Differential 2 **Marks: 60**

ANSWER KEY

Maths

Q.1 D	Q.2 A	Q.3 A	Q.4 C	Q.5 A	Q.6 B	Q.7 B	Q.8 B
Q.9 D	Q.10 A	Q.11 B	Q.12 D	Q.13 D	Q.14 C	Q.15 B	Q.16 A
Q.17 A	Q.18 D	Q.19 A	Q.20 D	Q.21 D	Q.22 A	Q.23 B	Q.24 C
Q.25 D	Q.26 A	Q.27 D	Q.28 C	Q.29 C	Q.30 D		

Maths

Q.1 The money invested in a company is compounded continuously. If ₹ 200 invested today becomes ₹ 400 in 6 years, then at the end of 33 years it will become ₹

Correct option: (D)

Here, Amount (A) = ₹ 400

Principal (P) = ₹ 200, N = 6 years

A = $P\left(1+\frac{R}{100}\right)^N$

⇒ 400 = $200\left(1+\frac{R}{100}\right)^6$

⇒ $\left(1+\frac{R}{100}\right)^6=2$

⇒ $1+\frac{R}{100}=2^{\frac{1}{6}}$

A = $P\left(1+\frac{R}{100}\right)^N$ = $200\left(1+\frac{R}{100}\right)^{33}$

= $200\left(2^{\frac{1}{6}}\right)^{33}$

= $200\left(2^5\cdot 2^{\frac{1}{2}}\right)$

= $200\left(32\sqrt{2}\right)$

= $6400\sqrt{2}$

Q.2 The solution of $\frac{dy}{dx}+1=e^{x+y}$ is

Correct option: (A)

$\frac{dy}{dx}=e^{x+y}-1$ …(i)

Put $x+y=v$ …(ii)

⇒ $1+\frac{dy}{dx}=\frac{dv}{dx}$

⇒ $\frac{dy}{dx}=\frac{dv}{dx}-1$ …(iii)

Substituting (ii) and (iii) in (i), we get

$\frac{dv}{dx}=e^v$

Integrating on both sides, we get

$\int e^{-v}dv=\int dx+c$

⇒ $-e^{-v}=x+c$

⇒ $x+e^{-v}+c=0$

⇒ $x+e^{-(x+y)}+c=0$

Q.3 Solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$ is

Correct option: (A)

$\frac{dy}{dx}+\frac{y}{x}=\sin x$

∴ I.F. = $e^{\int\frac{1}{x}dx}=e^{\log x}=x$

∴ The solution of the given equation is

$y\,x=\int x\ \sin x\ dx+c$

⇒ $yx=-x\cos x+\sin x+c$

⇒ $x(y+\cos x)=\sin x+c$

Q.4 The particular solution of $3e^x\tan y\,dx+(1+e^x)\sec^2 y\,dy=0$, when $x=0$, $y=\pi$ is

Correct option: (C)

$3e^x\tan y\,dx+(1+e^x)\sec^2 y\,dy=0$

∴ $\frac{3e^x}{1+e^x}dx+\frac{\sec^2 y}{\tan y}dy=0$

Integrating on both sides, we get

$3\int\frac{e^x}{1+e^x}dx+\int\frac{\sec^2 y}{\tan y}dy=\log|c|$

∴ $3\log|1+e^x|+\log|\tan y|=\log|c|$

⇒ $\log|(1+e^x)^3(\tan y)|=\log|c|$

⇒ $(1+e^x)^3\tan y=c$

When $x=0$, $y=\pi$

∴ $(1+e^0)^3\tan\pi=c$ ⇒ $c=0$

∴ $(1+e^x)^3\tan y=0$

Q.5 The differential equation of all parabolas whose axis is Y-axis is

Correct option: (A)

Axis of parabola = Y-axis

Vertex = (0, m)

∴ Equation of parabola is

$(x-0)^2=4a(y-m)$

⇒ $x^2=4ay-4am$

Differentiating w.r.t. x, we get

$2x = 4a \frac{dy}{dx}$

$\Rightarrow \frac{1}{x}\frac{dy}{dx} = \frac{1}{2a} \Rightarrow \frac{1}{x}\frac{d^2y}{dx^2} - \frac{dy}{dx}\frac{1}{x^2} = 0$

$\Rightarrow x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Q.6 A body cools according to Newton's law of cooling from 100°C to 60°C in 15 minutes. If the temperature of the surrounding is 20°C, then the temperature of the body after cooling down for one hour is

Correct option: (B)

Let θ be the temperature of the body at any time t.

$\therefore \frac{d\theta}{dt} \propto (\theta - 20)$

$\Rightarrow \frac{d\theta}{dt} = -k(\theta - 20), k > 0$

Integrating on both sides, we get

$\log|\theta - 20| = -kt + c$

When $t = 0, \theta = 100°$

$\therefore \log 80 = -k(0) + c$

$\Rightarrow c = \log 80$

$\therefore \log|\theta - 20| = -kt + \log 80 \quad \ldots(i)$

When $t = 15, \theta = 60°$

$\therefore \log 40 = -15k + \log 80$

$\Rightarrow k = \frac{-1}{15}\log\frac{1}{2}$

$\therefore \log|\theta - 20| = \frac{t}{15}\log\frac{1}{2} + \log 80 \quad \ldots$[From (i)]

When t = 1 hour = 60 minutes,

$\log|\theta - 20| = \frac{60}{15}\log\frac{1}{2} + \log 80$

$\Rightarrow \log\left(\frac{\theta - 20}{80}\right) = 4\log\frac{1}{2}$

$\Rightarrow \frac{\theta - 20}{80} = \left(\frac{1}{2}\right)^4 \Rightarrow \theta = 5 + 20 = 25°C$

Q.7 If the half life of substance is 5 years, then the total amount of the substance left after 15 years, when initial amount is 64 gms is

Correct option: (B)

Initial amount = 64 gms

Half life period = 5 years

$\therefore$ Amount of substance left after 5 years = $\frac{64}{2}$ = 32 gms

Amount of substance left after 10 years = $\frac{32}{2}$ = 16 gms

Amount of substance left after 15 years = $\frac{16}{2}$ = 8 gms

Q.8 The general solution of the differential equation $\frac{dy}{dx} = \cot x \cdot \cot y$ is

Correct option: (B)

$\frac{dy}{dx} = \cot x \cot y$

$\Rightarrow \cot x\, dx - \tan y\, dy = 0$

Integrating on both sides, we get

$\log(\sin x) - \log(\sec y) = \log c$

$\Rightarrow \log\left(\frac{\sin x}{\sec y}\right) = \log c \Rightarrow \sin x = c \sec y$

Q.9 The solution of the differential equation $\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$ is

Correct option: (D)

$\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$

Here, $P = \frac{3x^2}{1+x^3} and\, Q = \frac{\sin^2 x}{1+x^3}$

$\therefore$ I.F. $= e^{\int \frac{3x^2}{1+x^3}dx} = e^{\log(1+x^3)} = 1 + x^3$

$\therefore$ solution of the given equation is

$y.(1 + x^3) = \int \frac{\sin^2 x}{1+x^3}(1 + x^3)dx$

$\Rightarrow y(1 + x^3) = \int \frac{1 - \cos 2x}{2}dx$

$\Rightarrow y(1 + x^3) = \frac{1}{2}x - \frac{\sin 2x}{4} + c$

Q.10 The general solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1+x^2}} = 0$ is

Correct option: (A)

$$\frac{dy}{dx} + \frac{1}{\sqrt{1+x^2}} = 0$$

$$\therefore \int dy + \int \frac{1}{\sqrt{1+x^2}}\, dx = 0$$

$$\Rightarrow y + \sin^{-1} x = c$$

Q.11 The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ **are respectively**

Correct option: (B)

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^3\right]^{7} = 7^3\left(\frac{d^2y}{dx^2}\right)^3$$

Here, the highest order derivative is $\frac{d^2y}{dx^2}$ with power 3.

$\therefore$ order = 2 and degree = 3

Q.12 The general solution of the differential equati $on\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)dx - \left[\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right]dy = 0$ **is**

Correct option: (D)

$$\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)dx - \left[\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right]dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)} \quad \ldots(i)$$

Put $y = vx$ …(ii)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} = \frac{v\cos v}{\frac{1}{v}\sin v + \cos v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v\sin v}{\sin v + v\cos v}$$

$$\Rightarrow \frac{\sin v + v\cos v}{v\sin v}dv = -\frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{v} + \cot v\right)dv = -\frac{dx}{x}$$

Integrating on both sides, we get

$\log v + \log \sin v = -\log x + \log k$

$$\Rightarrow \log \frac{y}{x} + \log \sin\left(\frac{y}{x}\right) = -\log x + \log k$$

$$\Rightarrow \log\left(y\sin\frac{y}{x}\right) = \log k$$

$$\Rightarrow y\sin\left(\frac{y}{x}\right) = k$$

Q.13 The solution of the differential equation $\frac{dy}{dx}$ **= (1 +** x**)(1 +** y^2**) is**

Correct option: (D)

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

Integrating on both sides, we get

$$\int \frac{dy}{1+y^2} = \int (1+x)\, dx + c$$

$$\Rightarrow \tan^{-1} y = \frac{x^2}{2} + x + c$$

$$\Rightarrow y = \tan\left(\frac{x^2}{2} + x + c\right)$$

Q.14 The general solution of $\mathbf{\frac{dy}{dx} = \frac{x+y}{x-y}}$ **is**

Correct option: (C)

$$\frac{dy}{dx} = \frac{x+y}{x-y} \quad \ldots(i)$$

Put $y = vx$ …(ii)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} = \frac{x+vx}{x-vx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \frac{1-v}{1+v^2}dv = \frac{dx}{x}$$

Integrating on both sides, we get

$\tan^{-1} v - \frac{1}{2}\log\left|1+v^2\right|$ = log |x| + c

⇒ $\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right|$ = log |x| + c

⇒ $\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right|$ = $\frac{1}{2}\log\left|x^2\right|$ + c

⇒ $\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|x^2+y^2\right|$ = c

Q.15 The differential equation $x^2(y + 1)$ dx + $y^2(x – 1)$ dy = 0 has the general solution given by (where C is a constant of integration.)

Correct option: (B)

$x^2(y + 1)$ dx + $y^2(x – 1)$ dy = 0

⇒ $\frac{x^2}{x-1}$dx + $\frac{y^2}{y+1}$dy = 0

Integrating on both sides, we get

$$\int \frac{x^2-1+1}{x-1}dx + \int \frac{y^2-1+1}{y+1}dy = c_1$$

⇒

$$\int (x+1)dx + \int \frac{1}{x-1}dx + \int (y-1)dy + \int \frac{1}{y+1}dy = c_1$$

⇒

$$\frac{x^2}{2} + x + \log(x-1) + \frac{y^2}{2} - y + \log(y+1) = c_1$$

⇒ $(x^2 + 2x + 1) + (y^2 – 2y + 1)$ + 2 log $[(x – 1)(y + 1)] = 2c_1 + 2$

⇒ $(x + 1)^2 + (y – 1)^2$ + 2 log $[(x – 1)(y + 1)]$ = C, where C = $2c_1 + 2$

Q.16 The solution of the differential equation $\frac{dy}{dx}$ = 3^{x+y} at x = 0 = y is

Correct option: (A)

$\frac{dy}{dx} = 3^{x+y}$

⇒ 3^xdx – $3^{–y}$dy = 0

Integrating on both sides, we get

$\frac{3^x}{\log 3} + \frac{3^{-y}}{\log 3} = c_1$

⇒ $3^x + 3^{–y}$ = c, where c = $c_1$log3

When x = 0 = y,

$3^0 + 3^0$ = c ⇒ c = 2

∴ $3^x + 3^{–y} = 2$

⇒ $3^x + 3^{–y} – 2 = 0$

Q.17 The solution of the differential equation $\frac{dy}{dx}$ = 3^{x+y} at x = 0 = y is

Correct option: (A)

$\frac{dy}{dx} = 3^{x+y}$

⇒ 3^xdx – $3^{–y}$dy = 0

Integrating on both sides, we get

$3^x + 3^{–y}$ = c

When x = 0 = y,

$3^0 + 3^0$ = c ⇒ c = 2

∴ $3^x + 3^{–y} = 2$

⇒ $3^x + 3^{–y} – 2 = 0$

Q.18 The integrating factor of the differential equation $(1 + x^2)$ dt = $(\tan^{–1} x – t)$ dx is

Correct option: (D)

$(1 + x^2)$ dt = $(\tan^{–1} x – t)$ dx

⇒ $\frac{dt}{dx} = \frac{\tan^{-1}x - t}{1+x^2}$

⇒ $\frac{dt}{dx} + \frac{t}{1+x^2} = \frac{\tan^{-1}x}{1+x^2}$

Here, P = $\frac{1}{1+x^2}$

∴ I. F. = $e^{\int \frac{1}{1+x^2}} = e^{\tan^{-1}x}$

Q.19 If $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}$ = – cos x, y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ = [IIT Screening 2004; Assam CCE 2017]

Correct option: (A) $\frac{1}{3}$

$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}$ = – cos x

⇒ $\frac{dy}{1+y} = \left(\frac{-\cos x}{2+\sin x}\right)$ dx

Integrating on both sides, we get

$\int \frac{dy}{1+y} + \int \frac{\cos x}{2+\sin x}$ dx = log c

⇒ log(1 + y) + log(2 + sin x) = log c

⇒ (y + 1) (2 + sin x) = c …(i)

Since y(0) = 1, i.e., y = 1 when x = 0

∴ (1 + 1) (2 + sin 0) = c ⇒ c = 4

∴ (y + 1) (2 + sin x) = 4 …[From (i)]

⇒ $y = \frac{4}{2+\sin x} - 1$

$\therefore y\left(\frac{\pi}{2}\right)= \frac{4}{2+\sin\frac{\pi}{2}} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$

Q.20 Bismath has half life period of 5 days. A sample originally has a mass of 1000mg, then the mass of Bismath after 30 days is

Correct option: (D)

Original mass = 1000 mg

Half life period = 5 days

$\therefore$ Mass of Bismath after 5 days = $\frac{1000}{2}$ = 500 mg

Mass of Bismath after 10 days = $\frac{500}{2}$ = 250 mg

Mass of Bismath after 15 days = $\frac{250}{2}$ = 125 mg

Mass of Bismath after 20 days = $\frac{125}{2}$ = 62.5 mg

Mass of Bismath after 25 days = $\frac{62.5}{2}$ = 31.25 mg

Mass of Bismath after 30 days = $\frac{31.25}{2}$ = 15.625 mg

Q.21 The curve satisfying the differential equation $ydx - \left(x+3y^2\right)\mathrm{dy} = 0$ **and passing through the point (1, 1) also passes through the point**

Correct option: (D)

$y\,dx - (x + 3y^2)\,dy = 0$

$\Rightarrow y\,dx = (x + 3y^2)\,dy$

$\Rightarrow \frac{dx}{dy} = \frac{x+3y^2}{y}$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y$

$\Rightarrow \frac{dx}{dy} - \left(\frac{1}{y}\right)x = 3y$

Which is a linear equation

$\therefore$ IF = $e^{\int \frac{-1}{y}dy} = e^{-\log y} = \frac{1}{y}$

$\therefore$ The required solution is

$x\frac{1}{y} = \int 3y \times \frac{1}{y}\,dy + c$

$\therefore \frac{x}{y} = 3y + c$

$\Rightarrow x = 3y^2 + cy$ …(i)

Curve passes through (1, 1)

$\Rightarrow 1 = 3 + c$

$\Rightarrow c = -2$

Equation (i) becomes,

$x = 3y^2 - 2y$

Option (D) i.e., $\left(\frac{-1}{3}, \frac{1}{3}\right)$

Satisfies above equation.

Q.22 If $y = e^{-x}\cos 2x$, then which of the following differential equations is satisfied ?

Correct option: (A)

$y = e^{-x}\cos 2x$

$\Rightarrow \frac{dy}{dx} = -2e^{-x}\sin 2x - e^{-x}\cos 2x$

$\Rightarrow \frac{d^2y}{dx^2} = 4e^{-x}\sin 2x - 3e^{-x}\cos 2x$

$\therefore \frac{d^2y}{dx^2} + \frac{2dy}{dx} + 5y = 0$

Q.23 The solution of $x\sin\left(\frac{y}{x}\right)$ dy = $\left[y\sin\left(\frac{y}{x}\right) - x\right]$ dx is

Correct option: (B)

$x\sin\left(\frac{y}{x}\right)dy = \left[y\sin\left(\frac{y}{x}\right) - x\right]dx$

$\therefore \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)}$ …(i)

Put $y = vx$ …(ii)

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ …(iii)

Substituting (ii) and (iii) in (i), we get

$v + x\frac{dv}{dx} = \frac{vx\sin v - x}{x\sin v}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{\sin v}$

$\Rightarrow -\sin v\,dv = \frac{1}{x}dx$

Integrating on both sides, we get

$-\int \sin v\,dv = \int \frac{1}{x}dx$

$\Rightarrow \cos v = \log|x| + c$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + c$

Q.24 The solution of $e^{y-x}\frac{dy}{dx} = \frac{y(\sin x+\cos x)}{(1+y\log y)}$ is

Correct option: (C)

$$e^{y-x}\frac{dy}{dx} = \frac{y(\sin x + \cos x)}{(1 + y\log y)}$$

$\therefore$ $e^y \frac{(1+y\log y)}{y}$ dy = e^x(sin x + cos x) dx

$\therefore$ $\int e^y\left(\log y + \frac{1}{y}\right)$ dy = $\int e^x$ (sin x + cos x)

dx

$\Rightarrow$ $e^y \log y = e^x \sin x + c$…

$$\left[\because \int e^x\left(f(x) + f'(x)\right)\,\mathrm{d}x = e^x f(x) + c\right]$$

Q.25 Solution of the differential equation $y' = \frac{(x^2+y^2)}{xy}$, where $y(1) = -2$ is given by

Correct option: (D)

$$\frac{dy}{dx} = \frac{x^2+y^2}{xy} \quad \ldots\text{(i)}$$

Put $y = vx$ …(ii)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots\text{(iii)}$$

Substituting (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x\cdot vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1}{v}$$

$$\Rightarrow v\,dv = \frac{dx}{x}$$

Integrating on both sides, we get

$\frac{v^2}{2}$ = log x + c

$\Rightarrow \frac{y^2}{2x^2}$ = log x + c

$\Rightarrow y^2 = 2x^2 \log x + 2cx^2$ …(i)

$y(1) = -2$ i.e. when $x = 1$, $y = -2$

$\therefore$ $(-2)^2 = 2(1)^2 \log 1 + 2c(1)^2$

$\Rightarrow$ 4 = 2 c $\Rightarrow$ c = 2

$\therefore$ $y^2 = 2x^2 \log x + 4x^2$ …[From (i)]

$\Rightarrow y^2 = x^2 \log x^2 + 4x^2$

Q.26 If $y = y(x)$ is the solution of the differential equation $\left(\frac{5+e^x}{2+y}\right)\frac{dy}{\mathrm{d}x} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log 13)$ is

Correct option: (A)

$$\left(\frac{5+e^x}{2+y}\right)\frac{dy}{dx} + e^x = 0$$

$$\Rightarrow \frac{dy}{2+y} = \frac{-e^x}{5+e^x}\,dx$$

Integrating on both sides, we get

log |2 + y| = –log|5 + e^x| + log |c|

$\Rightarrow$ log |2 + y| = log$\left|\frac{c}{5+e^x}\right|$

Since y(0) = 1 i.e., y = 1 when x = 0

$\therefore$ log 3 = log$\left|\frac{c}{6}\right|$

$\Rightarrow 3 = \frac{c}{6}$

$\Rightarrow$ c = 18

$\therefore$ log |2 + y| = log$\left|\frac{18}{5+e^x}\right|$ …[From (i)]

$\Rightarrow 2 + y = \frac{18}{5+e^x}$

$\Rightarrow y = \frac{18}{5+e^x} - 2$

$\Rightarrow y(\log 13) = \frac{18}{5+e^{\log 13}} - 2 = \frac{18}{5+13} - 2 =$

–1

Q.27 The general solution of the differential equation $\frac{1}{x}\frac{dy}{dx} = \tan^{-1}x$ is

Correct option: (D)

$$\frac{1}{x}\frac{dy}{dx} = \tan^{-1}x$$

$dy = x\cdot\tan^{-1}x\,dx$

Integrating on both sides we get,

$$y = \int x\cdot\tan^{-1}x\,dx$$

y =

$$\tan^{-1}x\int x\,dx - \int\left(\frac{d}{dx}\tan^{-1}x\cdot\int x\,dx\right)dx$$

$y = \tan^{-1} x \cdot \frac{x^2}{2} - \int \left(\frac{1}{1+x^2} \times \frac{x^2}{2}\right) dx$

$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} \int \left(\frac{x^2}{1+x^2}\right) dx$

$= \frac{x^2 \cdot \tan^{-1} x}{2} - \frac{1}{2}\left(\int \frac{x^2+1}{x^2+1} - \int \frac{1}{1+x^2} dx\right)$

$\Rightarrow y = \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + c$

Q.28 The integrating factor of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+y+2}$ is

Correct option: (C)

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+y+2}$

$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = x + y + 2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} - x = y + 2$

$\therefore$ I.F. $= e^{\int - \mathrm{d}y} = e^{-y}$

Q.29 The integrating factor of the differential equation
$x\frac{dy}{dx} + y \log x = x.\, e^x x^{-\frac{1}{2}} \log x (x > 0)$

is

Correct option: (C)

$x\frac{dy}{dx} + y \log x = x.\, e^x x^{-\frac{1}{2}} \log x$

$\Rightarrow \frac{dy}{dx} + \frac{\log x}{x} \cdot y = e^x x^{-\frac{1}{2}} \log x$

$\therefore$ I.F. $= e^{\int \frac{\log x}{x} dx} = e^{\frac{1}{2}(\log x)^2}$

$= \left(e^{\frac{1}{2}\log x}\right)^{\log x}$ …[$\because (a^m)^n = a^{mn}$]

$= \left(\sqrt{x}\right)^{\log x}$

Q.30 The differential equation, having general solution as $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is

Correct option: (D)

$Ax^2 + By^2 = 1$

Differentiating w.r.t. x, we get

$2Ax + 2By\frac{dy}{dx} = 0$ …(i)

Again, differentiating w.r.t. x, we get

$2A + 2B\left[\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right] = 0$ ….(ii)

Solving (i) and (ii), we get

$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$