\cos\alpha}\right)$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{1 + \left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)^2}\frac{d}{dx}\left(\frac{x\sin\alpha}{1 - x\cos\alpha}\right)$$

$$= \frac{1}{\frac{1-2x\cos\alpha+x^2\cos^2\alpha+x^2\sin^2\alpha}{(1-x\cos\alpha)^2}} \times$$

$$\frac{(1 - x\cos\alpha)\sin\alpha + (x\sin\alpha)\cos\alpha}{(1 - x\cos\alpha)^2}$$

$$= \frac{\sin\alpha - x\sin\alpha\cos\alpha + x\sin\alpha\cos\alpha}{1 + 2(-\cos\alpha)x + x^2}$$

$$= \frac{\sin\alpha}{x^2 + 2(-\cos\alpha)x + 1}$$

$$= \frac{m}{x^2 + 2nx + 1} \quad \ldots[\text{Given}]$$

$\Rightarrow n = -\cos\alpha$ and $m = \sin\alpha$

$\Rightarrow m^2 + n^2 = 1$

Q.38 **If $\sin^2 x + \cos^2 y = 1$, then $\frac{dy}{dx}$ =**

Correct option: (C)

$\sin^2 x + \cos^2 y = 1$

Differentiating w.r.t. x, we get

$\Rightarrow 2\sin x \cdot \cos x + 2\cos y\,(-\sin y)\frac{dy}{dx} = 0$

$\Rightarrow \sin 2x + \sin 2y\frac{dy}{dx}$

$$\therefore \quad \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

Q.39 **Rolle's theorem is true for the function $f(x) = x^2 - 4$ in the interval**

Correct option: (B)

If Rolle's theorem is true for any function f(x) in [a,b].

Then f(a) = f(b)

Only option [B] satisfies this condition.

Q.40 **The approximate value of $f(x) = 3x^2 + 5x + 3$ at $x = 3.02$ is**

Correct option: (B)

$f(x) = 3x^2 + 5x + 3$

$\therefore \quad f'(x) = 6x + 5$

KUNAL ACADEMY

Here, a = 3 and h = 0.02

$\therefore$ f(a + h) $\approx$ f(a) + hf ′(a)

$\approx 3(3)^2 + 5(3) + 3 + 0.02(3(6) + 5)$

$\approx 45 + 0.46$

≈ 45.46

Q.41 **If x = a sec^2 θ, y = a tan^2 θ, then $\frac{d^2y}{dx^2}$ =**

Correct option: (D)

x = a sec^2 θ, y = a tan^2 θ

$\therefore \frac{dx}{d\theta}$ = 2a sec^2 θ tan θ

and $\frac{dy}{d\theta}$ = 2a tan θ sec^2 θ

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\tan\theta\sec^2\theta}{2a\sec^2\theta\tan\theta} = 1$

$\therefore \frac{d^2y}{dx^2} = 0$

Q.42 **If $y = \left[(\tan x)^{\tan x}\right]^{\tan x}$, then at $x = \frac{\pi}{4}$, the value of $\frac{dy}{dx}$ =**

Correct option: (C)

$y = \left[(\tan x)^{\tan x}\right]^{\tan x}$

Taking logarithm on both sides, we get

log y = tan x log(tan x)$^{\tan x}$

$\Rightarrow$ log y = (tan x)^2 log (tan x)

Differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = (\tan x)^2 \frac{1}{\tan x}\sec^2 x$ + log (tan x) . 2 tan x . sec^2 x

$\Rightarrow \frac{dy}{dx} = \left[(\tan x)^{\tan x}\right]^{\tan x}$. tan x sec^2 x [1 + 2 log (tan x)]

$\therefore \left(\frac{dy}{dx}\right)_{\left(x=\frac{\pi}{4}\right)} = 1.1.\left(\sqrt{2}\right)^2 (1 + 0) = 2$

Q.43 **For $x \in$ R, f(x) = |log 2 – sin x| and g(x) = f(f(x)), then**

Correct option: (C)

Given: f(x) = |log 2 – sin x| and g(x) = f(f(x))

As $x \to 0$, log 2 > sin x

$\therefore$ f(x) = log 2 – sin x and f(0) = log 2

$\Rightarrow$ f ′(x) = –cos x and f ′(0) = –1

Now, g(x) = f(f(x))

$\Rightarrow$ g ′(x) = f ′(f(x)) · f ′(x)

$\Rightarrow$ g ′(0) = f ′(f(0)) · f ′(0)

$\Rightarrow$ g ′(0) = f ′(log 2) (–1)

$\Rightarrow$ g ′(0) = –(–cos (log 2))

$\Rightarrow$ g ′(0) = cos (log 2)

Q.44 **If x = sinθ, y = sin^{3}θ, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is**

Correct option: (C)

x = sin θ and y = sin^3 θ

$\therefore$ $y = x^3$

$\therefore$ $\frac{dy}{dx} = 3x^2$

$\therefore$ $\frac{d^2y}{dx^2} = 6x$

At $\theta = \frac{\pi}{6}$, $x = \sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{6}} = \left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} = 6\left(\frac{1}{2}\right) = 3$

Q.45 **The curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n$ = 2, n $\in$ N touches the line at the point (a, b), then the equation of the lines is**

Correct option: (B)

The equation of curve is

$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

Differentiating w.r.t. x, we get

$n\left(\frac{x}{a}\right)^{n-1}\left(\frac{1}{a}\right) + n\left(\frac{y}{b}\right)^{n-1}\left(\frac{1}{b}\right)\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{n}{a}\right)\left(\frac{x}{a}\right)^{n-1} + \left(\frac{n}{b}\right)\left(\frac{y}{b}\right)^{n-1}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-\left(\frac{n}{a}\right)\left(\frac{x}{a}\right)^{n-1}}{\left(\frac{n}{b}\right)\left(\frac{y}{b}\right)^{n-1}} = -\left(\frac{b}{a}\right)\left(\frac{bx}{ay}\right)^{n-1}$

$\therefore \left(\frac{dy}{dx}\right)_{(a,\,b)} = \left(\frac{-b}{a}\right)\left(\frac{ab}{ab}\right)^{n-1} = \frac{-b}{a}$

$\therefore$ Equation of required line is

y – b = $\frac{-b}{a}$ (x – a)

$\Rightarrow$ ay – ab = –bx + ab

$\Rightarrow$ bx + ay = 2ab

KUNAL ACADEMY

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$

Q.46 10 is divided into two parts such that the sum of double of the first and square of the other is minimum, then the numbers are respectively

Correct option: (A)

Let one number be $(10 - x)$ and then another is x.

$\therefore$ $f(x) = 2(10 - x) + x^2$

$= x^2 - 2x + 20$

$\therefore$ $f'(x) = 0$

$\Rightarrow 2x - 2 = 0$

$\Rightarrow x = 1$

Here, $f''(x) = 2 > 0$

$\therefore$ Function is minimum at $x = 1$.

So the numbers are 9 and 1.

Q.47 If the function $f(x) = x^3 - 12ax^2 + 36a^2x - 4 (a > 0)$ attains its maximum and minimum at $x = p$ and $x = q$ respectively, and if $3p = q^2$, then a is equal to

Correct option: (A)

$f(x) = x^3 - 12ax^2 + 36a^2x - 4$

Differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12a(2x) + 36a^2(1)$

$= 3x^2 - 24ax + 36a^2$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 24ax + 36a^2 = 0$

$\therefore x^2 - 8ax + 12a^2 = 0$

$\therefore (x - 2a)(x - 6a) = 0$

$\therefore x = 2a$ or $x = 6a$

Also, $f''(x) = 6x - 24a$

$[f''(x)]_{x=2a} = 12a - 24a = -12a < 0$

$[f''(x)]_{x=6a} = 36a - 24a = 12a > 0$

$\therefore$ Maxima at $p = 2a$ and minima at $q = 6a$

$3p = q^2$ ….(given)

$\therefore 3 \times 2a = (6a)^2$

$\therefore 6a = 36a^2$

$\therefore a = \frac{1}{6}$

Q.48 A stone is dropped into a pond. Waves in the form of circles are generated and radius of outermost ripple increases at the rate of 5 cm/sec. Then area increased after 2 seconds is

Correct option: (A)

Given that $\frac{dr}{dt} = 5$ cm/sec

Area = $A = \pi r^2$

Differentiating w.r.t. 't', we get

$\frac{dA}{dt} = 2\pi r . \frac{dr}{dt}$

After 2 seconds, radius = $r = 2 \times 5 = 10$

$\left.\frac{dA}{dt}\right|_{t=2} = 2\pi \times 10 \times 5 = 100\ \pi\ cm^2/sec$

Q.49 The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when the side is 10 cm is

Correct option: (C)

Let 'a' be the side of the equilateral triangle and 'A' be its area.

The formula for the area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}a^2$.

We are given that the sides are increasing at the rate of 2 cm/sec, which means $\frac{da}{dt} = 2$ cm/sec.

We need to find the rate at which the area increases, i.e., $\frac{dA}{dt}$, when the side $a = 10$ cm.

Differentiate the area formula with respect to time 't' using the chain rule:

$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{\sqrt{3}}{4}a^2\right)$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}a\frac{da}{dt}$

Now, substitute the given values: $a = 10$ cm and

$\frac{da}{dt} = 2$ cm/sec.

$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times 10 \times 2$

$\frac{dA}{dt} = 10\sqrt{3}$ sq. cm/sec.

Thus, the rate at which the area increases is $10\sqrt{3}$ sq. unit/sec.

The correct option is (C).

KUNAL ACADEMY

Q.50 **If $y = \tan^{-1}\left(\frac{1-\cos 3x}{\sin 3x}\right)$, then $\frac{dy}{dx}$=**

Correct option: (D)

Given : $y = \tan^{-1}\left(\frac{1-\cos 3x}{\sin 3x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{2\sin^2\frac{3x}{2}}{2\sin\left(\frac{3x}{2}\right).\cos\left(\frac{3x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{3x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{3x}{2}\right)\right)$

$\Rightarrow y = \frac{3x}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2}$

Q.51 **If $y = 2^{ax}$ and $\left(\frac{dy}{dx}\right)_{x=1}$ = log 256, then**

a =

Correct option: (B)

$y = 2^{ax}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = 2^{ax}\cdot(\log 2)\cdot(a)$

$\left(\frac{dy}{dx}\right)_{x=1}$ = log 256 …[Given]

$\Rightarrow 2^{a(1)}(\log 2)(a) = \log 256$

$\Rightarrow 2^{a}\cdot a(\log 2) = 8\log 2$

$\Rightarrow 2^{a}\cdot a = 8$

$\Rightarrow a = 2$

Q.52 **If u = $\cos^3 x$, v = $\sin^3 x$, then $\left(\frac{dv}{du}\right)_{x=\frac{\pi}{4}}$ is**

equal to

Correct option: (D)

u = $\cos^3 x$ and v = $\sin^3 x$

$\frac{du}{dx} = 3\cos^2 x(-\sin x)$ and $\frac{dv}{dx} = 3\sin^2 x(\cos x)$

$\therefore \frac{dv}{du} = \frac{\frac{dv}{dx}}{\frac{du}{dx}} = \frac{3\sin^2 x(\cos x)}{3\cos^2 x(-\sin x)} = -\tan x$

$\therefore \left(\frac{dv}{du}\right)_{\text{at } x=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = -1$

Q.53 **If $y = \cos(2x + 45)$, then $\frac{dy}{dx}$ is**

Correct option: (D)

$y = \cos(2x + 45)$

$\therefore \frac{dy}{dx} = -\sin(2x + 45)\cdot\frac{d}{dx}(2x + 45)$

$= -2\sin(2x + 45)$

Q.54 **The approximate value of the function $f(x) = x^3 - 3x + 5$ at $x = 1.99$ is**

Correct option: (D)

$f(x) = x^3 - 3x + 5$

$f'(x) = 3x^2 - 3$

Here, a = 2 and h = –0.01

$\therefore$ f(a) = f(2) = 7 and

$f'(a) = f'(2) = 9$

$\therefore$ $f(a + h) \approx f(a) + h f'(a)$

$\approx 7 - (0.01)(9) \approx 6.91$

Q.55 **If $\sqrt{r} = ae^{\theta\cot\alpha}$, where a and α are real numbers, then $\frac{d^2r}{d\theta^2} - 4r\cot^2\alpha$ is**

Correct option: (D)

Given, $\sqrt{r} = a.e^{\theta(\cot\alpha)}$

$\Rightarrow r = a^2.e^{2\theta(\cot\alpha)}$

$\Rightarrow \frac{dr}{d\theta} = a^2.2\cot\alpha.e^{2\theta(\cot\alpha)}$

$\Rightarrow \frac{dr}{d\theta} = 2a^2\cot\alpha.e^{2\theta(\cot\alpha)}$

$\Rightarrow \frac{d^2r}{d\theta^2} = 4a^2\cot^2\alpha.e^{2\theta(\cot\alpha)}$

$\therefore \frac{d^2r}{d\theta^2} - 4r\cot^2\alpha$

$= 4a^2\cot^2\alpha.e^{2\theta(\cot\alpha)} - 4a^2\cot^2\alpha.e^{2\theta(\cot\alpha)}$

$= 0$

Q.56 **The approximate value of $(1.002)^{300}$ using differentiation is**

Correct option: (B)

Let $f(x) = x^{300}$

$\therefore$ $f'(x) = = 300x^{299}$

Here, a = 1 and h = 0.002

$f(a) = f(1) = (1)^{300} = 1$ and

$f'(a) = f'(1) = 300 \times (1)^{299} = 300$

$\therefore$ $f(a + h) \approx f(a) + hf'(a)$

$\therefore$ $(1.002)^{300} = 1 + (0.002)(300)$

$= 1 + 0.6$

$\therefore$ $(1.002)^{300} = 1.6$

KUNAL ACADEMY

Q.57 **If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then**

$\frac{dy}{dx}$ **= [Kerala (Engg.) 2005]**

Correct option: (B)

$y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$

$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}(\tan x)$

$\therefore y = \tan^{-1}\left(\frac{a}{b}\right) - x$

$\therefore \frac{dy}{dx} = -1$

Q.58 **If the function $y = \frac{ax+b}{(x-4)(x-1)}$ has an extreme at P(2, –1), then the values of a and b are**

Correct option: (C)

$y = \frac{ax+b}{(x-4)(x-1)} = \frac{ax+b}{x^2-5x+4}$

$\therefore \frac{dy}{dx} = \frac{(x^2-5x+4)a - (ax+b)(2x-5)}{(x^2-5x+4)^2}$

For extreme (i.e., maximum or minimum)

$\frac{dy}{dx} = 0$

$\therefore a(x^2 - 5x + 4) - (ax + b)(2x - 5) = 0$

Since y has an extreme at P(2, –1),

$x = 2$ satisfies above equation

$a(4 - 10 + 4) - (2a + b)(-1) = 0$

$\Rightarrow -2a + 2a + b = 0$

$\Rightarrow b = 0$

$x = 2$, $y = -1$ satisfies the equation of the curve

$\therefore -1 = \frac{a(2)+b}{4-10+4}$

$\therefore -1 = \frac{2a+0}{-2} = -a$

$\Rightarrow a = 1$

$\therefore a = 1, b = 0$

Q.59 **If $y = \sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$, then $\frac{dy}{dx}$ is equal to**

Correct option: (B)

$y = \sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$

$= \sin^{-1}\left(\frac{3x}{2} - 4\left(\frac{x}{2}\right)^3\right)$

Put $\frac{x}{2} = \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{2}\right)$

$\therefore y = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$\therefore y = 3\sin^{-1}\left(\frac{x}{2}\right)$

$\therefore \frac{dy}{dx} = 3 \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{3}{\sqrt{4 - x^2}}$

Q.60 **If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax - 6$ for $x \in [1, 3]$, then a + 4b =**

Correct option: (C)

$f(x) = x^3 + bx^2 + ax - 6$

$f(1) = 1 + b + a - 6 = a + b - 5$

$f(3) = 27 + 9b + 3a - 6 = 3a + 9b + 21$

$f(1) = f(3)$

$\therefore a + b - 5 = 3a + 9b + 21$

$\Rightarrow 2a + 8b = -26$

$\Rightarrow a + 4b = -13$

KUNAL ACADEMY