Matrices **Marks: 100**

ANSWER KEY

Maths

Q.1 A	Q.2 B	Q.3 C	Q.4 C	Q.5 D	Q.6 C	Q.7 B	Q.8 B
Q.9 C	Q.10 D	Q.11 D	Q.12 D	Q.13 D	Q.14 C	Q.15 C	Q.16 A
Q.17 B	Q.18 C	Q.19 D	Q.20 C	Q.21 C	Q.22 A	Q.23 A	Q.24 D
Q.25 C	Q.26 A	Q.27 B	Q.28 A	Q.29 C	Q.30 A	Q.31 B	Q.32 B
Q.33 D	Q.34 B	Q.35 C	Q.36 A	Q.37 A	Q.38 B	Q.39 B	Q.40 D
Q.41 C	Q.42 B	Q.43 C	Q.44 A	Q.45 D	Q.46 B	Q.47 C	Q.48 D
Q.49 A	Q.50 B						

Maths

Q.1 **If A = $\begin{bmatrix} 5 & 9 & 6 \\ -5 & 7 & 2 \\ 3 & -8 & 4 \end{bmatrix}$, then $(A^2 - 7A)A^{-1}$ =**

Correct option: (A)

$(A^2 - 7A)\,A^{-1} = A.A.A^{-1} - 7A\,.\,A^{-1}$

$= A - 7I$

$=$

$\begin{bmatrix} 5 & 9 & 6 \\ -5 & 7 & 2 \\ 3 & -8 & 4 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -2 & 9 & 6 \\ -5 & 0 & 2 \\ 3 & -8 & -3 \end{bmatrix}$

Q.2 **If A $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ then $\left(A^2 - 5A\right)^{-1}$ is**

Correct option: (B)

$A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ 42 & 23 \end{bmatrix}$

$\therefore \quad A^2 - 5A = \begin{bmatrix} 1 & 1 \\ 7 & 3 \end{bmatrix}$

$|A^2 - 5A| = 3 - 7 = -4$

$\therefore \quad (A^2 - 5A)^{-1} = \left(-\frac{1}{4}\right)\begin{bmatrix} 3 & -1 \\ -7 & 1 \end{bmatrix} =$

$\frac{1}{4}\begin{bmatrix} -3 & 1 \\ 7 & -1 \end{bmatrix}$

Q.3 **The sum of three numbers is 6. Thrice the third number when added to the first number gives 7. On adding three times first number to the sum of second and third numbers we get 12. The product of these numbers is**

Correct option: (C)

Let the three numbers be x, y and z.

According to the first condition,

$x + y + z = 6$ …(i)

According to the second condition,

$x + 3z = 7$ …(ii)

According to the third condition,

$3x + y + z = 12$ …(iii)

From (i) and (iii), we get

$x = 3$

Substituting $x = 3$ in (ii), we get

$z = \frac{4}{3}$

Substituting $x = 3$, $z = \frac{4}{3}$ in (i), we get

$y = \frac{5}{3}$

$\therefore \quad xyz = (3)\left(\frac{5}{3}\right)\left(\frac{4}{3}\right) = \frac{20}{3}$

Q.4 **For a invertible matrix A if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| =**

Correct option: (C)

$A\,(\text{adj } A) = |A|\, I_n$

$\Rightarrow \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = |A|\, I_n$

$\Rightarrow 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|\, I_n$

$\Rightarrow 10\, I_n = |A|\, I_n$

$\Rightarrow |A| = 10$

Q.5 **If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then $A^4 A^{-1}$ =**

Correct option: (D)

If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then $A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$

and $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

$\therefore \quad A^4 = \begin{bmatrix} 2^4 & 0 & 0 \\ 0 & (-2)^4 & 0 \\ 0 & 0 & (-1)^4 \end{bmatrix} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{-1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\therefore \quad A^4A^{-1} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{-1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix} =$

$\begin{bmatrix} 8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Q.6 Let A and B be two matrices of order n × n. Let A be non-singular and B be singular. Consider the following:

1. AB is singular.

2. AB is non-singular.

3. A^{-1} B is singular.

4. A^{-1} B is non-singular.

Which of the above is/are correct?

Correct option: (C) 1 and 3

Given, |A| ≠ 0 and |B| = 0

∴ |AB| = |A| |B| = 0

and $|A^{-1} B| = |A^{-1}|\,|B|$

$= \frac{1}{|A|}|B| \qquad .... \left[\because \ |A^{-1}| = \frac{1}{|A|}\right]$

$= 0$

∴ AB and A^{-1} B are singular.

Q.7 For the matrix A = $\begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$, the matrix of cofactors is

Correct option: (B)

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 1(0) = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} = (-1)(8) = -8$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = (1)(4) = 4$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (-1)(1) = -1$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = (1)(3) = 3$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = (-1)(2) = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (1)(1) = 1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = (-1)(7) = -7$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} = (1)(2) = 2$

∴ The matrix of the cofactors is

$\begin{bmatrix} 0 & -8 & 4 \\ -1 & 3 & -2 \\ 1 & -7 & 2 \end{bmatrix}$

Q.8 If A = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then the value of $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ = where A_{11}, A_{12}, A_{13} are co-factors of a_{11}, a_{12}, a_{13} respectively

Correct option: (B)

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= \cos\theta(\cos\theta - 0) + \sin\theta[-(-\sin\theta - 0)] + 0(0-0)$

$= \cos^2\theta + \sin^2\theta = 1$

Q.9 If matrix $A = \frac{1}{11}\begin{bmatrix} -1 & 7 & -24 \\ 2 & a & 4 \\ 2 & -3 & 15 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ b & -1 & c \end{bmatrix}$, then the values of a, b, c respectively are ......

Correct option: (C)

$AA^{-1} = I$

$\Rightarrow \frac{1}{11}\begin{bmatrix} -1 & 7 & -24 \\ 2 & a & 4 \\ 2 & -3 & 15 \end{bmatrix} \begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ b & -1 & c \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \frac{1}{11}\begin{bmatrix} 11-24b & 0 & 24-24c \\ 6+2a+4b & 2-3a & 8+4a+4c \\ 15b & 0 & -4+15c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By equality of matrices, we get

$15b = 0 \Rightarrow b = 0$

$2 - 3a = 11 \Rightarrow a = -3$

$24 - 24c = 0 \Rightarrow c = 1$

Q.10 **If A =** $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ **and** $(AB)^{-1}$ **=** $\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{4} & 0 \end{bmatrix}$,

then $B^{-1}.A^{-1}$ **=**

Correct option: (D)

$(AB)^{-1} = B^{-1} A^{-1}$

$\therefore B^{-1} A^{-1} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{4} & 0 \end{bmatrix}$

Q.11 **Which of the following matrix is invertible?**

$A_1 = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$,

$A_2 = \begin{bmatrix} -1 & -2 & 3 \\ 4 & 5 & 7 \\ 2 & 4 & -6 \end{bmatrix}$,

$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 1 \\ 7 & 2 & 1 \end{bmatrix}$,

$A_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

Correct option: (D)

$|A_1| = \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = 0$

$|A_2| = \begin{vmatrix} -1 & -2 & 3 \\ 4 & 5 & 7 \\ 2 & 4 & -6 \end{vmatrix}$

$= -1(-58) + 2(-38) + 3(6) = 0$

$|A_3| = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} = 0$

$|A_4| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$

$= 1(-4) - 0 + 1(-2) = -6 \neq 0$

$\therefore$ A_4 is invertible.

Q.12 **If ω is a complex cube root of unity and** **A =** $\begin{bmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, then** A^{-1} **=**

Correct option: (D)

$A = \begin{bmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ $|A| = \omega^3 = 1 \neq 0$

$\therefore$ adj A = $\begin{bmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^3 \end{bmatrix}^T$

$= \begin{bmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^3 \end{bmatrix}$

$= \begin{bmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{|A|}$ adj A = $\begin{bmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q.13 **For the matrix A =** $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$ **and**

where A_{ij} **is co-factor of elements** a_{ij}**,**

then $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ **=**

Correct option: (D)

$a_{11} = 3$, $a_{12} = 2$, $a_{13} = 4$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 4 & 1 \\ 6 & 3 \end{vmatrix} = 1(6) = 6$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (-1)(1) = -1$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix} = (1)(-2) = -2$

$\therefore$ $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 3(6) + 2(-1) + 4(-2)$

= 8

Q.14 If A and B are non-singular matrices of order 2 such that

$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 3 \end{bmatrix}$ **and**

$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$ **then** $B^{-1} =$

Correct option: (C)

$$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 3 \end{bmatrix}$$

$$\Rightarrow B^{-1} A^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 3 \end{bmatrix}$$

$$\Rightarrow B^{-1} \left(\frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\right) = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 3 \end{bmatrix}$$

Only option (C) satisfies the above condition.

Q.15 If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}_{3\times3}$ **, then** $A^{-1} =$

Correct option: (C)

$$|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$= A^{-1}$

Q.16 If A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$ **and B =** $\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$**, then**

$(AB)^{-1}$ =

Correct option: (A)

$$AB = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 10 & 17 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} 5 & 9 \\ 10 & 17 \end{vmatrix} = -5 \neq 0$$

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and ad – bc ≠ 0, then

$$A^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore (AB)^{-1} = \frac{1}{-5}\begin{bmatrix} 17 & -9 \\ -10 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-17}{5} & \frac{9}{5} \\ 2 & -1 \end{bmatrix}$$

Q.17 The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ **is**

Correct option: (B)

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = -2 \neq 0$$

$\therefore A^{-1}$ exists.

Here, $A_{11} = 1, A_{12} = -2, A_{13} = 1$

$A_{21} = -2, A_{22} = 2, A_{23} = -2$

$A_{31} = -1, A_{32} = 2, A_{33} = -3$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & -2 \\ -1 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = -\frac{1}{2}\begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 2 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 1 & \frac{1}{2} \\ 1 & -1 & -1 \\ -\frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$$

The element in the second row and third column is –1.

Q.18 Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix A – $3I_3$ is

Correct option: (C)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - 3I_3 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A - 3I_3| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$= 0$

∴ the matrix A – $3I_3$ is non-invertible.

Q.19 If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T, then 5a + b =

Correct option: (D)

$$\text{adj } A = \begin{bmatrix} 2 & -3 \\ b & 5a \end{bmatrix}^T = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$

Given, A adj A = AA^T

$$\Rightarrow \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10a+3b & 0 \\ 0 & 3b+10a \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$$

∴ By the equality of matrices,

15a – 2b = 0 and 3b + 10a = 13

$\Rightarrow a = \frac{2}{5}$ and b = 3

$\therefore 5a + b = 5\left(\frac{2}{5}\right) + 3 = 2 + 3 = 5$

Q.20 If the inverse of product of the matrix B = $\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ with a matrix A is C = $\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, then A^{-1} equals

Correct option: (C)

$(BA)^{-1} = C$

$\Rightarrow A^{-1}B^{-1} = C \Rightarrow A^{-1} = CB$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$$

Q.21 If A = $\begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, where I is unit matrix of order 2, then the values of x and y are respectively

Correct option: (C)

$$|A| = \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix} = 11 \neq 0$$

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and ad – bc ≠ 0, then

$$A^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$$

$A^{-1} = xA + yI$

$$\therefore \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{11} & \frac{-2}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} x+y & 2x \\ -5x & x+y \end{bmatrix}$$

∴ By the equality of matrices,

$2x = \frac{-2}{11}$ and $x + y = \frac{1}{11}$

$\therefore x = \frac{-1}{11}$ and $\frac{-1}{11} + y = \frac{1}{11}$

$\therefore\ x = \frac{-1}{11}$ and $y = \frac{2}{11}$

Q.22 **For a matrix A = $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, if U_1 and U_2 are 2 × 1 column matrices satisfying $AU_1 = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, and U is 2 × 2 matrix whose columns are U_1 and U_2, then sum of the elements of U^{-1} is**

Correct option: (A)

Let $U_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$ and $U_2 = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

$AU_1 = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$\therefore \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$\therefore \begin{bmatrix} 5a_1 + 4b_1 \\ 3a_1 + 2b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

∴ by the equality of matrices,

$5a_1 + 4b_1 = 0$ and $3a_1 + 2b_1 = 6 \Rightarrow a_1 = 12,\ b_1 = -15$

Similarly, $a_2 = 10$ and $b_2 = -12$

$\therefore U = \begin{bmatrix} 12 & 10 \\ -15 & -12 \end{bmatrix}$

$|U| = \begin{vmatrix} 12 & 10 \\ -15 & -12 \end{vmatrix} = 6$

$\therefore U^{-1}$ exists

$\therefore U^{-1} = \frac{1}{6} \begin{bmatrix} -12 & -10 \\ 15 & 12 \end{bmatrix}$

∴ Sum of elements of $U^{-1} = \frac{1}{6}(-12 + 15 - 10 + 12) = \frac{5}{6}$

Q.23 **If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, then $(AB)^{-1}$ =**

Correct option: (A)

$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}$

$\therefore\ |AB| = \begin{vmatrix} 11 & 3 \\ 7 & 2 \end{vmatrix} = 1 \neq 0$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$, then

$$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\therefore\ (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

Q.24 **If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 3 & 3 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that AX = B, then the value of $x_1 + x_2 + x_3$ =**

Correct option: (D)

AX = B

$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 3 & 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$,

$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 6 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

Applying $R_3 \rightarrow R_3 - 6R_2$,

$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$

$\therefore\ x_1 - x_2 + x_3 = 1$ …(i)

$x_2 - 2x_3 = -1$ …(ii)

$5x_3 = 5 \Rightarrow x_3 = 1$

From (ii),

$x_2 - 2(1) = -1 \Rightarrow x_2 = 1$

From (i),

$x_1 - 1 + 1 = 1 \Rightarrow x_1 = 1$

$\therefore\ x_1 + x_2 + x_3 = 1 + 1 + 1 = 3$

Q.25 **For a 3 × 3 matrix A, if A(adj A) = $\begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}$, then the value of determinant of A is**

Correct option: (C)

$\because$ A(adj A) = |A| I

$\therefore$ |A| = –10

Q.26 **If A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$, then $A_{31} + A_{32} + A_{33}$ = where A_{ij} is cofactor of a_{ij}, where A**

$= [a_{ij}]_{3\times3}$

Correct option: (A)

$A_{31} = (-1)^{3+1}\begin{vmatrix}1 & 1\\1 & -3\end{vmatrix} = -4$

$A_{32} = (-1)^{3+2}\begin{vmatrix}1 & 1\\2 & -3\end{vmatrix} = 5$

$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 1\\2 & 1\end{vmatrix} = -1$

$\therefore A_{31} + A_{32} + A_{33} = -4 + 5 - 1 = 0$

Q.27 **If A = $\begin{bmatrix}1 & 2 & 3\\0 & 4 & 2\\0 & 0 & 6\end{bmatrix}$, then the minor of the element a_{31} is**

Correct option: (B)

$M_{31} = \begin{vmatrix}2 & 3\\4 & 2\end{vmatrix}$ …[By leaving R_3 and C_1]

$= -8$

Q.28 **If $A = \begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ and $A^{-1} = \alpha I + \beta A$ α, β ∈ R where I is the identity matrix of order 2, then 4(α + β) =**

Correct option: (A)

If A = $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ and ad – bc ≠ 0, then

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

$|A| = \begin{vmatrix}1 & 2\\-1 & 4\end{vmatrix} = 6 \neq 0$

$\therefore \; A^{-1} = \frac{1}{6}\begin{bmatrix}4 & -2\\1 & 1\end{bmatrix}$

$A^{-1} = \alpha I + \beta A$

$\Rightarrow \frac{1}{6}\begin{bmatrix}4 & -2\\1 & 1\end{bmatrix} = \alpha\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \beta\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\frac{4}{6} & \frac{-2}{6}\\\frac{1}{6} & \frac{1}{6}\end{bmatrix} = \begin{bmatrix}\alpha+\beta & 2\beta\\-\beta & \alpha+4\beta\end{bmatrix}$

∴ by the equality of matrices,

$\alpha + \beta = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow 4(\alpha + \beta) = 4\left(\frac{2}{3}\right) = \frac{8}{3}$

Q.29 **Matrix A is non-singular matrix and $(A-3I)(A-5I) = 0$, then $\frac{15}{8}A^{-1} = \ldots\ldots$**

Correct option: (C)

$(A - 3I)(A - 5I) = 0$

$\Rightarrow A^2 - 8A + 15I = 0$

$\Rightarrow A^2A^{-1} - 8AA^{-1} + 15IA^{-1} = 0$

$\Rightarrow A - 8I + 15A^{-1} = 0$

$\Rightarrow 15A^{-1} = 8I - A$

$\Rightarrow \frac{15}{8}A^{-1} = I - \frac{1}{8}A$

Q.30 **Let A be a non-singular matrix of order n and $|A| = k$, then $(\text{adjA})^{-1}$ is**

Correct option: (A)

$|A| = k \neq 0$ ...[Given]

$\therefore \; A^{-1} = \frac{1}{|A|}$ (adj A)

$A^{-1} = \frac{(adjA)}{k}$

$k.A^{-1}$ = (adj A)

Taking inverse on both sides, we get

$(kA^{-1})^{-1} = (\text{adj }A)^{-1}$

$\Rightarrow k^{-1}(A^{-1})^{-1} = (\text{adj }A)^{-1}$

$\Rightarrow k^{-1}A = (\text{adj }A)^{-1}$ …($\because (A^{-1})^{-1} = A$)

$\Rightarrow \frac{A}{k} = (adjA)^{-1}$

Q.31 **Let A be a 2 × 2 matrix**

Statement-1 : adj(adj A) = A

Statement-2 : |adj A| = |A|

Correct option: (B)

$|\text{adj }A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\text{Adj}(\text{adj }A) = |A|^{n-2}A = |A|^0A = A$

∴ option [B] is the correct answer.

Q.32 **The sum of the cofactors of the elements of second row of the matrix $\begin{bmatrix}1 & 3 & 2\\-2 & 0 & 1\\5 & 2 & 1\end{bmatrix}$ is**

Correct option: (B)

Let A = $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix}$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = (-1)(-1) = 1$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = (1)(-9) = -9$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = (-1)(-13) = 13$

∴ Sum of cofactors of 2^{nd} row = 1 – 9 + 13 = 5

Q.33 **If** $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ **and A.adj A = AA^T,**

then 5a + b =

Correct option: (D)

$A_{11} = (-1)^{1+1}(2) = 2$, $A_{12} = (-1)^{1+2}(3) = -3$,

$A_{21} = (-1)^{2+1}(-b) = b$, $A_{22} = (-1)^{2+2}(5a) = 5a$

∴ adj A = $\begin{bmatrix} 2 & -3 \\ b & 5a \end{bmatrix}^T = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$

Given, A adj A = AA^T

$\Rightarrow \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 10a + 3b & 0 \\ 0 & 3b + 10a \end{bmatrix} =$

$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

∴ by the equality of matrices,

15a – 2b = 0 and 3b + 10a = 13

$\Rightarrow a = \frac{2}{5}$ and b = 3

∴ $5a + b = 5\left(\frac{2}{5}\right) + 3 = 2 + 3 = 5$

Q.34 **If A =** $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ **and A.adj A =**

$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$**, then k is equal to**

Correct option: (B)

Since A(adj A) = $|A|$. I,

$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = (\cos^2\alpha + \sin^2\alpha) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⇒ k = 1

Q.35 **If A =** $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ **and $A^{-1} = \alpha A$, then α =**

Correct option: (C)

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$,ad – bc ≠ 0

$\therefore A^{-1} = \frac{1}{-7} \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{7} A$

$\therefore \alpha = \frac{1}{7}$

Q.36 **If matrix A =** $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ **is such that AX = I,**

where I is 2 × 2 unit matrix, then X =

Correct option: (A)

A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

$AX = I \Rightarrow A^{-1}AX = A^{-1}I$

$\Rightarrow X = A^{-1}$

|A| = –5 ≠ 0

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and ad – bc ≠ 0, then

$$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\therefore A^{-1} = \frac{-1}{5} \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$

Q.37 **If A^{-1} =** $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 5 & 5 \end{bmatrix}$**, then A =**

Correct option: (A)

Option (A):

$AA^{-1} = \begin{bmatrix} -5 & 20 & -2 \\ -1 & 3 & 0 \\ 3 & -11 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 5 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= I

∴ Option (A) is the correct answer.

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Use AA^{-1} = I and substitute each of the provided options to identify the correct answer.

Q.38 **If A = $\begin{bmatrix}1 & 1 & 0\\2 & 1 & 5\\1 & 2 & 1\end{bmatrix}$, then $a_{11}A_{21} + a_{12}A_{22}$ $+ a_{13}A_{23}$ =**

Correct option: (B)

$a_{11} = 1, a_{12} = 1, a_{13} = 0$

$A_{21} = (-1)^{2+1}\begin{vmatrix}1 & 0\\2 & 1\end{vmatrix} = -1$

$A_{22} = (-1)^{2+2}\begin{vmatrix}1 & 0\\1 & 1\end{vmatrix} = 1$

$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 1\\1 & 2\end{vmatrix} = -1$

$\therefore\ a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 1 \times (-1) + 1 \times 1 + 0 \times (-1)$

$= 0$

Q.39 **If A = $\begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$ and A(adj A) = $\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$, then k is equal to**

Correct option: (B)

Given, A(adj A) = $\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$

Since A(adj A) = $|A|$. I

$\therefore \begin{bmatrix}k & 0\\0 & k\end{bmatrix} = (\cos^2\alpha + \sin^2\alpha)\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}k & 0\\0 & k\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow k = 1$

Q.40 **If A = $\begin{bmatrix}1 & 2 & 1\\1 & 1 & 1\\1 & 1 & 0\end{bmatrix}$, then $[\text{adj}(\text{adj } A)]^{-1}$ =**

Correct option: (D)

$|A| = \begin{vmatrix}1 & 2 & 1\\1 & 1 & 1\\1 & 1 & 0\end{vmatrix}$

$= 1 \neq 0$

adj (adj A) = $(1)^{3-2}$ A ...[adj (adj A) = $|A|^{n-2}$. A]

= A

$\therefore$ $[\text{adj (adj A)}]^{-1} = A^{-1}$

Q.41 **If A^{-1} = $\begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix}$, then adj A =**

Correct option: (C)

$AA^{-1} = I$

$\therefore A\begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$\therefore A\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$

$A = \begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$

$|A| = 1$

$A^{-1} = \frac{1}{|A|}$ adj A

$\therefore A^{-1}$ = adj A

Q.42 **Let A be a 3 × 3 non-singular matrix with |A| = α. If A^{-1}(adj (adj A)) = kI, then the value of k is**

Correct option: (B) α

Since A(adj A) = |A|.I

Replacing A by adj A, we get

adj A (adj(adj A)) = |adj A|I

$\Rightarrow A^{-1}.|A|$ (adj(adj A)) = |adj A|I ….

$\left[\because\ A^{-1} = \frac{1}{|A|}(adj\ A)\right]$

$\Rightarrow \alpha\ A^{-1}$ (adj (adj A)) = $|A|^2$.I ….[$\because$ |adj A| = $|A|^{n-1}$]

$\Rightarrow \alpha\ A^{-1}$(adj (adj A)) = α^2I

$\Rightarrow A^{-1}$ (adj (adj A)) = αI

Given, A^{-1}(adj (adj A)) = kI

$\therefore$ k = α

Q.43 **Let M be a 3 × 3 matrix satisfying M $\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ and M $\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$. Then the sum of the diagonal entries of M is**

Correct option: (C)

Let M = $\begin{bmatrix} a & b & c \\ x & y & z \\ l & m & n \end{bmatrix}$, then

M $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} b \\ y \\ m \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

∴ by the equality of matrices,

b = –1, y = 2, m = 3

M $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} a-b \\ x-y \\ l-m \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

∴ by the equality of matrices,

a – b = 1, $x – y$ = 1, l – m = –1

⇒ a = 0, x = 3, l = 2

M $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow \begin{bmatrix} a+b+c \\ x+y+z \\ l+m+n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

∴ by the equality of matrices,

a + b + c = 0, $x + y$ + z = 0, l + m + n = 12

⇒ c = 1, z = –5, n = 7

∴ sum of diagonal elements of M = a + y + n

= 0 + 2 + 7 = 9

Q.44 **If A = $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3$ =**

Correct option: (A)

|A| = $\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3 \neq 0$

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and ad – bc ≠ 0, then

$$A^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

∴ $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$(A^{-1})^3 = \frac{1}{27}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}^3$

$= \frac{1}{27}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$= \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$

Q.45 **If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$, $B = adj\ A$ and $C = 5A$, then $\frac{|adj\ B|}{|C|}$ =**

Correct option: (D) 1

A = $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$

B = adj A = $\begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

∴ adj B = $\begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$ = 5A

∴ adj B = C ….[∵ C = 5A(given)]

⇒ |adj B| = |C|

⇒ $\frac{|adj\ B|}{|C|} = 1$

Q.46 **If A is non-singular matrix and (A + I) (A – I) = 0, then A + A^{-1} =**

Correct option: (B)

(A + I) (A – I) = 0

⇒ $A^2 - I^2 = 0$

Pre-multiplying by A^{-1}, we get

A – A^{-1} = 0

⇒ A = A^{-1}

∴ A + A^{-1} = A + A = 2A

Q.47 **If A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$, then $(adj(adj\ A))^{-1}$ =**

Correct option: (C)

A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$

adj A = $\begin{bmatrix} 13 & 4 & -5 \\ -9 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$

adj(adj A) = $\begin{bmatrix} 6 & 6 & 12 \\ 12 & 18 & 6 \\ 18 & 30 & 36 \end{bmatrix}^T = \begin{bmatrix} 6 & 12 & 18 \\ 6 & 18 & 30 \\ 12 & 6 & 36 \end{bmatrix}$

Let B = $\begin{bmatrix} 6 & 12 & 18 \\ 6 & 18 & 30 \\ 12 & 6 & 36 \end{bmatrix}$

$$|B| = \begin{vmatrix} 6 & 12 & 18 \\ 6 & 18 & 30 \\ 12 & 6 & 36 \end{vmatrix} = 1296 = 6^4 \neq 0$$

$$\text{adjB} = \begin{bmatrix} 468 & 144 & -180 \\ -324 & 0 & 108 \\ 36 & -72 & 36 \end{bmatrix}^T =$$

$$\begin{bmatrix} 468 & -324 & 36 \\ 144 & 0 & -72 \\ -180 & 108 & 36 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{6^4} \begin{bmatrix} 468 & -324 & 36 \\ 144 & 0 & -72 \\ -180 & 108 & 36 \end{bmatrix} =$$

$$\frac{1}{36} \begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

Q.48 **If A =** $\begin{bmatrix} \frac{k}{2} & 0 & 0 \\ 0 & \frac{l}{3} & 0 \\ 0 & 0 & \frac{m}{4} \end{bmatrix}$ **and** A^{-1} **=** $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ **then k +** ***l*** **+ m =**

Correct option: (D)

If B = $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then B^{-1} = $\begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

$$\therefore A^{-1} = \begin{bmatrix} \frac{2}{k} & 0 & 0 \\ 0 & \frac{3}{l} & 0 \\ 0 & 0 & \frac{4}{m} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$\Rightarrow \frac{2}{k} = \frac{1}{2} \Rightarrow k = 4,$

$\frac{3}{l} = \frac{1}{3} \Rightarrow l = 9$ and

$\frac{4}{m} = \frac{1}{4} \Rightarrow m = 16$

$\therefore k + l + m = 4 + 9 + 16 = 29$

Q.49 **If** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$**, then 2*x* – *y* + z =**

Correct option: (A)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

Applying $R_2 \to R_2 + 2R_1$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_3$,

$$\begin{bmatrix} 0 & -2 & 0 \\ 3 & 0 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 4 \end{bmatrix}$$

$\therefore\ -2y = -4 \Rightarrow y = 2$

$3x = 3 \Rightarrow x = 1$

$x + 3y + z = 4 \Rightarrow z = -3$

$\therefore\ 2x - y + z = 2 - 2 - 3 = -3$

Q.50 **If A =** $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$**, B =** $\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$**, then** $(A + B)^{-1}$ **=**

Correct option: (B)

$$A + B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\therefore\ |A + B| = \begin{vmatrix} 5 & 2 \\ 4 & 3 \end{vmatrix} = 7 \neq 0$$

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and ad – bc ≠ 0, then

$$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore\ (A + B)^{-1} = \frac{1}{7} \begin{bmatrix} 3 & -2 \\ -4 & 5 \end{bmatrix}$$