Differential Equations **Marks: 100**

ANSWER KEY

Maths

Q.1 B	Q.2 A	Q.3 D	Q.4 C	Q.5 A	Q.6 A	Q.7 A	Q.8 A
Q.9 D	Q.10 A	Q.11 A	Q.12 B	Q.13 D	Q.14 A	Q.15 C	Q.16 A
Q.17 C	Q.18 A	Q.19 A	Q.20 C	Q.21 A	Q.22 B	Q.23 D	Q.24 B
Q.25 B	Q.26 C	Q.27 A	Q.28 D	Q.29 B	Q.30 C	Q.31 C	Q.32 C
Q.33 D	Q.34 B	Q.35 C	Q.36 B	Q.37 A	Q.38 B	Q.39 A	Q.40 A
Q.41 B	Q.42 A	Q.43 B	Q.44 B	Q.45 C	Q.46 D	Q.47 B	Q.48 B
Q.49 D	Q.50 B						

KUNAL ACADEMY

Maths

Q.1 The solution of $\frac{dy}{dx}$ = $\frac{xy}{x^2-y^2}$ is

Correct option: (B)

$\frac{dy}{dx} = \frac{xy}{x^2-y^2}$ …(i)

Put y = vx …(ii)

⇒ $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ …(iii)

Substituting (ii) and (iii) in (i), we get

$v + x\frac{dv}{dx} = \frac{x.vx}{x^2-v^2x^2} = \frac{v}{1-v^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{v}{1-v^2} - v \Rightarrow x\frac{dv}{dx} = \frac{v^3}{1-v^2}$

$\Rightarrow \frac{1-v^2}{v^3}dv = \frac{dx}{x}$

Integrating on both sides, we get

$\int v^{-3}\,dv - \int \frac{1}{v}\,dv = \int \frac{dx}{x} + \log|k|$

$\Rightarrow \frac{v^{-2}}{-2} - \log|v| = \log|x| + \log|k|$

$\Rightarrow \frac{1}{v^2} = \log|x\,v\,k|^{-2}$

$\Rightarrow e^{\frac{1}{v^2}} = \frac{1}{(x\,v\,k)^2} \Rightarrow e^{\frac{x^2}{y^2}} = \frac{1}{c\,y^2}$ …[c = k^2]

Q.2 The order and degree of the differential equation are respectively

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} \cdot \left(x+\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$$

Correct option: (A)

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} \cdot \left(x+\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$$

$$\Rightarrow \left[\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} \cdot \left(x+\frac{dy}{dx}\right)^{\frac{1}{2}}\right]^6 = (0)^6$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 \cdot \left(x+\frac{dy}{dx}\right)^3 = 0$$

Here, the highest order derivative is $\frac{d^2y}{dx^2}$ with power 2.

∴ order = 2 and degree = 2

Q.3 The sum of the degree and order of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[5]{\frac{dy}{dx}-5}$ is

Correct option: (D)

$$\sqrt{\frac{d^2y}{dx^2}} = \sqrt[5]{\frac{dy}{dx}-5}$$

Taking tenth power of both sides, we get

$$\left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{dy}{dx}-5\right)^2$$

∴ By definition,

order = 2 and degree = 5

∴ Order + degree = 2 + 5 = 7

Q.4 The solution of (cosec x log y)dy + (x^2y)dx = 0 is

Correct option: (C)

(cosec x log y)dy + (x^2y)dx = 0

$\Rightarrow \frac{1}{y}\log y\,dy + x^2 \sin x\,dx = 0$

Integrating on both sides, we get

$$\frac{(\log y)^2}{2} + [x^2(-\cos x) + \int 2x\cos x\,dx] = c$$

$$\Rightarrow \frac{(\log y)^2}{2} - x^2\cos x + 2(x\sin x + \cos x) = c$$

$$\Rightarrow \frac{(\log y)^2}{2} + (2 - x^2)\cos x + 2x\sin x = c$$

Q.5 The differential equation of $y = ae^{bx}$ is

Correct option: (A)

$y = ae^{bx}$ …(i)

$\Rightarrow \frac{dy}{dx} = abe^{bx}$

$\Rightarrow \frac{dy}{dx} = by$ …(ii)[From (i)]

$$\Rightarrow \frac{d^2y}{dx^2} = b\frac{dy}{dx} \Rightarrow y\frac{d^2y}{dx^2} = by\frac{dy}{dx}$$

$\Rightarrow y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$ …[From (ii)]

Q.6 The order and degree of the differential equation $\left(\frac{d^2s}{dt^2}\right)^2 + 3\left(\frac{ds}{dt}\right)^3 + 4 = 0$ are

Correct option: (A)

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Here, the highest order derivative is $\frac{d^2s}{dt^2}$ with power 2.

∴ order = 2 and degree = 2

Q.7 The differential equation of the family of curves $y = e^x$ (A cos x + B sin x). Where A and B are arbitary constants is

Correct option: (A)

$y = e^x (A\cos x + B\sin x)$

$\Rightarrow y' = e^x (A\cos x + B\sin x) + e^x (B\cos x - A\sin x)$

$\Rightarrow y' = y + e^x (B\cos x - A\sin x)$ …(i)

$\therefore y'' = y' + e^x (B\cos x - A\sin x) - e^x (A\cos x + B\sin x)$

$\Rightarrow y'' = y' + (y' - y) - y$ …[From (i)]

$\Rightarrow y'' - 2y' + 2y = 0$

Q.8 The particular solution of differential equation $(x + y)dy + (x - y)\,dx = 0$ at $x = y = 1$ is

Correct option: (A)

$(x + y)dy + (x - y)\,dx = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y-x}{y+x}$ ….(i)

Put $y = vx$ ….(ii)

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ….(iii)

Substituting (ii) and (iii) in (i), we get

$v + x\frac{dv}{dx} = \frac{vx - x}{vx + x} = \frac{v-1}{v+1}$

$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{-(v^2+1)}{v+1}$

Integrating on both sides, we get

$\int \frac{v+1}{v^2+1}dv = -\int \frac{dx}{x} + c_1$

$\Rightarrow \frac{1}{2}\int \frac{2v}{v^2+1}dv + \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x} + c_1$

$\Rightarrow \frac{1}{2}\log\left|v^2+1\right| + \tan^{-1} v = -\log|x| + c_1$

$\Rightarrow \log\left|\frac{y^2+x^2}{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = -2\log|x| + 2c_1$

$\Rightarrow \log|x^2 + y^2| - 2\log|x| + 2\tan^{-1}\left(\frac{y}{x}\right) = -2\log|x| + 2c_1$

$\Rightarrow \log|x^2 + y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = c$, where $c = 2c_1$

When $x = y = 1$,

$\log 2 + 2\tan^{-1}(1) = c$

$\Rightarrow \log 2 + 2\left(\frac{\pi}{4}\right) = c$

$\Rightarrow c = \log 2 + \frac{\pi}{2}$

$\therefore \log|x^2 + y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = \log 2 + \frac{\pi}{2}$

$\Rightarrow \log\left|\frac{x^2+y^2}{2}\right| = \frac{\pi}{2} - 2\tan^{-1}\left(\frac{y}{x}\right)$

Q.9 The general solution of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$ is

Correct option: (D)

$(3xy + y^2)dx + (x^2 + xy)dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy}$ …(i)

Put $y = vx$ …(ii)

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ …(iii)

Substituting (ii) and (iii) in (i), we get

$v + x\frac{dv}{dx} = -\frac{3x.\,vx + v^2x^2}{x^2 + x.\,vx} = -\frac{x^2(3v + v^2)}{x^2(1+v)}$

$\therefore x\frac{dv}{dx} = \frac{-3v - v^2}{1+v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{-4v - 2v^2}{1+v}$

$\Rightarrow x\frac{dv}{dx} = -\frac{2v(2+v)}{1+v}$

Integrating on both sides, we get

$\int \frac{v+1}{v(v+2)}dv + \int \frac{2}{x}dx = \log c_1$

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$\Rightarrow \frac{1}{2}\int\left(\frac{1}{v}+\frac{1}{v+2}\right)dv + 2\int \frac{1}{x}dx = \log c_1$

$\Rightarrow \frac{1}{2}$ [log v + log (v + 2)] + 2 log x = log c_1

$\Rightarrow$ log [v(v + 2)] + 4 log x = 2 log c_1

$\Rightarrow$ log [x^4.v(v + 2)] = logc_1^2

$\Rightarrow x^4$.v (v + 2) = c_1^2

$\Rightarrow x^4 \cdot \frac{y}{x}\left(\frac{y}{x}+2\right) = c_1^2$

$\Rightarrow x^2(y^2 + 2xy)$ = c, where c = c_1^2

Q.10 The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+1}{2(x-2y)}$ is

Correct option: (A)

$\frac{dy}{dx} = \frac{x-2y+1}{2(x-2y)}$ …(i)

Put $x - 2y$ = v …(ii)

$\Rightarrow 1 - 2\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(1-\frac{dv}{dx}\right)$ …(iii)

Substituting (ii) and (iii) in (i), we get

$\frac{1}{2}\left(1-\frac{dv}{dx}\right) = \frac{v+1}{2v}$

$\Rightarrow \frac{dv}{dx} = -\frac{1}{v}$

Integrating on both sides, we get

$\int v dv = -\int dx + c_1$

$\Rightarrow \frac{v^2}{2} = -x + c_1$

$\Rightarrow (x - 2y)^2 = -2x + 2c_1$

$\Rightarrow (x - 2y)^2 + 2x$ = c, where c = $2c_1$

Q.11 The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ is respectively

Correct option: (A)

$\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

$\Rightarrow \left(\sqrt{\frac{dy}{dx}}\right)^2 = \left(4\frac{dy}{dx}+7x\right)^2$

$\Rightarrow \frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 56x\frac{dy}{dx} + 49x^2$

This is a differential equation of order 1 and degree 2.

Q.12 A function y = f(x) satisfies (x + 1) f ′(x) – 2 ($x^2 + x$) f(x) = $\frac{e^{x^2}}{x+1}$ ∀ x > –1. If f(0) = 5, then f(x) is

Correct option: (B)

The given equation is

$(x + 1) f'(x) - 2(x^2 + x) f(x) = \frac{e^{x^2}}{x+1}$

If y = f(x), the equation is

$\frac{dy}{dx} - 2xy = \frac{e^{x^2}}{(x+1)^2}$, which is a linear equation

$\therefore$ I.F. = $e^{-\int 2x dx} = e^{-x^2}$

$\therefore$ the required solution is

$y.\, e^{-x^2} = \int \frac{1}{(x+1)^2} dx + c = -\frac{1}{x+1} + c$

When x = 0, y = 5

$\therefore$ c = 6

$\therefore y.\, e^{-x^2} = -\frac{1}{x+1} + 6$

$= \frac{-1+6x+6}{x+1} = \frac{6x+5}{x+1}$

$\therefore y = f(x) = \left(\frac{6x+5}{x+1}\right)e^{x^2}$

Q.13 Degree of the differential equation $e^{\frac{dy}{dx}} + \left(\frac{dy}{dx}\right)^3 = x$ is

Correct option: (D)

$e^{\frac{dy}{dx}} + \left(\frac{dy}{dx}\right)^3 = x$

Here, the highest order derivative is $\frac{dy}{dx}$. So order is 1. But degree is not defined due to presence of the term $e^{\frac{dy}{dx}}$.

Q.14 The solution of the differential equation $x\frac{dy}{dx} - y = 3$ represents a family of

Correct option: (A)

$x\frac{dy}{dx} - y = 3$

$\Rightarrow x\frac{dy}{dx} = 3 + y$

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$\Rightarrow \int \frac{1}{3+y} dy = \int \frac{1}{x} dx$

$\Rightarrow \log|y + 3| = \log|x| + \log c$

$\Rightarrow y + 3 = xc$

$\Rightarrow y = xc - 3$

This is the equation of family of straight line.

Q.15 If $\frac{dy}{dx} = \frac{xy+y}{xy+x}$, then the solution of the differential equation is

Correct option: (C)

$\frac{dy}{dx} = \frac{xy+y}{xy+x}$

$\Rightarrow \left(\frac{1+y}{y}\right) dy = \left(\frac{1+x}{x}\right) dx$

Integrating on both sides, we get

$\log y + y = \log x + x + \log A$

$\Rightarrow \log\left(\frac{y}{Ax}\right) = x - y \Rightarrow y = Axe^{x-y}$

Q.16 The differential equation representing the family of parabolas having vertex at origin and axis along positive direction of X-axis is

Correct option: (A)

The differential equation representing the family of parabolas having vertex at origin is

$y^2 = 4ax$ …(i)

Differentiating w.r.t. x, we get

$2y\frac{dy}{dx} = 4a$

$\Rightarrow 2y\frac{dy}{dx} = \frac{y^2}{x}$ …[From (i)]

$\Rightarrow 2yx\frac{dy}{dx} = y^2$

$\Rightarrow y^2 - 2xy\frac{dy}{dx} = 0$

Q.17 The particular solution of $\sec^2 y \tan x\, dy + \sec^2 x \tan y\, dx = 0$, when $x = y = \frac{\pi}{4}$, is

Correct option: (C)

$\sec^2 y \tan x\, dy + \sec^2 x \tan y\, dx = 0$

$\therefore \frac{\sec^2 y}{\tan y} dy + \frac{\sec^2 x}{\tan x} dx = 0$

Integrating on both sides, we get

Log $|\tan y| + \log|\tan x| = \log|c|$

$\therefore \log|\tan y.\tan x| = \log|c|$

$\Rightarrow \tan x \tan y = c$

When $x = y = \frac{\pi}{4}$,

$\tan\frac{\pi}{4}\tan\frac{\pi}{4} = c \Rightarrow c = 1$

$\therefore \tan x \tan y = 1$

Q.18 The integrating factor of differential equation$(1 + y + x^2 y)\, dx + (x + x^3)\, dy = 0$ is

Correct option: (A)

$(1 + y + x^2 y)\, dx + (x + x^3)\, dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1+y+x^2y}{x+x^3}$

$\Rightarrow \frac{dy}{dx} = -\frac{1+y(1+x^2)}{x(1+x^2)}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = -\frac{1}{x(1+x^2)}$

$\therefore$ I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Q.19 The order and degree of $\left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^{4/5} = \left(\frac{m}{m+1}\right)\frac{d^3y}{dx^3}$ are respectively

Correct option: (A)

$\left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^{4/5} = \left(\frac{m}{m+1}\right)\frac{d^3y}{dx^3}$

$\Rightarrow \left\{\left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^{4/5}\right\}^5 =$

$\left(\frac{m}{m+1}\right)^5\left(\frac{d^3y}{dx^3}\right)^5$

$\Rightarrow \left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^4 = \left(\frac{m}{m+1}\right)^5\left(\frac{d^3y}{dx^3}\right)^5$

Here, the highest order derivative is $\frac{d^3y}{dx^3}$ with power 5.

$\therefore$ order = 3 and degree = 5

Q.20 The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1, c_2 are arbitrary constants is

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Correct option: (C)

$y = c_1 e^{c_2 x}$

Differentiating w.r.t. x, we get

$y' = c_1 c_2 e^{c_2 x}$

$\Rightarrow y' = c_2 y$ …(i)

Differentiating w.r.t. x, we get

$y'' = c_2 y'$

$\Rightarrow y'' = \dfrac{(y')^2}{y}$ …[From (i)]

$\Rightarrow y\,y'' = (y')^2$

Q.21 The solution of the differential equation $\dfrac{dy}{dx} = 3^{x+y}$ at $x = 0 = y$ is

Correct option: (A)

$\dfrac{dy}{dx} = 3^{x+y}$

$\Rightarrow 3^x dx - 3^{-y} dy = 0$

Integrating on both sides, we get

$\dfrac{3^x}{\log 3} + \dfrac{3^{-y}}{\log 3} = c_1$

$\Rightarrow 3^x + 3^{-y} = c$, where $c = c_1 \log 3$

When $x = 0 = y$,

$3^0 + 3^0 = c \Rightarrow c = 2$

$\therefore\ 3^x + 3^{-y} = 2$

$\Rightarrow 3^x + 3^{-y} - 2 = 0$

Q.22 The solution of the equation $x^2 y - x^3 \dfrac{dy}{dx} = y^4 \cos x$, where $y(0) = 1$, is

Correct option: (B)

$x^2 y - x^3 \dfrac{dy}{dx} = y^4 \cos x$

$\Rightarrow x^3 \dfrac{dy}{dx} - x^2 y = -y^4 \cos x$

$\Rightarrow \dfrac{-1}{y^4} \cdot \dfrac{dy}{dx} + \dfrac{1}{xy^3} = \dfrac{1}{x^3} \cos x$ …(i)

Put $\dfrac{1}{y^3} = v$

$\Rightarrow \dfrac{-1}{y^4} \cdot \dfrac{dy}{dx} = \dfrac{1}{3} \cdot \dfrac{dv}{dx}$

$\therefore\ \dfrac{1}{3} \cdot \dfrac{dv}{dx} + \dfrac{v}{x} = \dfrac{1}{x^3} \cos x$ …[From (i)]

$\Rightarrow \dfrac{dv}{dx} + \dfrac{3v}{x} = \dfrac{3}{x^3} \cos x$

$\therefore$ I.F. $= e^{\int \frac{3}{x} dx} = e^{3 \log x} = e^{\log x^3} = x^3$

$\therefore$ Solution of the given equation is

$v.\, x^3 = \int \dfrac{3}{x^3} \cos x .\, x^3 dx + c$

$\Rightarrow \dfrac{x^3}{y^3} = 3 \int \cos x\, dx + c$

$\Rightarrow \dfrac{x^3}{y^3} = 3 \sin x + c$

$y(0) = 1$ i.e. when $x = 0$, $y = 1$

$\Rightarrow \dfrac{0}{1} = 3 \sin 0 + c$

$\Rightarrow c = 0$

$\therefore\ \dfrac{x^3}{y^3} = 3 \sin x$

$\Rightarrow x^3 = 3y^3 \sin x$

Q.23 A particular solution of $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$ with $y(1) = \dfrac{\pi}{4}$ is

Correct option: (D)

Given, $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

$\Rightarrow 3e^x \tan y\, dx = -(1 - e^x) \sec^2 y\, dy$

$\Rightarrow \dfrac{3e^x dx}{(1 - e^x)} = -\dfrac{\sec^2 y}{\tan y} dy$

Integrating on both sides, we got

$\int \dfrac{3e^x}{(1 - e^x)} dx = -\int \dfrac{\sec^2 y}{\tan y} dy$

$-3 \int \dfrac{-e^x}{1 - e^x} dx = -\int \dfrac{\sec^2 y}{\tan y} dy$

$\therefore\ -3 \log(1 - e^x) = -\log(\tan y) + \log c$

$\Rightarrow 3 \log(1 - e^x) = \log(\tan y) - \log c$

$\Rightarrow \log(1 - e^x)^3 = \log\left(\dfrac{\tan y}{c}\right)$

$\Rightarrow (1 - e^x)^3 = \dfrac{\tan y}{c}$ …(i)

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Put, $y = \frac{\pi}{4}$ and $x = 1$ ...[Given]

We get

$$(1-e)^3 = \frac{\tan\frac{\pi}{4}}{c}$$

$$\Rightarrow c = \frac{1}{(1-e)^3}$$

Substituting value of 'c' in equation (i), we get

$$(1-e^x)^3 = \frac{\tan y}{\frac{1}{(1-e)^3}}$$

$$\Rightarrow \tan y = \left(\frac{1-e^x}{1-e}\right)^3$$

Q.24 The solution of r dx + (x – r^2) dr = 0 is

Correct option: (B)

$r\,dx + (x - r^2)\,dr = 0$

$$\Rightarrow \frac{dx}{dr} + \frac{x - r^2}{r} = 0$$

$$\Rightarrow \frac{dx}{dr} + \frac{x}{r} = r$$

∴ I.F. $= e^{\int \frac{1}{r} dr} = e^{\log r} = r$

∴ Solution of the given equation is

$$x \cdot r = \int r.\,rdr + c$$

$$\Rightarrow rx = \frac{r^3}{3} + c$$

Q.25 For the differential equation $\left[1 - \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{3}} = 8\frac{d^2y}{dx^2}$, the order and degree are ________ respectively.

Correct option: (B)

$$\left[1 - \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{3}} = 8\frac{d^2y}{dx^2}$$

$$\Rightarrow \left[1 - \left(\frac{dy}{dx}\right)^2\right]^5 = 8^3\left(\frac{d^2y}{dx^2}\right)^3$$

Here, the highest order derivative is $\frac{d^2y}{dx^2}$ with power 3.

∴ order = 2 and degree = 3

Q.26 The general solution of the differential equation $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$ is (where C is a constant of integration.)

Correct option: (C)

$(x^2 + y^2)dx = 2xydy$

$$\Rightarrow \frac{dy}{dx} = \frac{1 + \left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)} \quad \text{…(i)}$$

Put $y = vx$ …(ii)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{…(iii)}$$

Substituting (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

Integrating on both sides, we get

$$\int \frac{dx}{x} - \int \frac{2v}{1 - v^2}\,dv = c_1$$

$\Rightarrow \log x + \log(1 - v^2) = c_1$

$\Rightarrow (1 - v^2).x = e^{c_1}$

$$\Rightarrow \left(1 - \frac{y^2}{x^2}\right)x = e^{c_1}$$

$\Rightarrow x^2 - y^2 = e^{c_1}.x$

$\Rightarrow x^2 - y^2 = Cx$, where $C = e^{c_1}$

Q.27 The solution of $\frac{dy}{dx} + y \tan x = \sec x$, $y(0) = 0$ is [KEAM 2017]

Correct option: (A)

$$\frac{dy}{dx} + y\tan x = \sec x$$

I.F. $= e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$

∴ Solution of the given differential equation is

$$y \sec x = \int \sec^2 x + c$$

$\Rightarrow y \sec x = \tan x + c$

$y(0) = 0 \Rightarrow c = 0$

∴ $y \sec x = \tan x$

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Q.28 The particular solution of $\frac{dy}{dx}$ = 1 + x + y^2 + xy^2, when $y(0) = 0$ is

Correct option: (D)

$\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \frac{dy}{dx} = (1 + x)(1 + y^2)$

Integrating on both sides, we get

$\int \frac{dy}{1+y^2} = \int (1+x)dx + c$

$\Rightarrow \tan^{-1}y = x + \frac{x^2}{2} + c$ …(i)

$y(0) = 0$ i.e. when $x = 0$, $y = 0$

$\therefore$ $\tan^{-1}(0) = 0 + c \Rightarrow c = 0$

$\therefore$ $\tan^{-1}y = x + \frac{x^2}{2}$ …[From (i)]

$\Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

Q.29 The solution of $x + y\frac{dy}{dx}$ = cosec ($x^2 + y^2$) is

Correct option: (B)

$x + y\frac{dy}{dx} = \text{cosec}(x^2 + y^2)$ …(i)

Put $x^2 + y^2 = u$ …(ii)

Differentiating w.r.t. x, we get

$2x + 2y\frac{dy}{dx} = \frac{du}{dx}$

$\therefore x + y\frac{dy}{dx} = \frac{1}{2}\cdot\frac{du}{dx}$ …(iii)

Substituting (ii) and (iii) in (i), we get

$\frac{1}{2}\cdot\frac{du}{dx} = \text{cosec } u$

$\therefore \frac{du}{\text{cosec } u} = 2\,dx$

Integrating on both sides, we get

$\int \sin u\, du = 2\int dx$

$\therefore -\cos u = 2x + c_1$

$\therefore \cos(x^2 + y^2) + 2x = c$, where $c = -c_1$

Q.30 The integrating factor of the differential equation $\frac{dy}{dx}$ ($x \log x$) + y = 4 log x is

Correct option: (C)

$\frac{dy}{dx}(x\log x) + y = 4\log x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{4}{x}$

Here, $P = \frac{1}{x\log x}$

$\therefore$ I.F. $= e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

Q.31 A body at an unknown temperature is placed in a room which is held at a constant temperature of 30°F. If after 10 minutes the temperature of the body is 0°F and after 20 minutes the temperature of the body is 15°F, then the expression for the temperature of the body at any time t is

Correct option: (C)

Let ‘T’ be the temperature of the body at time ‘t’. Then,

$\frac{dT}{dt} = -k(T - 30)$, where k > 0

$\Rightarrow \frac{dT}{T-30} = -k\,dt$

Integrating on both sides, we get

log |T – 30| = – kt + c …(i)

When t = 10, T = 0

$\therefore$ log |0 – 30| = –10k + c

$\Rightarrow$ log 30 + 10k = c …(ii)

When t = 20, T = 15

$\therefore$ log |15 – 30| = –20k + c

$\Rightarrow$ log 15 + 20k = c …(iii)

From (ii) and (iii), we get

$k = \frac{1}{10}\log 2$

$\therefore$ $\log 30 + \frac{10}{10}\log 2 = c$ …[From (ii)]

$\Rightarrow$ log 30 + log 2 = c

$\Rightarrow$ c = log 60

$\therefore$ $\log|T - 30| = \frac{-t}{10}\log 2 + \log 60$ …[From (i)]

$\Rightarrow \log\left|\frac{T-30}{60}\right| = \frac{-t}{10}\log 2$

$\Rightarrow \log\left|\frac{T-30}{60}\right| = \frac{-t}{10}(0.6931)$

$\Rightarrow \frac{T-30}{60} = e^{-0.06931t}$

$\Rightarrow T = 60\,e^{-0.06931t} + 30$

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Q.32 If the slope of the tangent of the curve at any point is equal to $-y + e^{-x}$, then the equation of the curve passing through origin is

Correct option: (C)

$\frac{dy}{dx} = -y + e^{-x}$

$\Rightarrow \frac{dy}{dx} + y = e^{-x}$

$\therefore$ I.F. $= e^{\int dx} = e^{x}$

$\therefore$ Solution of the given equation is

$ye^{x} = \int e^{x} \cdot e^{-x} dx + c$

$\Rightarrow ye^{x} = \int dx + c$

$\Rightarrow ye^{x} = x + c$ …(i)

Since the curve passes through (0, 0).

$\therefore$ 0 = 0 + c

$\Rightarrow$ c = 0

$\therefore$ $ye^{x} = x$ …[From (i)]

$\Rightarrow ye^{x} - x = 0$

Q.33 $y = mx + \frac{2}{m}$ is the general solution of

Correct option: (D)

$y = mx + \frac{2}{m}$ …(i)

$\Rightarrow \frac{dy}{dx} = m$

Putting m = $\frac{dy}{dx}$ in (i), we get

$y = x\frac{dy}{dx} + \frac{2}{\frac{dy}{dx}} \Rightarrow y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^{2} + 2$

Q.34 The general solution of differential equation $xdy - ydx = 0$ represents

Correct option: (B)

$xdy - ydx = 0$

$\Rightarrow \frac{1}{y}$ dy $= \frac{1}{x}$ dx

Integrating on both sides, we get

$\log y = \log x + \log c$

$\Rightarrow \log y = \log (xc)$

$\Rightarrow y = xc$

It represent equation of line passing through origin.

Q.35 A firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$.

If the firm employs 25 more workers, then the new level of production of items is

Correct option: (C)

$\frac{dP}{dx} = 100 - 12\sqrt{x}$

Integrating on both sides, we get

$P = 100x - 12 \cdot \frac{2}{3}x\sqrt{x} + c$ …(i)

When $x = 0$, P = 2000

$\therefore$ c = 2000

$\Rightarrow P = 100x - 8x\sqrt{x} + 2000$ …[From (i)]

When $x = 25$, we have

$P = 100(25) - 8(25)\sqrt{25} + 2000$

$\Rightarrow$ P = 2500 – 1000 + 2000

$\Rightarrow$ P = 3500

Q.36 If $\frac{dy}{dx} = \frac{y}{x-y^{2}}$, $y > 0$ and $y(1) = 1$, then $y(-3)$ is equal to

Correct option: (B)

$\frac{dy}{dx} = \frac{y}{x-y^{2}}$

$\Rightarrow \frac{dx}{dy} = \frac{x-y^{2}}{y} \Rightarrow \frac{dx}{dy} + \left(-\frac{1}{y}\right)x = -y$

$\therefore$ I.F. $= e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$

$\therefore$ solution of the given equation is

$x \cdot \frac{1}{y} = \int -y \cdot \frac{1}{y} dy + c$

$\Rightarrow \frac{x}{y} = -y + c$ …(i)

Since $y(1) = 1$, i.e., $y = 1$ when $x = 1$

$\therefore$ 1 = –1 + c $\Rightarrow$ c = 2

$\therefore \frac{x}{y} = -y + 2$ …[From (i)]

Putting $x = -3$, we get

$-\frac{3}{y} = -y + 2$

$\Rightarrow y^{2} - 2y - 3 = 0$

$\Rightarrow (y - 3)(y + 1) = 0$

Since $y(x) > 0$, $y = 3$

Q.37 The assets of a person are reduced in his business such that the rate of reduction

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is proportional to the square root of the existing assets. If the assets were initially ₹ 10,00,000 and due to loss they reduce to ₹ 10,000 after
3 years, then the number of years required for the person to go bankrupt will be
Correct option: (A)

Let x be the asset at time t.

$\therefore \frac{dx}{dt} \propto \sqrt{x}$

$\Rightarrow \frac{dx}{dt} = -k\sqrt{x}$, where k > 0

$\Rightarrow \frac{dx}{\sqrt{x}} = -kdt$

Integrating on both sides, we get

$2\sqrt{x} = -kt + c$

When t = 0, x = 10,00,000

$\therefore 2\sqrt{1000000} = -k(0) + c$

$\Rightarrow c = 2(1000) = 2000$

$\therefore 2\sqrt{x} = -kt + 2000$ …(i)

When t = 3, x = 10,000

$\therefore 2\sqrt{10000} = -3k + 2000$

$\Rightarrow 2(100) = -3k + 2000$

$\Rightarrow 3k = 1800$

$\Rightarrow k = 600$

$\therefore 2\sqrt{x} = -600t + 2000$ …[From (i)]

Time to go bankrupt = T

When t = T, x = 0

$\therefore 0 = -600T + 2000$

$\Rightarrow T = \frac{2000}{600} = \frac{10}{3}$ years

Q.38 The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1-\left(\frac{dy}{dx}\right)^4}$ are respectively
Correct option: (B)

$$\frac{d^2y}{dx^2} = \sqrt[3]{1-\left(\frac{dy}{dx}\right)^4}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 1 - \left(\frac{dy}{dx}\right)^4$$

Here, the highest order derivative is $\frac{d^2y}{dx^2}$ with power 3.

$\therefore$ order = 2 and degree = 3

Q.39 The rate of increase of bacteria in a certain culture is proportional to the number present. If it doubles in 6 hours, then in 18 hours its number would be
Correct option: (A)

Let P_0 be the initial population and let the population after t years be P. Then,

$\frac{dP}{dt} = kP \Rightarrow \frac{dP}{P} = k\,dt$

Integrating on both sides, we get

log P = kt + c

When t = 0, $P = P_0$

$\therefore \log P_0 = 0 + c \Rightarrow c = \log P_0$

$\therefore \log P = kt + \log P_0$

$\Rightarrow \log \frac{P}{P_0} = kt$ …(i)

When t = 6 hrs, $P = 2P_0$

$\therefore \log \frac{2P_0}{P_0} = 6k$

$\Rightarrow k = \frac{\log 2}{6}$

$\therefore \log \frac{P}{P_0} = \frac{\log 2}{6}t$ …[From (i)]

When t = 18 hrs, we have

$\log \frac{P}{P_0} = \frac{\log 2}{6} \times 18 = 3 \log 2 = \log 8$

$\therefore P = 8P_0$

Q.40 The differential equation of an ellipse whose major axis is twice its minor axis, is
Correct option: (A)

The equation of ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4b^2} + \frac{y^2}{b^2} = 1 \quad \ldots[\because a = 2b]$$

$\Rightarrow x^2 + 4y^2 = 4b^2$

Differentiating w.r.t. 'x', we get

$2x + 8y\frac{dy}{dx} = 0$

$\Rightarrow x + 4y\frac{dy}{dx} = 0$

Q.41 The general solution of the differential equation (2y – 1)dx – (2x + 3)dy = 0 is
Correct option: (B)

$(2y - 1)\,dx - (2x + 3)\,dy = 0$

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Integrating on both sides, we get

$$\int \frac{dx}{2x+3} - \int \frac{dy}{2y-1} = \log c_1$$

$\Rightarrow \frac{1}{2}\log(2x + 3) - \frac{1}{2}\log(2y - 1) = \log c_1$

$\Rightarrow \log(2x + 3) - \log(2y - 1) = 2\log c_1$

$\Rightarrow \log\left(\frac{2x+3}{2y-1}\right) = \log {c_1}^2$

$\Rightarrow \frac{2x+3}{2y-1} = c$, where $c = {c_1}^2$

Q.42 The solution of the differential equation $\frac{dy}{dx}$ = (be^{ax} + c sin(nx)) is

Correct option: (A)

$\frac{dy}{dx} = (be^{ax} + c\sin(nx))$

Integrating on both sides, we get

$\int dy = \int (be^{ax} + c\sin(nx))dx + k$

$\Rightarrow y = \frac{be^{ax}}{a} + \frac{-c\cos(nx)}{n} + k$

Q.43 The solution of the differential equation $\mathbf{\frac{dx}{dy} + \frac{1+x}{1-y} = 0}$ is

Correct option: (B)

$$\frac{dx}{dy} + \frac{1+x}{1-y} = 0$$

$\Rightarrow \frac{dx}{dy} = \frac{1+x}{y-1}$

$\Rightarrow \frac{1}{1+x}dx = \frac{1}{y-1}dy$

Integrating on both sides, we get

$$\int \frac{1}{x+1}dx = \int \frac{1}{y-1}dy$$

$\Rightarrow \log(x + 1) = \log(y - 1) + \log c_1$

$\Rightarrow \log(x + 1) = \log[(y - 1)c_1]$

$\Rightarrow \frac{x+1}{1-y} = c$, where $c = -c_1$

Q.44 The solution of the differential equation $(1 + y^2)\tan^{-1}x\,dx + (1 + x^2)\,2y\,dy = 0$ is

Correct option: (B)

$(1 + y^2)\tan^{-1}x\,dx + (1 + x^2)\,2y\,dy = 0$

$\Rightarrow \frac{\tan^{-1}x\,dx}{1+x^2} + \frac{2y}{1+y^2}dy = 0$

Integrating on both sides, we get

$$\frac{(\tan^{-1}x)^2}{2} + \log|1 + y^2| = c_1$$

$\Rightarrow (\tan^{-1}x)^2 + 2\log|1 + y^2| = c$, where $c = 2c_1$

Q.45 The solution of $\frac{dy}{dx}$ = e^x(sin x + cos x) is

Correct option: (C)

$\frac{dy}{dx} = e^x(\sin x + \cos x)$

Integrating on both sides, we get

$\int dy = \int e^x(\sin x + \cos x)dx + c$

$\Rightarrow y = e^x\sin x + c$

Q.46 The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is

Correct option: (D)

$y = c_1e^{c_2x}$

$\Rightarrow y' = c_2c_1e^{c_2x}$

$\Rightarrow y' = c_2y$ …(i)

$\Rightarrow y'' = c_2y'$

$\Rightarrow y'' = \frac{(y')^2}{y}$ …[From (i)]

$\Rightarrow yy'' = (y')^2$

Q.47 The population of a city increases at a rate proportional to the population at that time. If the population of the city increase from 20 lakhs to 40 lakhs in 30 years, then after another 15 years the population is

Correct option: (B)

Let P be the population at time t.

Then, $\frac{dP}{dt} = kP$

$\Rightarrow \frac{dP}{P} = k\,dt$

Integrating on both sides, we get

log P = kt + c

When t = 0, P = 20

$\therefore$ log 20 = k (0) + c

$\Rightarrow$ c = log 20

$\therefore$ log P = kt + log 20

$\Rightarrow \log\frac{P}{20} = kt$ …(i)

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When t = 30, P = 40

$\therefore \log\left(\frac{40}{20}\right) = 30k$

⇒ log 2 = 30k

⇒ k = $\frac{1}{30}$log 2

$\therefore \log\frac{P}{20} = \frac{t}{30}\log 2$ …[From (i)]

When t = 45, we have

$\log\frac{P}{20} = \frac{45}{30}\log 2$

⇒ $\log\frac{P}{20} = \frac{3}{2}\log 2$

⇒ $\frac{P}{20} = 2^{\frac{3}{2}}$

⇒ P = $40\sqrt{2}$ lakhs

Q.48 The differential equation satisfied by the family of curves $y = ax\cos\left(\frac{1}{x} + b\right)$,

where a, b are parameters, is

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Correct option: (B)

$y = ax\cos\left(\frac{1}{x} + b\right)$ …(i)

$\Rightarrow \frac{dy}{dx} = -ax\sin\left(\frac{1}{x} + b\right).\left(-\frac{1}{x^2}\right) + a\cos\left(\frac{1}{x} + b\right)$

$\Rightarrow \frac{dy}{dx} = \frac{a}{x}.\sin\left(\frac{1}{x} + b\right) + a\cos\left(\frac{1}{x} + b\right)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{a}{x}\cos\left(\frac{1}{x} + b\right).\left(-\frac{1}{x^2}\right) - \frac{a}{x^2}\sin\left(\frac{1}{x} + b\right) - a\sin\left(\frac{1}{x} + b\right).\left(-\frac{1}{x^2}\right)$

$= -\frac{a}{x^3}\cos\left(\frac{1}{x} + b\right) = -\frac{ax}{x^4}\cos\left(\frac{1}{x} + b\right)$

$\therefore \frac{d^2y}{dx^2} = -\frac{y}{x^4}$ …[From (i)]

$\Rightarrow x^4\frac{d^2y}{dx^2} + y = 0 \Rightarrow x^4y_2 + y = 0$

Q.49 The solution of $(1 + x^2)\frac{dy}{dx} = 1$ is

Correct option: (D)

$(1 + x^2)\frac{dy}{dx} = 1$

Integrating on both sides, we get

$\int dy = \int \frac{1}{1 + x^2}dx + c$

$\Rightarrow y = \tan^{-1}x + c$

Q.50 If order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^5}{\left(\frac{d^3y}{dx^3}\right)} + \frac{d^3y}{dx^3} = \sin x,$$

are m and n respectively, then the value of $(m^2 + n^2)$ is equal to

Correct option: (B)

Given differential equation is

$$\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^5}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = \sin x$$

$$\left(\frac{d^2y}{dx^2}\right)^5\cdot\frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^5 + \left(\frac{d^3y}{dx^3}\right)^2 = \sin x\frac{d^3y}{dx^3}$$

Here, order = 3 and degree = 2

$\therefore$ m = 3, n = 2

$\therefore m^2 + n^2 = 3^2 + 2^2 = 13$

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