Integration 2 **Marks: 240**

ANSWER KEY

Maths

Q.1 C	Q.2 D	Q.3 B	Q.4 A	Q.5 B	Q.6 A	Q.7 D	Q.8 A
Q.9 A	Q.10 D	Q.11 A	Q.12 C	Q.13 A	Q.14 B	Q.15 D	Q.16 B
Q.17 B	Q.18 B	Q.19 C	Q.20 C	Q.21 D	Q.22 A	Q.23 D	Q.24 A
Q.25 A	Q.26 A	Q.27 A	Q.28 D	Q.29 D	Q.30 A	Q.31 A	Q.32 A
Q.33 D	Q.34 D	Q.35 C	Q.36 C	Q.37 A	Q.38 C	Q.39 C	Q.40 A
Q.41 A	Q.42 C	Q.43 B	Q.44 A	Q.45 B	Q.46 B	Q.47 B	Q.48 A
Q.49 A	Q.50 A	Q.51 B	Q.52 B	Q.53 B	Q.54 B	Q.55 A	Q.56 B
Q.57 C	Q.58 B	Q.59 C	Q.60 D	Q.61 D	Q.62 B	Q.63 B	Q.64 A
Q.65 C	Q.66 D	Q.67 A	Q.68 A	Q.69 A	Q.70 C	Q.71 D	Q.72 A
Q.73 D	Q.74 B	Q.75 B	Q.76 A	Q.77 C	Q.78 C	Q.79 B	Q.80 A
Q.81 A	Q.82 A	Q.83 C	Q.84 D	Q.85 D	Q.86 B	Q.87 B	Q.88 D
Q.89 B	Q.90 D	Q.91 C	Q.92 B	Q.93 C	Q.94 B	Q.95 D	Q.96 B
Q.97 B	Q.98 A	Q.99 A	Q.100 A	Q.101 D	Q.102 B	Q.103 C	Q.104 C
Q.105 C	Q.106 C	Q.107 B	Q.108 A	Q.109 B	Q.110 D	Q.111 A	Q.112 D
Q.113 C	Q.114 D	Q.115 B	Q.116 B	Q.117 D	Q.118 B	Q.119 D	Q.120 B

KUNAL ACADEMY

Maths

Q.1 $\int \frac{x^3 \sin\left[\tan^{-1}\left(x^4\right)\right]}{1+x^8} dx$=

Correct option: (C)

Let I = $\int \frac{x^3 \sin\left[\tan^{-1}\left(x^4\right)\right]}{1+x^8} dx$

Put x^4 = t ⇒ $4x^3$ dx = dt

I = $\frac{1}{4}\int \frac{\sin(\tan^{-1} t)}{1+t^2} dt$

Put $\tan^{-1}$ t = z ⇒ $\frac{1}{1+t^2} dt$ = dz

∴ I = $\frac{1}{4}\int \sin z dz$ = $\frac{1}{4}$ (–cos z) + c

= – $\frac{1}{4}$cos($\tan^{-1}$t) + c = $\frac{-1}{4}$cos[$\tan^{-1}$(x^4)] + c

Q.2 **If f(x) = $\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}$dx, $x \geq 0$ and f(0) = 0, then value of f(1) is**

Correct option: (D)

f(x) = $\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2} dx$

= $\int \frac{\left(\frac{5}{x^6}+\frac{7}{x^8}\right)}{\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)^2} dx$ …. [Dividing N[r] and D[r] by x^{14}]

Put $\frac{1}{x^5}+\frac{1}{x^7}+2$= t ⇒ $-\left(\frac{5}{x^6}+\frac{7}{x^8}\right)dx$ = dt

∴ f(x) = $\int \frac{-\text{dt}}{t^2}$

= $\frac{1}{t}+c$ = $\frac{1}{\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)}+c$

f(x) = $\frac{x^7}{\left(x^2+1+2x^7\right)}+c$

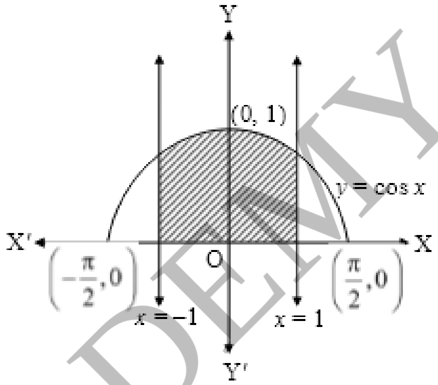
∴ f(0) = 0 ⇒ c = 0

f(x) = $\frac{x^7}{x^2+1+2x^7}$

∴ f(1) = $\frac{1}{4}$

Q.3 **The area bounded by y = cos$x$, y = 0 and |x| = 1 is given by**

Correct option: (B)



|x| = 1 ⇒ x = 1 or x = –1

Required Area = $\int_{-1}^{1} \cos x \, dx$

= 2 $\int_{0}^{1} \cos x \, dx$ …[∵ cos x is an even function]

= 2$[\sin x]_0^1$

= 2(sin 1 – 0)

= 2 sin 1

Q.4 $\int \frac{dx}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = Ax^{\frac{1}{2}}+Bx^{\frac{1}{3}}+Cx^{\frac{1}{6}}+\text{Dlog}\left(x^{\frac{1}{6}}+1\right)+k$

(where k is the integration constant) then values of A, B, C and D are respectively,

Correct option: (A)

Let I = $\int \frac{dx}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$

Put $x^{\frac{1}{6}}$ = t

⇒ x = t^6

⇒ dx = $6t^5$dt

∴ I = $\int \frac{6t^5}{t^3+t^2}$ dt

= $\int \frac{6t^5}{t^2(t+1)}$ dt = 6$\int \frac{t^3}{t+1}$ dt

KUNAL ACADEMY

$$= 6\int \frac{t^3+1-1}{t+1}\,dt = 6$$

$$\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$$

$$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log\left|t+1\right|\right] + k$$

$$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log|\sqrt[6]{x} + 1| + k$$

∴ A = 2, B = – 3, C = 6, D = – 6

Q.5 If $\int \frac{\cos 4x+1}{\cot x - \tan x}\,dx = k\cos 4x + c$, then

Correct option: (B)

$\int \frac{\cos 4x+1}{\cot x-\tan x}\,dx$

$= \int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x} \cdot \sin x \cdot \cos x\ dx$

$= \int \frac{\cos^2 2x\ \sin 2x}{\cos 2x}\,dx$

$= \frac{1}{2}\int 2\cos 2x\ \sin 2x\ dx$

$= \frac{1}{2}\int \sin 4x\ dx$

$\therefore \int \frac{\cos 4x+1}{\cot x-\tan x}\,dx = -\frac{1}{8}\cos 4x + c$

Comparing with the given condition, we get

$k = -\frac{1}{8}$

Q.6 $\int_0^3 \frac{3x+1}{x^2+9}$ **dx =**

Correct option: (A)

$$\int_0^3 \frac{3x+1}{x^2+9}dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}dx + \int_0^3 \frac{dx}{x^2+9}$$

$$= \left[\frac{3}{2}\log(x^2+9) + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= \frac{3}{2}(\log 18 - \log 9) + \frac{1}{3}\left(\frac{\pi}{4}\right)$$

$$= \frac{3}{2}\log 2 + \frac{\pi}{12} = \log(2\sqrt{2}) + \frac{\pi}{12}$$

Q.7 $\int_0^2 \frac{x^3\,dx}{(x^2+1)^{\frac{3}{2}}} =$

Correct option: (D)

Put $x^2 + 1 = t \Rightarrow 2x\,dx = dt$

When $x = 0$, t = 1 and when $x = 2$, t = 5

$$\therefore \int_0^2 \frac{x^3}{(x^2+1)^{\frac{3}{2}}}dx = \frac{1}{2}\int_1^5 \frac{(t-1)}{t^{\frac{3}{2}}}dt$$

$$= \frac{1}{2}\int_1^5 \left(t^{\frac{-1}{2}} - t^{\frac{-3}{2}}\right)dt$$

$$= \frac{1}{2}\left[2\sqrt{t} + 2\frac{1}{\sqrt{t}}\right]_1^5$$

$$= \frac{1}{2}\left[2\sqrt{5} + \frac{2}{\sqrt{5}} - 2 - 2\right]$$

$$= \sqrt{5} + \frac{1}{\sqrt{5}} - 2 = \frac{6 - 2\sqrt{5}}{\sqrt{5}}$$

Q.8 $\int \frac{2a\ \sin\ x + b\ \sin\ 2x}{(b + a\ \cos\ x)^3}\,dx =$

Correct option: (A)

Let I $= \int \frac{2a\ \sin\ x + b\ \sin\ 2x}{(b+a\ \cos\ x)^3}\,dx$

$= 2\int \frac{(a + b\ \cos\ x)}{(b + a\ \cos\ x)^3} \cdot \sin\ x\ dx$

Put b + a cos x = t

∴ – a sin x dx = dt

∴ sin x dx = $-\frac{1}{a}$dt

$\therefore I = 2\int \frac{a + b\left(\frac{t-b}{a}\right)}{t^3}\left(-\frac{1}{a}\right)dt$

$= -\frac{2}{a}\int \frac{a^2 + bt - b^2}{at^3}\,dt$

$= -\frac{2}{a^2}\int \left[\left(a^2 - b^2\right)t^{-3} + bt^{-2}\right]dt$

$= -\frac{2}{a^2}\left[\frac{a^2-b^2}{-2t^2} + \frac{b}{-t}\right] + c$

$= \frac{1}{a^2} \cdot \frac{a^2 - b^2}{t^2} + \frac{2b}{a^2 t} + c$

Q.9 $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\,dx =$

Correct option: (A)

$\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}\,dx$

$= \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha}\,dx$

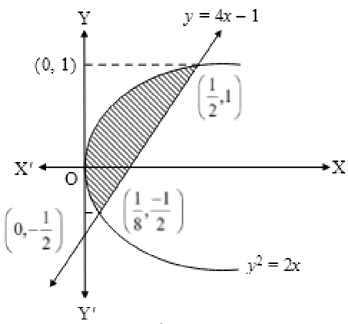
$= \int \frac{2(\cos^2 x - \cos^2\alpha)}{\cos x - \cos\alpha}\,dx$

$= 2\int (\cos x + \cos\alpha)\,dx$

= 2(sin x + x cos α) + c

Q.10 The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x$ and $y \ge 4x - 1\}$ is

Correct option: (D)



Putting $x = \frac{y^2}{2}$ in $y = 4x - 1$, we get

KUNAL ACADEMY

$y = 4\left(\frac{y^2}{2}\right) - 1 \Rightarrow 2y^2 - y - 1 = 0$

$\Rightarrow (y-1)(2y+1) = 0 \Rightarrow y = 1, \frac{-1}{2}$

∴ Required area

$= \int_{-1/2}^{1} \left(\frac{y+1}{4}\right) dy - \int_{-1/2}^{1} \frac{y^2}{2} dy$

$= \frac{1}{4}\left[\frac{y^2}{2} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$

$= \frac{1}{4}\left[\left(\frac{1}{2} - \frac{1}{8}\right) + \left(1 + \frac{1}{2}\right)\right] - \frac{1}{2}\left(\frac{1}{3} + \frac{1}{24}\right)$

$= \frac{1}{4}\left(\frac{15}{8}\right) - \frac{1}{2}\left(\frac{9}{24}\right)$

$= \frac{15}{32} - \frac{3}{16}$

$= \frac{9}{32}$

Q.11 Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x - [3x]| dx$ is

Correct option: (A)

Let I = $\int_1^2 |2x - [3x]| dx$

Put $3x$ = t ⇒ 3dx = dt

When x = 1, t = 3 and when x = 2, t = 6

∴ I = $\int_3^6 \left|\frac{2t}{3} - [t]\right| \frac{dt}{3}$

$= \frac{1}{9} \int_3^6 |2t - 3[t]| dt$

$= \frac{1}{9}\left[\int_3^4 |2t - 9| dt + \int_4^5 |2t - 12| dt + \int_5^6 |2t - 15| dt\right]$

$= \frac{1}{9}\left[\int_3^4 (9 - 2t) dt + \int_4^5 (12 - 2t) dt + \int_3^6 (15 - 2t) dt\right]$

$= \frac{1}{9}\left\{\left[9t - t^2\right]_3^4 + \left[12t - t^2\right]_4^5 + \left[15t - t^2\right]_5^6\right\}$

$= \frac{1}{9}$ [9(1) – 7 + 12 (1) – 9 + 15 (1) – 11]

$= \frac{1}{9}$ (2 + 3 + 4)

∴ I = 1

Q.12 $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx =$

Correct option: (C)

$\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx = \int \frac{x^5 - x^4}{x^3 - x^2} dx$

$= \int \frac{x^4(x-1)}{x^2(x-1)} dx = \int x^2 dx = \frac{x^3}{3} + c$

Q.13 $\int_1^3 \left[\tan^{-1}\left(\frac{x}{x^2-1}\right) + \tan^{-1}\left(\frac{x^2-1}{x}\right)\right] dx =$

Correct option: (A)

$\int_1^3 \left[\tan^{-1}\left(\frac{x}{x^2-1}\right) + \tan^{-1}\left(\frac{x^2-1}{x}\right)\right] dx$

$= \int_1^3 \left[\tan^{-1}\left(\frac{x}{x^2-1}\right) + \cot^{-1}\left(\frac{x}{x^2-1}\right) dx\right]$

$= \int_1^3 \frac{\pi}{2} dx$

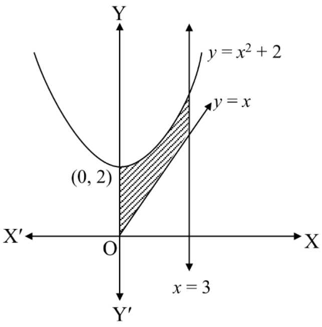
$= \frac{\pi}{2}[x]_1^3$

$= \frac{\pi}{2}$ (3 – 1) = π

Q.14 Area of the region bounded by the curve $y = x^2 + 2$ and the lines $y = x$, $x = 0$ and $x = 3$ is

Correct option: (B)

KUNAL ACADEMY



Required area = $\int_0^3 (x^2 + 2 - x)dx$

= $\left[\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_0^3$

= $9 + 6 - \frac{9}{2} - 0$

= $\frac{21}{2}$ sq. units

Q.15 $\int \frac{\log\left(t + \sqrt{1+t^2}\right)}{\sqrt{1+t^2}} \text{dt} = \frac{1}{2}[g(t)]^2 + c$

, (where c is a constant of integration) then g(2) is

Correct option: (D)

Put log $\left(t + \sqrt{1+t^2}\right) = y$

$\Rightarrow \left[\frac{1}{t+\sqrt{1+t^2}}\left(1 + \frac{t}{\sqrt{1+t^2}}\right)\right]\text{dt} = dy$

$\Rightarrow \frac{1}{\sqrt{1+t^2}}\text{dt} = dy$

$\therefore \int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}\text{dt} = \int y\,dy$

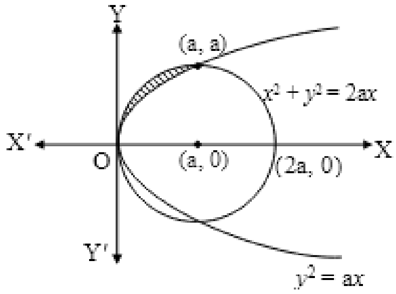
= $\frac{y^2}{2} + c = \frac{\left[\log\left(t+\sqrt{1+t^2}\right)\right]^2}{2} + c$

$\therefore$ g(t) = $\log\left(t + \sqrt{1+t^2}\right)$

$\Rightarrow$ g(2) = $\log\left(2 + \sqrt{1+2^2}\right)$ = $\log\left(2+\sqrt{5}\right)$

Q.16 The area of the region lying above X-axis, and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$, a > 0 is

Correct option: (B)



Required area = $\frac{\pi a^2}{4} - \int_0^a \sqrt{ax}\,dx$

= $\frac{\pi a^2}{4} - \frac{2\sqrt{a}}{3}\left[x^{\frac{3}{2}}\right]_0^a$

= $\frac{\pi a^2}{4} - \frac{2a^2}{3}$

= $a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$ sq. units.

Q.17 Let f : [–1, 2] → [0, ∞) be a continuous function such that f(x) = f(1 – x), ∀ x ∈ [–1, 2]

Let R_1 = $\int_{-1}^{2} x$ f(x) dx and R_2 be the area of the region bounded by y = f(x), x = –1, x = 2 and the X-axis, then R_2 is

Correct option: (B)

Given that f(x) = f(1 – x) and $R_1 = \int_{-1}^{2} x\ f(x)\ dx$

$\therefore\ R_1 = \int_{-1}^{2}(1-x)\ f(1-x)\ dx$ …

$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx\right]$

$\therefore\ R_1 = \int_{-1}^{2} f(x)\ dx - \int_{-1}^{2} x\ f(x)\ dx$ …

$[\because f(x) = f(1-x)]$

$\therefore\ R_1 = \int_{-1}^{2} f(x)\ dx - R_1$

KUNAL ACADEMY

$\therefore\ 2R_1 = \int_{-1}^{2} f(x)\, dx$

Note that $R_2 = \int_{-1}^{2} f(x)\, dx = 2R_1$

Q.18 **The value of $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^{103}x.\cos^{101}x dx$ is**

Correct option: (B)

Since $\sin^{103} x \cos^{101} x$ is an odd function.

$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^{103}x \cos^{101}x dx = 0$

Q.19 **The area of the region bounded by the curve $y = 4x^3 - 6x^2 + 4x + 1$ and the lines $x = 1, x = 5$ and X-axis is**

Correct option: (C)

Required area

$= \int_1^5 y\, dx = \int_1^5 \left(4x^3 - 6x^2 + 4x + 1\right) dx$

$= \left[\frac{4}{4}\left(x^4\right) - \frac{6}{3}\left(x^3\right) + \frac{4}{2}\left(x^2\right) + x\right]_1^5$

$= [((5)^4 - 2(5)^3 + 2(5)^2 + 5) - (1 - 2 + 2 + 1)]$

$= 625 - 250 + 50 + 5 - 2$

$= 428$ sq. units

Q.20 $\int \frac{(\log x - 1)^2}{\left[1 + (\log x)^2\right]^2} = dx$ **, (where C is a constant of integration.)**

Correct option: (C)

Let $I = \int \left[\frac{1 - \log x}{1 + (\log x)^2}\right]^2 dx$

Put $\log x = t$

$\therefore\ x = e^t$

$dx = e^t dt$

$\therefore\ I = \int \left(\frac{1 - t}{1 + t^2}\right)^2 e^t dt$

$= \int \left(\frac{1 - 2t + t^2}{(1 + t^2)^2}\right) e^t dt$

$= \int e^t \left(\frac{1 + t^2 - 2t}{(1 + t^2)^2}\right)$

$= \int e^t \left[\frac{1}{1 + t^2} - \frac{2t}{(1 + t^2)^2}\right] dt$

$= e^t \left(\frac{1}{1 + t^2}\right) + c \quad \ldots$

$\left[\because \int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + c\right]$

$= \frac{x}{1 + (\log x)^2} + C$

Q.21 **The area of the region bounded by the curve $y = 4x - x^2$ and the X -axis in the first quadrant is**

Correct option: (D)

For X-axis, $y = 0$

$\therefore\ 4x - x^2 = 0$

$\Rightarrow x(4 - x) = 0 \quad \Rightarrow x = 0, 4$

$\therefore$ Required area $= \int_0^4 (4x - x^2) dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= 32 - \frac{64}{3} = \frac{32}{3}$ sq. units

Q.22 $\int \frac{\log \sqrt{x}}{3x} dx$ **is equal to**

Correct option: (A)

Put $x = t^2 \Rightarrow dx = 2t\, dt$

$\therefore \int \frac{\log \sqrt{x}}{3x} dx = \int \frac{\log t}{3t^2}(2t dt)$

$= \frac{2}{3} \int \frac{\log t}{t} dt$

$= \frac{2}{3} \cdot \frac{(\log t)^2}{2} + c$

$= \frac{(\log \sqrt{x})^2}{3} + c$

Q.23 **If $\int \sin^{13}x \cos^3 x\, dx = A \sin^{14} x + B \sin^{16} x + c$, then A + B =**

Correct option: (D)

Let $I = \int \sin^{13}x \cos^3 x\, dx$

KUNAL ACADEMY

$$= \int \sin^{13}x\left(1-\sin^2 x\right)\cos x\,dx$$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$

$\therefore I = \int t^{13}\left(1-t^2\right)dt$

$$= \frac{t^{14}}{14} - \frac{t^{16}}{16} + c =$$

$$\frac{1}{14}\sin^{14}x - \frac{1}{16}\sin^{16}x + c$$

$\therefore A + B = \frac{1}{14} - \frac{1}{16} = \frac{1}{112}$

Q.24 **The value of** $\lim\limits_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$ **is equal to**

Correct option: (A)

$$\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$$

$$= \lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt + \int_a^{x+y} e^{\sin^2 t}dt\right]$$

$$= \lim_{x\to 0}\frac{\int_a^{x+y} e^{\sin^2 t}dt}{x}$$

$$= \lim_{x\to 0}\frac{e^{\sin^2(x+y)}\times 1 - e^{\sin^2 y}\times 0}{1}$$ ...[By L'Hospital Rule]

$= e^{\sin^2 y}$

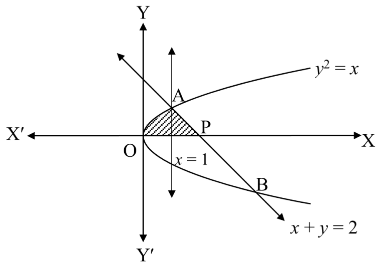
Q.25 **The area bounded by the parabola $y^2 = x$ and the line $x + y = 2$ in the first quadrant is**

Correct option: (A)

$y^2 = x$ and $x + y = 2$

Solving these equations, we get

$x = 1$ or 4



Required area $= \int\limits_0^1 \sqrt{x}\,dx + \int\limits_1^2 (2-x)dx$

$$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{2}{3}(1) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{2}{3} + 2 - \frac{3}{2} = \frac{7}{6}\text{ sq. units}$$

Q.26 $\int x^{51}\left(\tan^{-1}x + \cot^{-1}x\right)$**dx =**

Correct option: (A)

$\int x^{51}\left(\tan^{-1}x + \cot^{-1}x\right)dx = \int x^{51}\cdot\frac{\pi}{2}dx$ $\ldots\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$= \frac{\pi}{2}\times\frac{x^{52}}{52} + c$

$= \frac{x^{52}}{52}\left(\tan^{-1}x + \cot^{-1}x\right) + c$

Q.27 $\int \sec^4 x\tan x\,dx =$

Correct option: (A)

$$\int \sec^4 x\tan x\,dx = \int \sec^3 x\,\sec x\,\tan x\,dx$$

Put $t = \sec x \Rightarrow dt = \sec x\tan x\,dx$

$$\therefore \int \sec^4 x\tan x\,dx = \int t^3 dt = \frac{t^4}{4} + c =$$

$$\frac{1}{4}\sec^4 x + c$$

Q.28 **The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the X-axis is**

Correct option: (D)

Required area $= \int\limits_1^3 |x-2|dx$

$$= \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$$

$$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= \frac{1}{2} + \frac{1}{2} = 1\text{ sq. unit}$$

Q.29 $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx =$ **(where $|x| < 1$)**

Correct option: (D)

KUNAL ACADEMY

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$dx = \sec^2\theta \, d\theta$

$$\therefore \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

$$= \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan\theta}\right)dx \sec^2\theta \cdot d\theta$$

$$= \int \sin^{-1}(\sin 2\theta)\cdot \sec^2\theta \cdot d\theta$$

$$= \int 2\theta \cdot \sec^2\theta d\theta$$

$$= 2\theta\cdot \tan\theta - \int 2\cdot \tan\theta d\theta$$

$$= 2\theta \tan\theta + 2\log|\cos\theta| + c$$

$$= 2\tan^{-1} x \cdot (x) + 2\log\left|\frac{1}{\sqrt{1+\tan^2\theta}}\right| + c$$

$$= 2x\tan^{-1} x + 2\log\left|\frac{1}{\sqrt{1+x^2}}\right| + c$$

$$= 2x\tan^{-1} x + 2\log\left|1+x^2\right|^{\frac{-1}{2}} + c$$

$$= 2x\tan^{-1}x - \log\left|1+x^2\right| + c$$

Q.30 **If $g(x) = \int_0^x \cos^4 t\, dt$, then g($x$ + π) equals**

Correct option: (A)

$$g(x+\pi) = \int_0^{x+\pi} \cos^4 t\, dt$$

$$= \int_0^x \cos^4 t\, dt + \int_0^{\pi+x} \cos^4 t\, dt$$

In 2nd integral,

Put t = π + z ⇒ dt = dz

$$\therefore g(x+\pi) = g(\pi) + \int_0^x \cos^4(\pi+z)$$

$$= g(\pi) + \int_0^x \cos^4 z$$

$= g(\pi) + g(x)$

Q.31 $\int \frac{dx}{2e^{2x}+3e^x+1} =$

Correct option: (A)

Let $I = \int \frac{dx}{2e^{2x}+3e^x+1}$

∴ Let $e^x = t$

$e^x dx = dt$

$\Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t}$

$$\therefore I = \int \frac{dt}{t(2t^2+3t+1)}$$

$$= \int \frac{dt}{t(2t+1)(t+1)}$$

Let $\frac{1}{t(2t+1)(t+1)} = \frac{A}{t} + \frac{B}{(2t+1)} + \frac{C}{t+1}$

⇒ 1 = A (2t + 1) (t + 1) + Bt (t + 1) + Ct (2t + 1) ...(i)

Put t = –1 in (i), we get

C = 1

Put $t = \frac{-1}{2}$ in (i), we get

B = –4

Put t = 0 in (i), we get

A = 1

$$\therefore \int \frac{dt}{t(2t+1)(t+1)}$$

$$= \int \frac{1}{t}dt - 4\int \frac{1}{2t+1}\,dt + \int \frac{1}{t+1}\,dt$$

$$= \log t - \frac{1}{2}\times 4\log|2t+1| + \log|t+1| + c$$

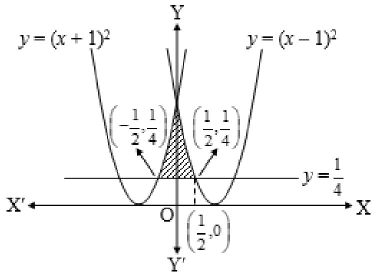
$$= \log e^x + \log|e^x+1| - 2\log|2e^x+1| + c$$

$= x + \log(e^x + 1) - 2\log(2e^x + 1) + c$

Q.32 **The area bounded by the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y = \frac{1}{4}$ is**

Correct option: (A)

KUNAL ACADEMY



Required area = $2\int_{0}^{\frac{1}{2}}\left[(x-1)^2-\frac{1}{4}\right]dx$

$= 2\left[\frac{(x-1)^3}{3}\right]_0^{\frac{1}{2}}-\frac{1}{2}[x]_0^{\frac{1}{2}}$

$= \frac{2}{3}\left(-\frac{1}{8}+1\right)-\frac{1}{2}\left(\frac{1}{2}-0\right)$

$= \frac{1}{3}$ sq. units

Q.33 $\int_{2}^{3}\frac{dx}{x^2+x}=$

Correct option: (D)

$\int_{2}^{3}\frac{dx}{x^2+x}=\int_{2}^{3}\frac{dx}{x(x+1)}$

$= \int_{2}^{3}\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$

$= [\log|x|-\log|x+1|]_2^3$

$= (\log 3-\log 4)-(\log 2-\log 3)$

$= \log\left(\frac{3}{4}\right)-\log\left(\frac{2}{3}\right)$

$= \log\left(\frac{3}{4}\times\frac{3}{2}\right)=\log\left(\frac{9}{8}\right)$

Q.34 $\int\cos\left(\frac{x}{16}\right)\cdot\cos\left(\frac{x}{8}\right)\cdot\cos\left(\frac{x}{4}\right)\cdot\sin\left(\frac{x}{16}\right)dx=$

Correct option: (D)

$\int\cos\frac{x}{16}\cos\frac{x}{8}\cos\frac{x}{4}\sin\frac{x}{16}\,dx$

$= \int\cos\frac{x}{16}\sin\frac{x}{16}\cos\frac{x}{8}\cos\frac{x}{4}\,dx$

$= \int\sin\frac{x}{8}\cos\frac{x}{8}\cos\frac{x}{4}\,dx$ …[∵ sin 2θ = 2 sin θ cos θ]

$= \frac{1}{4}\int\sin\frac{x}{4}\cos\frac{x}{4}dx$

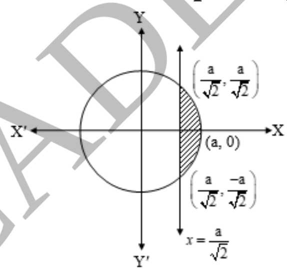
$= \frac{1}{8}\int\sin\frac{x}{2}dx$

$= \left(-\cos\frac{x}{2}\right)+c$

$= \frac{-1}{4}\cos\frac{x}{2}+c$

Q.35 The area (in sq. units) of the smaller part of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{\sqrt{2}}$ is

Correct option: (C)



Substitute $x=\frac{a}{\sqrt{2}}$ in $x^2+y^2=a^2$, we get

$\frac{a^2}{2}+y^2=a^2\Rightarrow y=\pm\frac{a}{\sqrt{2}}$

∴ Required area

$= 2\int_{\frac{a}{\sqrt{2}}}^{a}\sqrt{a^2-x^2}\,dx$

$= 2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{\frac{a}{\sqrt{2}}}^{a}$

$= 2\left\{\left[0+\frac{a^2}{2}\times\frac{\pi}{2}\right]-\left[\frac{a}{2\sqrt{2}}\sqrt{a^2-\frac{a^2}{2}}+\frac{a^2}{2}\times\frac{\pi}{4}\right]\right\}$

$= 2\left[\frac{a^2\pi}{4}-\frac{a^2}{4}-\frac{a^2\pi}{8}\right]$

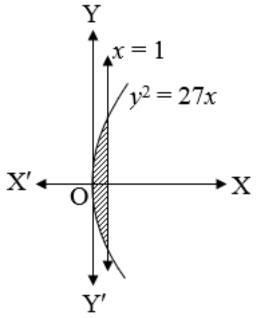
$= \frac{a^2}{2}\left|\pi-1-\frac{\pi}{2}\right|$

$= \frac{a^2}{2}\left|\frac{\pi}{2}-1\right|$

KUNAL ACADEMY

Q.36 The area of the region bounded by the parabola $y^2 = 27x$ and the line $x = 1$ is _____ sq.units.

Correct option: (C)



Required area

$= 2\int_0^1 y dx$

$= 2\int_0^1 \sqrt{27x} dx$

$= 6\sqrt{3}\int_0^1 x^{\frac{1}{2}} dx$

$= 6\sqrt{3}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= 4\sqrt{3}\left(1^{\frac{3}{2}} - 0\right)$

$= 4\sqrt{3}$ sq. units

Q.37 $\int \frac{1 + x^2}{\sqrt{1 - x^2}} dx =$

Correct option: (A)

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$\therefore \int \frac{1+x^2}{\sqrt{1-x^2}} dx = \int \frac{1+\sin^2\theta}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta\, d\theta$

$= \int (1+\sin^2\theta) d\theta$

$= \int d\theta + \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$

$= \theta + \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right) + c$

$= \frac{3}{2}\theta - \frac{\sin\theta\, \cos\theta}{2} + c$

$= \frac{3}{2}\theta - \frac{\sin\theta\sqrt{1-\sin^2\theta}}{2} + c$

$= \frac{3}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2} + c$

Q.38 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx =$

Correct option: (C)

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx$ ….[∵ $\sin^2 x$ is an even function]

$\int_0^{\frac{\pi}{2}} \sin^n x dx =$

$\begin{cases} \frac{(n-1)(n-3)\ldots.2}{n(n-2)\ldots..3}, when\ n\ is\ odd \\ \frac{(n-1)(n-3)\ldots.1}{n(n-2)\ldots..2}\cdot\frac{\pi}{2}, when\ n\ is\ even \end{cases}$

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx = 2\left(\frac{1}{2}\cdot\frac{\pi}{2}\right) = \frac{\pi}{2}$

Q.39 $\int \left(f(x)g''(x) - f''(x)g(x)\right)dx$ **is equal to**

Correct option: (C)

$\int [f(x)g''(x) - f''(x)g(x)] dx$

$= f(x)g'(x) - \int f'(x)g'(x)dx - g(x)f'(x) + \int f'(x)g'(x)dx$

$= f(x)g'(x) - g(x)f'(x) + c$

Q.40 $\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x} dx =$

Correct option: (A)

Put $\sin^2 x = t \Rightarrow 2\sin x\cos x\, dx = dt$

When $x = 0$, $t = 0$ and when $x = \frac{\pi}{2}$, $t = 1$

$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x} dx = \frac{1}{2}\int_0^1 \frac{1}{1+t^2} dt$

KUNAL ACADEMY

$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$

$= \frac{1}{2}\left(\tan^{-1} 1 - 0\right)$

$= \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}$

Q.41 $\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ **is equal to**

Correct option: (A)

Let I = $\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$\therefore I = \int_0^{\pi/6} \frac{\cos\theta\, d\theta}{\left(1+\sin^2\theta\right)\cos\theta}$

$= \int_0^{\pi/6} \frac{\sec^2\theta}{\sec^2\theta + \tan^2\theta}\, d\theta$

$= \int_0^{\pi/6} \frac{\sec^2\theta}{1 + 2\tan^2\theta}\, d\theta$

$= \int_0^{\pi/6} \frac{\sec^2\theta}{1 + \left(\sqrt{2}\tan\theta\right)^2}\, d\theta$

$= \frac{1}{\sqrt{2}}\int_0^{\pi/6} \frac{\sqrt{2}\sec^2\theta}{1 + \left(\sqrt{2}\tan\theta\right)^2}\, d\theta$

$= \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{2}\tan\theta\right)\right]_0^{\pi/6}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\sqrt{\frac{2}{3}}$

Q.42 **Let** $I_1 = \int_a^{\pi-a} x\, f(\sin x)\, dx$,

$I_2 = \int_a^{\pi-a} f(\sin x)\, dx$**, then** I_2 **is equal to**

Correct option: (C)

$I_1 = \int_a^{\pi-a} x\, f(\sin x)\, dx$

$= \int_a^{\pi-a} (\pi - x)\, f(\sin(\pi - x))dx$

$\ldots\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

$= \int_a^{\pi-a} (\pi - x)\, f(\sin x)dx$

$\therefore I_1 = \int_a^{\pi-a} \pi\, f(\sin x)dx - I_1$

$\Rightarrow 2I_1 = \pi I_2 \Rightarrow I_2 = \frac{2}{\pi} I_1$

Q.43 $\int_{-1}^{1} \sin^2 x \cos^2 x\, dx =$

Correct option: (B)

Let $f(x) = \sin^2 x \cos^2 x$

$\therefore f(-x) = [\sin(-x)]^2\, [\cos(-x)]^2$

$= \sin^2 x \cos^2 x$

$= f(x)$

$\therefore f(x)$ is an even function.

$\therefore \int_{-1}^{1} \sin^2 x \cos^2 x\, dx$

$= 2\int_0^1 \sin^2 x \cos^2 x\, dx$

$= \frac{2}{4}\int_0^1 (2\sin x \cos x)^2 dx$

$= \frac{1}{2}\int_0^1 \sin^2 2x\, dx$

$= \frac{1}{4}\int_0^1 (1 - \cos 4x)\, dx$

$= \frac{1}{4}[x]_0^1 - \frac{1}{16}[\sin 4x]_0^1$

$= \frac{1}{4} - \frac{\sin 4}{16}$

Q.44 $\int_0^a \frac{x-a}{x+a} dx =$

Correct option: (A)

Let I = $\int_0^a \frac{x-a}{x+a} dx$

Let $x + a = t$

KUNAL ACADEMY

$\Rightarrow x = t - a$

$\therefore \quad dx = dt$

If $x = 0$, then $t = a$

If $x = a$, then $t = 2a$

$$\therefore \quad I = \int_a^{2a} \frac{t - 2a}{t} dt$$

$$= \int_a^{2a} 1 dt - 2a \int_a^{2a} \frac{1}{t} dt = [t]_a^{2a} - 2a[\log t]_a^{2a}$$

$= a - 2a (\log 2a - \log a)$

$= a - 2a \log 2$

Q.45 $\int \tan(3x - 5) \sec(3x - 5) \, dx =$

Correct option: (B)

Put $t = 3x - 5 \Rightarrow dt = 3dx$

$$\therefore \int \tan(3x - 5) \sec(3x - 5) \, dx = \frac{1}{3} \int \tan t . \sec t \, dt = \frac{\sec t}{3} + c = \frac{\sec(3x - 5)}{3} + c$$

Q.46 $\int \frac{dx}{\sin x + \sqrt{3} \cos x} =$

Correct option: (B)

$$\int \frac{dx}{\sin x + \sqrt{3}\cos x} = \frac{1}{2} \int \frac{dx}{\frac{\sin x}{2} + \frac{\sqrt{3}}{2}\cos x} = \frac{1}{2} \int \frac{dx}{\sin x \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos x}$$

$$= \frac{1}{2} \int \frac{dx}{\sin\left(x + \frac{\pi}{3}\right)}$$

$$= \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{3}\right) dx = \frac{1}{2} \log \left| \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) \right| + c$$

Q.47 **Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x) \sin 2\alpha + c$, (where c is a constant of integration), then functions $A(x)$ and $B(x)$ are respectively**

Correct option: (B)

$$\text{Let } I = \int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$$

$$= \int \frac{\frac{\sin x}{\cos x} + \frac{\sin \alpha}{\cos \alpha}}{\frac{\sin x}{\cos x} - \frac{\sin \alpha}{\cos \alpha}} dx$$

$$= \int \frac{\sin x \cos \alpha + \sin \alpha \cos x}{\sin x \cos \alpha - \sin \alpha \cos x} dx$$

$$= \int \frac{\sin(x + \alpha)}{\sin(x - \alpha)} dx$$

Let $x - \alpha = t$

$$\therefore \quad I = \int \frac{\sin(t + 2\alpha)}{\sin t} dt$$

$$= \int \frac{\sin(t) \cos 2\alpha + \cos(t) \sin 2\alpha}{\sin(t)} dx$$

$$= \cos 2\alpha \int 1 \, dt + \sin 2\alpha \int \cot(t) \, dt$$

$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \log |\sin(t)| + c$

$\therefore \quad I = (x - \alpha) \cos 2\alpha + \log |\sin(x - \alpha)| \sin 2\alpha + c$

But $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x) \sin 2\alpha + c$…[Given]

$\Rightarrow A(x) = x - \alpha, B(x) = \log |\sin(x - \alpha)|$

Q.48 **The area of the region bounded by the parabola $x^2 = 16y$, $y = 1$, $y = 4$ and the Y-axis lying in the first quadrant is**

Correct option: (A)

$x^2 = 16y \Rightarrow x = 4\sqrt{y}$, $y = 1$, $y = 4$

$$\text{Required area} = \int_1^4 4\sqrt{y} \, dy$$

$$= 4 \int_1^4 y^{\frac{1}{2}} dy$$

KUNAL ACADEMY

$= \frac{4 \times 2}{3} \left[y^{\frac{3}{2}}\right]_1^4$

$= \frac{8}{3}[8-1] = \frac{56}{3}$ sq. units

Q.49 $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx =$

Correct option: (A)

Let I = $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

When $x = 0$, t = 0 and when $x = 1$, t = $\frac{\pi}{4}$

$\therefore$ I = $8\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \cdot sec^2\theta\, d\theta$

$= 8\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{\sec^2\theta} \cdot sec^2\theta\, d\theta$

$\therefore$ I = $8\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$

$\Rightarrow$ I = $8\left(\frac{\pi}{8}\log 2\right)$

$= \pi \log 2$

Q.50 **The area (in sq. units) bounded by the curve $y = x|x|$, X-axis and the lines $x = -1$ and $x = 1$ is**

Correct option: (A)

$y = x|x|$ …[Given]

Required area

$= \int_{-1}^{1} x|x|\, dx$

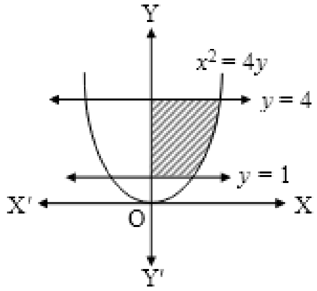
$= 2\int_0^1 x^2\, dx$ …[$\because$ Area is always positive]

$= 2 \times \left[\frac{x^3}{3}\right]_0^1$

$= 2 \times \left(\frac{1}{3} - 0\right) = \frac{2}{3}$ sq.units

Q.51 **The area of the region bounded by $x^2 = 4y, y = 1, y = 4$ and the Y-axis lying in the first quadrant is _______ square units.**

Correct option: (B)



Required area = $\int_1^4 x\, dy$

$= \int_1^4 2\sqrt{y}\, dy$

$= 2\left(\frac{2}{3}\right)\left[y^{3/2}\right]_1^4$

$= \frac{28}{3}$ sq. units

Q.52 **The value of the integral** $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^2 + \log\frac{\pi - x}{\pi + x}\right)\cos x dx$ **is equal to**

Correct option: (B)

Let I = $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left[x^2 + \log\left(\frac{\pi - x}{\pi + x}\right)\right]\cos x\, dx$

I =

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{\pi - x}{\pi + x}\right)\cos x\, dx$

Let I = $I_1 + I_2$

Where $I_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx$ and

$I_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{\pi - x}{\pi + x}\right)\cos x dx$

Consider

KUNAL ACADEMY

$$I_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x \mathrm{d}x$$

$$= 2\int_0^{\frac{\pi}{2}} x^2 \cos x \mathrm{d}x$$ …[$x^2 \cos x$ is an even function]

$$= 2\left[x^2 \int cosx\ \mathrm{d}x - \int \frac{d}{dx}(x^2)\left(\int \cos x\ \mathrm{d}x\right)\ \mathrm{d}x\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[x^2 \cdot \sin x - \int 2x \cdot \sin x\ \mathrm{d}x\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[x^2 \sin x - 2\int x \cdot \sin x\ \mathrm{d}x\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[x^2 \sin x - 2\left(x(-\cos x) - \int(-\cos x)dx\right)\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi^2}{4}\sin\frac{\pi}{2} + 2\times\frac{\pi}{2}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} - 0^2\sin 0 - 2\times 0\times\cos 0 + 2\sin 0\right]$$

$$= 2\left[\frac{\pi^2}{4} - 2 - 0 - 0 - 0\right]$$

$$= 2\left[\frac{\pi^2}{4} - 2\right] = \frac{\pi^2}{2} - 4$$

Consider

$$I_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{\pi - x}{\pi + x}\right)\cos x dx$$

$\therefore$ $I_2 = 0$ …

$$\left[\log\left(\frac{\pi - x}{\pi + x}\right)\cos x \text{ is an odd function}\right]$$

$\therefore$ $I = I_1 + I_2$

$\Rightarrow I = \frac{\pi^2}{2} - 4$

Q.53 $\int \frac{\sqrt{\cot x}}{\sin x \cdot \cos x}$ **dx =**

Correct option: (B)

Let $I = \int \frac{\sqrt{\cot x}}{\sin x \cdot \cos x}\ \mathrm{d}x$

Dividing numerator and denominator by $\sin^2 x$, we get

$I = \int \frac{\sqrt{\cot x}}{\cot x} \cdot \mathrm{cosec}^2 x\ \mathrm{d}x$

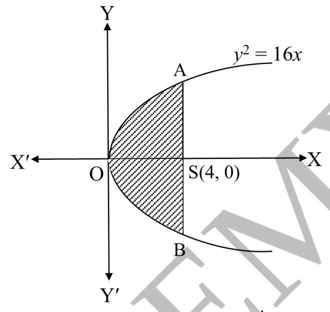
$= -\int \frac{-\mathrm{cosec}^2 x}{\sqrt{\cot x}}\ \mathrm{d}x$

$= -2\sqrt{\cot x} + c$ …

$\left[\because\ \int \frac{f'(x)}{\sqrt{f(x)}}\ dx\ =\ 2\sqrt{f\ (x)}\ +\ c\right]$

Q.54 **The area of the region bounded by parabola $y^2 = 16x$ and its latus rectum is ______ square units.**

Correct option: (B)



Required area $= 2\int_0^4 4\sqrt{x}\,dx$

$= 8\int_0^4 \sqrt{x}\,dx$

$= 8\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$

$= \frac{16}{3}\left[4^{\frac{3}{2}} - 0\right]$

$= \frac{16}{3} \times 8 = \frac{128}{3}$ sq. units

Q.55 $\int \frac{x}{(x-2)(x-1)}\ dx$ **=**

Correct option: (A)

Let $\frac{x}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$

$\therefore x = A(x - 1) + B(x - 2)$ …(i)

Putting $x = 1$ in (i), we get

B = –1

Putting $x = 2$ in (i), we get

A = 2

$\therefore \int \frac{x}{(x-2)(x-1)}\ \mathrm{d}x =$

$\int\left(\frac{2}{x-2} - \frac{1}{x-1}\right)\ \mathrm{d}x$

$= 2\log|x - 2| - \log|x - 1| + c$

KUNAL ACADEMY

$$= \log\left|\frac{(x-2)^2}{(x-1)}\right| + c$$

Q.56 $\displaystyle\int_0^3 \frac{dx}{(x+2)\sqrt{x+1}} =$

Correct option: (B)

Let I = $\displaystyle\int_0^3 \frac{dx}{(x+2)\sqrt{x+1}}$

Put $x + 1 = t^2$

$\Rightarrow dx = 2t\,dt$

When $x = 0$, t = 1 and when $x = 3$, t = 2

$\therefore\ \ I = \displaystyle\int_1^2 \frac{2t\,dt}{(t^2+1)\sqrt{t^2}}$

$= 2\displaystyle\int_1^2 \frac{t\,dt}{(t^2+1)t}$

$= 2\displaystyle\int_1^2 \frac{dt}{t^2+1}$

$= 2\left[\tan^{-1}t\right]_1^2$

$= 2\left(\tan^{-1}2 - \tan^{-1}1\right)$

$= 2\left[\tan^{-1}\left(\dfrac{2-1}{1+(2)(1)}\right)\right]\ldots$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)\right]$

$= 2\tan^{-1}\left(\dfrac{1}{3}\right)$

Q.57 $\displaystyle\int \tan^{-1}\left(\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right) dx =$

Correct option: (C)

$\displaystyle\int \tan^{-1}\left(\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right) dx = \int \tan^{-1}\left(\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}\right) dx$

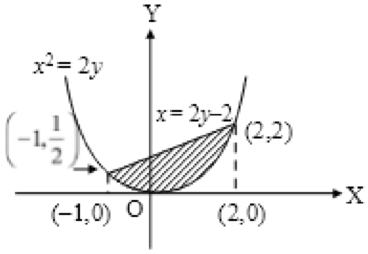
$= \displaystyle\int \tan^{-1}(\tan x)\,dx$

$= \displaystyle\int x\,dx = \frac{x^2}{2} + c$

Q.58 **Area bounded by the curve $x^2 = 2y$ and the straight line $x = 2y - 2$ is**

Correct option: (B)

The points of intersection of $x^2 = 2y$ and $x = 2y - 2$ are (2, 2) and $\left(-1, \frac{1}{2}\right)$.



Required area = $\displaystyle\int_{-1}^{2} \frac{1}{2}(x+2)\,dx - \int_{-1}^{2} \frac{1}{2}x^2\,dx$

$= \frac{1}{2}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{9}{4}$ sq. units

Q.59 $\displaystyle\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2} =$

Correct option: (C)

$\displaystyle\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}$

$= \displaystyle\int_0^{\pi} \frac{dx}{(1+a^2)\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) - 2a\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}$

$= \displaystyle\int_0^{\pi} \frac{dx}{(1-a)^2\cos^2\frac{x}{2} + (1+a)^2\sin^2\frac{x}{2}}$

$= \displaystyle\int_0^{\pi} \frac{\sec^2\frac{x}{2}}{(1-a)^2 + (1+a^2)\tan^2\frac{x}{2}}\,dx$

$= \dfrac{2}{(1+a)^2}\displaystyle\int_0^{\infty} \frac{dt}{\left\{\frac{(1-a)}{(1+a)}\right\}^2 + t^2}\ \ldots.$

$\left[\text{Put } t = \tan\dfrac{x}{2} \Rightarrow dt = \dfrac{1}{2}\sec^2\dfrac{x}{2}dx\right]$

$= \dfrac{2}{(1+a)^2}\dfrac{(1+a)}{(1-a)}\left[\tan^{-1}\left(\dfrac{1+a}{1-a}.t\right)\right]_0^{\infty}$

$= \dfrac{2}{(1+a)^2}\left(\tan^{-1}\infty - \tan^{-1}0\right) = \dfrac{\pi}{1-a^2}$

KUNAL ACADEMY

Q.60 $\int_{0}^{\pi/4} (cosx - sinx)$**dx +**

$\int_{\pi/4}^{5\pi/4} (sinx - cosx)$ **dx +**

$\int_{2\pi}^{\pi/4} (cosx - sinx)$**dx is equal to**

Correct option: (D)

$\int_{0}^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx$

$+ \int_{2\pi}^{\frac{\pi}{4}} (\cos x - \sin x)dx$

$=[\sin x + \cos x]_0^{\pi/4} - [\cos x + \sin x]_{\pi/4}^{5\pi/4} +$

$[\sin x + \cos x]_{2\pi}^{\pi/4}$

$=$

$\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

$+\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right)$

$= \left(\sqrt{2} - 1\right) - \left(-\sqrt{2} - \sqrt{2}\right) + \left(\sqrt{2} - 1\right)$

$= 4\sqrt{2} - 2$

Q.61 $\int_{-1}^{4} (f(x))\, dx = 4$ **and**

$\int_{2}^{4} (3 - f(x))\; dx = 7$**, then** $\int_{-1}^{2} [f(x)]\; dx$

is

Correct option: (D)

$\int_{2}^{4} (3 - f(x))\; dx = 7$

$\Rightarrow \int_{2}^{4} 3\, dx - \int_{2}^{4} f(x)\; dx = 7$

$\Rightarrow 3[x]_2^4 - 7 = \int_{2}^{4} f(x)\; dx$

$\Rightarrow \int_{2}^{4} f(x)dx = 3\,(4 - 2) - 7$

$= -1$

$\int_{-1}^{4} f(x)\; dx = \int_{-1}^{2} f(x)\; dx + \int_{2}^{4} f(x)\; dx \;\ldots$

$\left[\because \int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx \quad if\, a < c < b\right]$

$\Rightarrow 4 = \int_{-1}^{2} f(x)\; dx - 1$

$\Rightarrow \int_{-1}^{2} f(x)\; dx = 5$

Q.62 $\int_{0}^{2\pi} (sinx + |sinx|)$ **dx =**

Correct option: (B)

$\int_{0}^{2\pi} (sinx + |sinx|)\, dx =$

$\int_{0}^{\pi} 2\sin\; x\; dx \; + \; \int_{\pi}^{2\pi} 0\; .\; dx$

$= 2[-\;\cos\; x]_0^{\pi} \; + \; 0$

$= -\; 2\; (\cos\; \pi \; - \; \cos\; 0)$

$= -\; 2\; (-\; 1 \; - \; 1) \; = \; 4$

Q.63 **The value of** $\int_1^4 \log[x]dx$, **where** $[x]$ **is the greatest integer function less than or equal to** ***x*** **is equal to**

Correct option: (B)

Let I = $\int_1^4 \log[x]dx$

KUNAL ACADEMY

$$\int_1^4 \log[x]dx = \int_1^2 0dx + \int_2^3 \log 2dx + \int_3^4 \log 3dx$$

= 0 + log 2 [3 – 2] + log3 [4 – 3]

= log 2 + log 3

= log (2 × 3) = log 6

Q.64 If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curve, line x = 4 and X-axis is 8 sq. units, then

Correct option: (A)

The given curve passes through (1, 2).

∴ 2 = a + b …(i)

According to the given condition,

$$\int_0^4 \left(a\sqrt{x} + bx\right) dx = 8$$

$$\Rightarrow \frac{2a}{3}[x^{3/2}]_0^4 + \frac{b}{2}[x^2]_0^4 = 8 \Rightarrow \frac{2a}{3}.8 + 8b = 8$$

⇒ 2a + 3b = 3 …(ii)

From (i) and (ii), we get

a = 3, b = –1

Q.65 $\int [1 + 2\tan x(\tan x + \sec x)]^{\frac{1}{2}} dx =$

Correct option: (C)

1 + 2 tan x (sec x + tan x)

= 1 + 2 tan x . sec x + 2 $\tan^2 x$

= (1 + $\tan^2 x$) + 2 sec x . tan x + $\tan^2 x$

= $\sec^2 x$ + 2 sec x . tan x + $\tan^2 x$

= $(\sec x + \tan x)^2$

$$\therefore \int [1 + 2\tan x(\tan x + \sec x)]^{\frac{1}{2}} dx$$

$$= \int (\sec x + \tan x)\ dx$$

= log |sec$x$ + tan x| – log |cos$x$| + c

$$= \log \frac{|\sec x + \tan x|}{(\cos x)} + c$$

= log [sec x (sec x + tan x)] + c

Q.66 $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\text{cosec}x \cdot \cot x}{1 + \text{cosec}^2 x} dx =$

Correct option: (D)

Let I = $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\text{cosec}x \cdot \cot x}{1 + \text{cosec}^2 x} dx$

Put cosec x = t

⇒ cosec x cot x dx = –dt

When $x = \frac{\pi}{6}$, t = 2 and when $x = \frac{\pi}{2}$, t = 1

$$\therefore \quad I = \int_2^1 \frac{1}{1 + t^2}(-\text{dt})$$

$$= \int_1^2 \frac{1}{1 + t^2} dt$$

$$= \left[\tan^{-1}(t)\right]_1^2$$

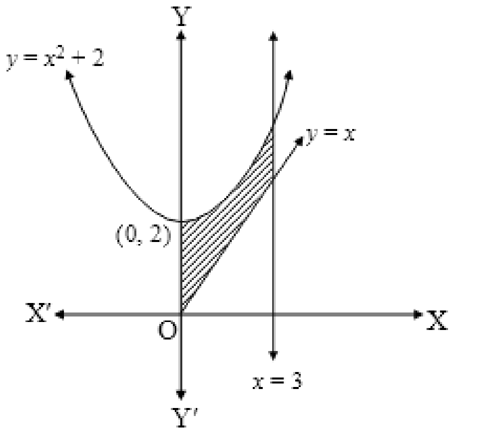
= $\tan^{-1} 2 - \tan^{-1} 1$

$$= \tan^{-1}\left(\frac{2 - 1}{1 + 2(1)}\right)$$

$$\therefore \quad I = \tan^{-1}\left(\frac{1}{3}\right)$$

Q.67 The area of the region bounded by the parabola $y = x^2 + 2$ and the lines $y = x$, x = 0 and x = 3 is

Correct option: (A)



Required area = $\int_0^3 (x^2 + 2 - x)dx$

$= \left[\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_0^3$

= 9 + 6 – $\frac{9}{2}$ – 0

= $\frac{21}{2}$ sq. unit

Q.68 For 0 ≤ x ≤ $\frac{\pi}{2}$, the area bounded by $y = x + \cos x$ and $y = x$, is

Correct option: (A)

The curves $y = x + \cos x$ and $y = x$ intersect at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

KUNAL ACADEMY

$\therefore$ Required area = $\int_0^{\frac{\pi}{2}} (x + cosx)\text{d}x - \int_0^{\frac{\pi}{2}} x\, \text{d}x$

$= \int_0^{\frac{\pi}{2}} cosx\, \text{d}x$

$= [\sin x]_0^{\frac{\pi}{2}}$

$= \sin \frac{\pi}{2} + \sin 0$

$= 1 + 0$

$= 1$

Q.69 If area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$, then the value of m is

Correct option: (A)

The two curves $y^2 = 4ax$ and $y = mx$ intersect at (0, 0) and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

According to the given condition,

$$\int_0^{\frac{4a}{m^2}} (\sqrt{4ax} - mx)\text{d}x = \frac{a^2}{3}$$

$\Rightarrow \frac{8}{3} \cdot \frac{a^2}{m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2$

Q.70 $\int \cos\sqrt{x}\, dx =$, (where C is a constant of integration.)

Correct option: (C)

Put $x = t^2$

$\Rightarrow dx = 2t\, dt$

$\therefore \int \cos\sqrt{x}\, dx = \int \cos t \cdot 2t\, dt$

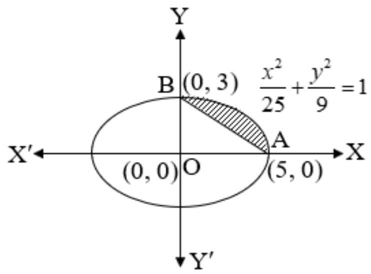
$= 2\int t \cdot \cos t\, dt$

$= 2\,[t \sin t + \cos t] + C$

$= 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C$

Q.71 AOB is the positive quadrant of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in which OA = 5, OB = 3. The area between the arc AB and the chord AB of the ellipse in sq. units is

Correct option: (D)



Total area of ellipse = πab

= π(5)(3)

= 15π sq. units

Area of a quarter ellipse = $\frac{1}{4}(15\pi) = \frac{15\pi}{4}$

Now,

Area of ΔAOB = $\frac{1}{2}$ × OA × OB

$= \frac{1}{2} \times 5 \times 3 = \frac{15}{2}$

$\therefore$ Required area

= Area of a quarter ellipse – Area of ΔAOB

$= \frac{15\pi}{4} - \frac{15}{2}$

$= \frac{15}{4}(\pi - 2)$ sq. units

Q.72 $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{sinx}{sinx+cosx}\, dx$

Correct option: (A)

Let I = $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{sinx}{sinx+\cos x}\, dx$ …(i)

$\therefore$ I = $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{cosx}{cosx+\sin x}\, dx$ …(ii)

$\left[\because \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right]$

Adding (i) and (ii), we get

2I = $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} 1.dx = [x]_{\pi/5}^{3\pi/10} = \frac{3\pi}{10} - \frac{\pi}{5} = \frac{\pi}{10}$

$\Rightarrow$ I = $\frac{\pi}{20}$

Q.73 $\int \frac{\log(\cot x)}{\sin 2x} dx =$

Correct option: (D)

Let I = $\int \frac{\log(\cot x)}{\sin 2x} dx$

Put log(cot$x$) = t

$\Rightarrow \left[\frac{1}{\cot x} \cdot (-\text{cosec}^2 x)\right] dx = \text{dt}$

KUNAL ACADEMY

$$\Rightarrow \left[\frac{-\sin x}{\sin^2 x.\cos x}\right]dx = \text{dt}$$

$$\Rightarrow \frac{dx}{\sin 2x} = \frac{-\text{dt}}{2}$$

$$\therefore \ \text{I} = \int t \cdot \left(-\frac{\text{dt}}{2}\right)$$

$$= \frac{-1}{4}\text{t}^2 + \text{c} = \frac{-1}{4}\ [\log(\cot x)]^2 + \text{c}$$

Q.74 The area of the region bounded by the curve $xy = 2$, X axis and the lines $x = 1$, $x = 4$ is

Correct option: (B)

Required area = $\int_1^4 y dx$

$= \int_1^4 \frac{2}{x} dx$

$= 2[\log x]_1^4$

$= 2(\log 4 - \log 1)$

$= 2 \log 4$ sq.units

Q.75 $\int xe^x\, dx$ is equal to

Correct option: (B) $(x - 1)e^x + c$

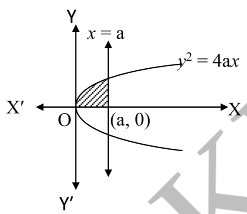
$\int xe^x dx = x\int e^x dx - \int\left[\frac{d}{dx}(x)\int e^x dx\right]dx$

$= x.e^x - \int 1.e^x dx$

$= xe^x - e^x + c = e^x(x-1) + c$

Q.76 The area bounded by the parabola $y^2 = 4ax$ and its latus-rectum $x = a$ is

Correct option: (A)



Required area = $2\int_0^a \sqrt{4ax}\ \text{d}x$

$= 4\sqrt{a}\int_0^a \sqrt{x}\ \text{d}x$

$= 4\sqrt{a}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$

$= 4\sqrt{a} \times \frac{2}{3} \times a^{\frac{3}{2}}$

$= \frac{8}{3}\ \text{a}^2$ sq. units

Q.77 If $\int_0^1 \tan^{-1} x dx =$ p, then

$\int_0^1 \tan^{-1}\left(\frac{1-x}{1+x}\right)dx =$

Correct option: (C)

$\int_0^1 \tan^{-1}\left(\frac{1-x}{1+x}\right)dx =$

$\int_0^1 \tan^{-1} 1 dx - \int_0^1 \tan^{-1} x dx$

$= (\tan^{-1} 1)[x]_0^1 - p = \frac{\pi}{4} - p$

Q.78 $\int (e^{a\log x} + e^{x\log a})dx =$

Correct option: (C)

$\int (e^{a\log x} + e^{x\log a})\,\text{d}x = \int (e^{\log_e x^a} + e^{\log_e a^x})\,\text{d}x$

$= \int (x^a + a^x)\, dx$

$= \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + \text{c}$

Q.79 $\int_0^{\frac{\pi}{2}} \log(\cot x)dx =$

Correct option: (B)

Let I = $\int_0^{\frac{\pi}{2}} \log(\cot x)dx$ …(i)

$= \int_0^{\frac{\pi}{2}} \log(\tan x)dx$ …(ii)

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

Adding (i) and (ii), we get

2I = $\int_0^{\frac{\pi}{2}} \log(\cot x\, tan\, x)dx$

∴ 2I = $\int_0^{\frac{\pi}{2}} \log 1\, dx$

⇒ 2I = 0 ⇒ I = 0

KUNAL ACADEMY

Q.80 $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$=

Correct option: (A)

Let I = $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Put x = tan t ⇒ dx = sec^2 t dt

When x = 0, t = 0 and when x = 1, t = $\frac{\pi}{4}$

∴ I = $8\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{1+\tan^2 t}\cdot \sec^2 t\,dt$ = 8

$\int_0^{\frac{\pi}{4}} \log(1+\tan\,t)\,dt$

= $8\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-t\right)\right]dt \ldots$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

= $8\int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan\,t}{1+\tan t}\right)dt$ = 8

$\int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right)dt$

= $8\int_0^{\frac{\pi}{4}} [\log 2 - \log(1+\tan t)]dt$

∴ I = $8\int_0^{\frac{\pi}{4}} (\log 2)dt\; - I$

∴ 2I = 8log $2[t]_0^{\pi/4}$

∴ I = π log 2

Q.81 Let f(x) be a non-negative continuous function such that the area bounded by the curve y = f(x), X-axis and the ordinates x = $\frac{\pi}{4}$ and x = β > $\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then f $\left(\frac{\pi}{2}\right)$ is [MP PET 2007]

Correct option: (A)

According to the given condition,

$$\int_{\frac{\pi}{4}}^{\beta} f(x)\,dx = \beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$$

Differentiating w.r.t. β, we get

f(β) = sin β + β cos β – $\frac{\pi}{4}\sin\beta+\sqrt{2}$

∴ $f\left(\frac{\pi}{2}\right)$= $\sin\frac{\pi}{2}+\frac{\pi}{2}\cos\frac{\pi}{2}-\frac{\pi}{4}\sin\frac{\pi}{2}+\sqrt{2}$ = 1 – $\frac{\pi}{4}$ + $\sqrt{2}$

Q.82 $\int \sin^{-1}x$ **dx is equal to**

Correct option: (A)

Put sin^{-1}x = t ⇒ x = sin t ⇒ dx = cos t dt

∴ $\int \sin^{-1}x\,dx = \int t\cos t\,dt$

= t sin t – $\int 1.\sin t$ dt

= t sin t + cos t + c

= t sin t + $\sqrt{1-\sin^2 t}+c$

=$x\sin^{-1}x+\sqrt{1-x^2}+c$

Q.83 $\int_3^7 \sqrt{(x-3)(7-x)}\,dx =$

Correct option: (C)

Since $\int_a^b \sqrt{(x-a)(b-x)}\,dx = \frac{\pi}{8}(b-a)^2$

∴ $\int_3^7 \sqrt{(x-3)(7-x)}dx = \frac{\pi}{8}(7-3)^2$

$= \frac{\pi}{8}\times 16 = 2\pi$

Q.84 $\int \frac{1}{x^6+x^4}dx$ **is equal to [BCECE 2014]**

Correct option: (D)

$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}x+c$

$\int\frac{dx}{x^6+x^4} = \int\frac{(x^2+1)\,dx}{x^4\,(x^2+1)} - \int\frac{x^2\,dx}{x^4\,(x^2+1)}$

$= \int\frac{1}{x^4}\,dx - \int\frac{dx}{x^2(x^2+1)}$

$= -\frac{1}{3x^3} - \int\frac{dx}{x^2} + \int\frac{dx}{x^2+1}$

$= \frac{-1}{3x^3}+\frac{1}{x}+\tan^{-1}x+c$

Q.85 $\int_0^{\pi}\frac{x\cos x\sin x}{\cos^3 x+\cos x}\,dx =$

Correct option: (D)

Let I = $\int_0^{\pi}\frac{x\cos x\sin x}{\cos^3 x+\cos x}\,dx$

= $\int_0^{\pi}\frac{x\sin x}{\cos^2 x+1}\,dx$ …(i)

KUNAL ACADEMY

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{\cos^2 x + 1}\, dx \quad \ldots$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$\therefore \quad I = \int_0^{\pi} \frac{\pi \sin x}{\cos^2 x + 1} dx - I \qquad \ldots$[From (i)]

$\Rightarrow 2I = \int_0^{\pi} \frac{\pi \sin x}{\cos^2 x + 1}\, dx$

Put cos x = t ⇒ –sin x dx = dt

When x = 0, t = 1 and when x = π, t = –1

$\therefore \quad 2I = -\int_1^{-1} \frac{\pi}{1 + t^2}\, dt$

$= \pi \int_{-1}^{1} \frac{dt}{1 + t^2} \quad \ldots$

$$\left[\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right]$$

$= \pi\left[\tan^{-1} t\right]_{-1}^{1}$

$= \pi\left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$

$= \pi\left(\frac{\pi}{4} + \frac{\pi}{4}\right)$

$\therefore \quad 2I = \frac{\pi^2}{2} \Rightarrow I = \frac{\pi^2}{4}$

Q.86 $\int \frac{\sin x \cos x}{3\sin^2 x + 5\cos^2 x}$ **dx =**

Correct option: (B)

Put $3\sin^2 x + 5\cos^2 x = t$

⇒ (3 × 2 sin x cos x – 5 × 2 sin x cos x) dx = dt

⇒ –4 sin x cos x dx = dt

⇒ sin x cos x dx = $\frac{dt}{-4}$

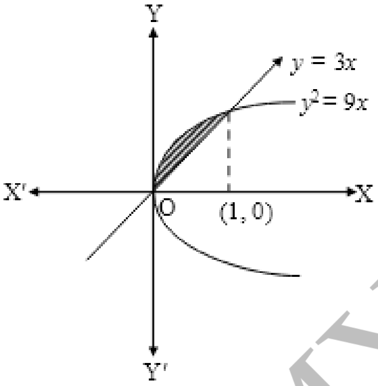
$\therefore \int \frac{\sin x \cos x}{3\sin^2 x + 5\cos^2 x} dx = \int \frac{dt}{(-4)t}$

$= -\frac{1}{4}\int \frac{1}{t} dt = -\frac{1}{4} \log |t| + c$

$= -\frac{1}{4} \log |3\sin^2 x + 5\cos^2 x| + c$

Q.87 The area of the region bounded by the curves $y^2 = 9x$ and $y = 3x$ is

Correct option: (B)



Required area = $\int_0^1 \left(\sqrt{9x} - 3x\right) dx$

$= 3\int_0^1 x^{1/2}\, dx - 3\int_0^1 x\, dx$

$= 3\left[\frac{2}{3} x^{3/2}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$

$= \frac{1}{2}$ sq. unit

Q.88 The value of the definite integral $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$, for 0 < α < π, is equal to

Correct option: (D)

$\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1} =$

$\int_0^1 \frac{dx}{(x + \cos\alpha)^2 + 1 - \cos^2\alpha}$

$= \int_0^1 \frac{dx}{(x + \cos\alpha)^2 + \sin^2\alpha}$

$= \left[\frac{1}{\sin\alpha} \tan^{-1}\left(\frac{x + \cos\alpha}{\sin\alpha}\right)\right]_0^1$

$= \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\cot\frac{\alpha}{2}\right) - \tan^{-1}(\cot\alpha)\right]$

$= \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)\right) - \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \alpha\right)\right)\right]$

KUNAL ACADEMY

$= \frac{\alpha}{2}(\sin \alpha)^{-1}$

Q.89 The area bounded by the parabola $x^2 = 2y$ and the line $y = 3x$ is

Correct option: (B)

The area bounded by $x^2 = 4ay$ and the line $y = mx$ is $\frac{8a^2m^3}{3}$.

Given, $x^2 = 2y \Rightarrow x^2 = 4\left(\frac{1}{2}\right)y$ and $y = 3x$

Here, $a = \frac{1}{2}$ and $m = 3$

$\therefore$ Required area $= \frac{8}{3} \times \frac{1}{4} \times 3 \times 3 \times 3 = 18$ sq. units

Q.90 The area of the triangle formed by the lines joining vertex of the parabola $x^2 = 12y$ to the extremities of its latus rectum is

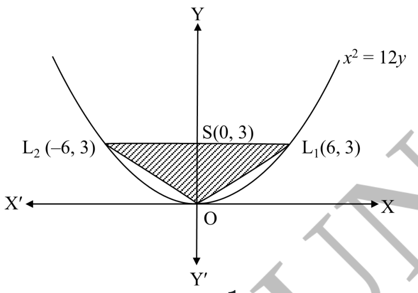
Correct option: (D)

$x^2 = 12y$

$\Rightarrow 4b = 12 \Rightarrow b = 3$

$\Rightarrow$ co-ordinates of latus rectum are $(\pm 2b, b) \equiv (\pm 6, 3)$

$L_1 \equiv (6, 3)$, $L_2 \equiv (-6, 3)$ and OS = 3



$A(\Delta OL_1L_2) = \frac{1}{2} \times (L_1L_2) \times (OS)$

$= \frac{1}{2} \times 12 \times 3$

= 18 sq. units

Q.91 $\int \frac{dx}{32 - 2x^2}$ = A log(4 – x) + B log(4 + x) + c, then the values of A and B are respectively (where c is a constant of integration)

Correct option: (C)

$$\int \frac{1}{32 - 2x^2}dx = \frac{1}{2}\int \frac{1}{(4)^2 - x^2}dx$$

$$= \frac{1}{2} \cdot \frac{1}{2(4)}\log\left|\frac{4+x}{4-x}\right| + c$$

$$= \frac{1}{16}\log(4 + x) - \frac{1}{16}\log|4 - x| + c$$

Comparing with A log (4 – x) + B log(4 + x) + c, we get

$A = \frac{-1}{16}$, $B = \frac{1}{16}$

Q.92 $\int e^{x\log a} \cdot e^x\, dx$ is equal to

Correct option: (B)

$$\int e^{x\log a} \cdot e^x\, dx = \int e^{\log a^x} \cdot e^x\, dx = \int a^x e^x\, dx$$

$$= \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + c$$

Q.93 $\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx =$

Correct option: (C)

Let $I = \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

$$= 2\int_0^1 \tan^{-1} x\, dx$$

$$= 2\left[x\tan^{-1}x\right]_0^1 - 2\int_0^1 \frac{1}{1+x^2} \cdot x\, dx$$

$$= 2\tan^{-1}(1) - \int_0^1 \frac{2x}{1+x^2}dx$$

$$= \frac{\pi}{2} - \left[\log\left|1+x^2\right|\right]_0^1 = \frac{\pi}{2} - \log 2$$

Q.94 Area of the region bounded by $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ is

Correct option: (B)

Here, a = 3, b = 2

$\therefore$ Required area $= \frac{1}{4}\pi(3)(2) - \frac{1}{2}(3)(2)$

KUNAL ACADEMY

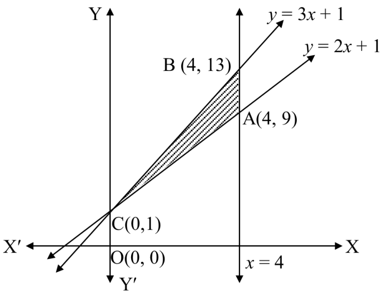
$= \frac{3}{2}(\pi - 2)$ sq.units

Shortcut

The area between $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$ is $\left(\frac{1}{4}\pi ab-\frac{1}{2}ab\right)$ sq.units.

Q.95 The area of the region bounded by the lines $y = 2x + 1$, $y = 3x + 1$ and $x = 4$ is

Correct option: (D)



Required area $= \int_0^4 (3x+1-2x-1)dx$

$= \int_0^4 x dx$

$= \left[\frac{x^2}{2}\right]_0^4$

= 8 sq. units

Q.96 If area bounded by $y^2 = 16x$ and $y = mx$ is $\frac{16}{3}$ sq. units, then the value of m is

Correct option: (B)

The parabola $y^2 = 16x$ and line $y = mx$ intersect at (0, 0) and $\left(\frac{16}{m^2}, \frac{16}{m}\right)$

According to the given condition,

$\int_0^{\frac{16}{m^2}} \left(4\sqrt{x} - mx\right) = \frac{16}{3}$

$\Rightarrow \frac{128}{3m^3} = \frac{16}{3}$

$\Rightarrow m^3 = 8$

$\Rightarrow m = 2$

Q.97 $\int_0^{\frac{\pi}{2}} \sin^2 x . \cos^3 x \, dx =$

Correct option: (B)

Let I $= \int_0^{\frac{\pi}{2}} \sin^2 x . \cos^3 x \, dx$

$= \int_0^{\frac{\pi}{2}} \sin^2 x \left(1 - \sin^2 x\right) \cos x \, dx$

Put $\sin x = t \Rightarrow \cos x \, dx = dt$

$\therefore I = \int_0^1 t^2\left(1 - t^2\right) dt$

$= \int_0^1 \left(t^2 - t^4\right) dt = \left[\frac{t^3}{3}\right]_0^1 - \left[\frac{t^5}{5}\right]_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

Q.98 $\int_0^{\pi/4} \tan^6 x \sec^2 x dx =$

Correct option: (A)

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

When $x = 0$, t = 0 and when $x = \frac{\pi}{4}$, t = 1

$\therefore \int_0^{\pi/4} \tan^6 x \sec^2 x dx = \int_0^1 t^6 dt = \frac{1}{7}\left[t^7\right]_0^1 = \frac{1}{7}$

Q.99 The area bounded by the X-axis and the curve $y = x(x - 2)(x + 1)$ is

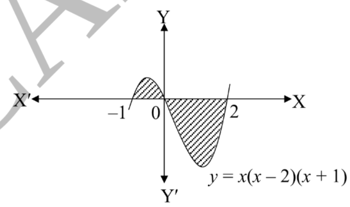
Correct option: (A)

For X-axis,

$y = 0$

$\therefore x(x - 2)(x + 1) = 0$

$\Rightarrow x = 0$ or $x = 2$ or $x = -1$



Required area $= \int_{-1}^0 y \, dx + \left|\int_0^2 y \, dx\right|$

$= \int_{-1}^0 x(x-2)(x+1)dx + \left|\int_0^2 x(x-2)(x+1)dx\right|$

$= \int_{-1}^0 \left(x^3 - x^2 - 2x\right)dx + \left|\int_0^2 \left(x^3 - x^2 - 2x\right)dx\right|$

$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^0 + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_0^2\right|$

$= \frac{5}{12} + \left|-\frac{8}{3}\right|$

$= \frac{37}{12}$ sq. units

Q.100 $\int \frac{\cot x}{\log \sin x} dx =$

KUNAL ACADEMY

Correct option: (A)

Put log sin x = t

$\Rightarrow$ cot x dx = dt

$$\therefore \quad \int \frac{\cot\ x}{\log\ \ \sin\ x} dx = \int \frac{dt}{t} = \log t + c$$

$$= \log(\log \sin x) + c$$

Q.101 $\displaystyle\int_{-4}^{4} \log\left(\frac{9-x}{9+x}\right) dx$ **equals**

Correct option: (D)

Let $f(x) = \log\left(\frac{9-x}{9+x}\right)$

$$\therefore f(-x) = \log\left(\frac{9-x}{9+x}\right)^{-1}$$

$$= -\log\left(\frac{9-x}{9+x}\right) = -f(x)$$

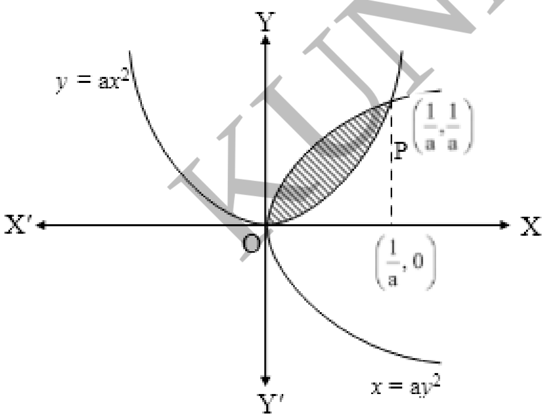
$\therefore$ f(x) is an odd function.

$$\therefore \int_{-4}^{4} \log\left(\frac{9-x}{9+x}\right) dx = 0$$

Q.102 If the area bounded by $y = ax^2$ and $x = ay^2$, a > 0, is 1, then a =

Correct option: (B)

The two curves intersect at O(0, 0) and P $\left(\frac{1}{a}, \frac{1}{a}\right)$.



According to the given condition,

$$\int_0^{\frac{1}{a}} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx = 1$$

$$\Rightarrow \left[\frac{2}{3\sqrt{a}} x^{3/2} - \frac{ax^3}{3}\right]_0^{1/a} = 1$$

$$\Rightarrow \frac{2}{3\sqrt{a}} \times \frac{1}{a^{3/2}} - \frac{a}{3} \times \frac{1}{a^3} = 1$$

$$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1 \Rightarrow \frac{1}{3a^2} = 1$$

$$\Rightarrow a = \frac{1}{\sqrt{3}} \ \ldots[\because a > 0]$$

Q.103 $\displaystyle\int_0^{\frac{\pi}{2}} \left(\frac{\sqrt[n]{\sec x}}{\sqrt[n]{\sec x} + \sqrt[n]{\operatorname{cosec} x}}\right) dx =$

Correct option: (C)

$$\text{Let I} = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt[n]{\sec x}}{\sqrt[n]{\sec x} + \sqrt[n]{\operatorname{cosec} x}}\right) dx \ \ldots\text{(i)}$$

$$\therefore \quad I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt[n]{cosecx}}{\sqrt[n]{cosecx} + \sqrt[n]{secx}}\right) dx \ \ \ldots\text{(ii)}$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

Q.104 $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(5+\sin x)(4+\sin x)}$ **dx =**

Correct option: (C)

Let I = $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(5+\sin x)(4+\sin x)}$ dx

Put sin x = t $\Rightarrow$ cos x dx = dt

$\therefore$ I = $\int_0^1 \frac{dt}{(5+t)(4+t)}$

$= -\int_0^1 \frac{1}{5+t}$dt + $\int_0^1 \frac{1}{4+t}$dt

$= -[\log|5+t|]_0^1 + [\log|4+t|]_0^1$

= – (log 6 – log 5) + (log 5 – log 4)

= – log $\frac{6}{5}$ + log $\frac{5}{4}$

$\therefore$ I = log $\left(\frac{25}{24}\right)$

Q.105 $\int$ sin **(log$x$) dx =**

Correct option: (C)

Let I = $\int$ sin (log x) dx

Put log x = t $\Rightarrow$ $x = e^t$ $\Rightarrow$ dx = e^t dt

KUNAL ACADEMY

$\therefore I = \int \sin t . e^t dt = \sin t . e^t - \int \cos t.\ e^t\ dt$

$= \sin t.e^t - \left[\cos t.\ e^t + \int \sin t.\ e^t\ dt\right]$

$\therefore I = \sin t . e^t - \cos t . e^t - I + c_1$

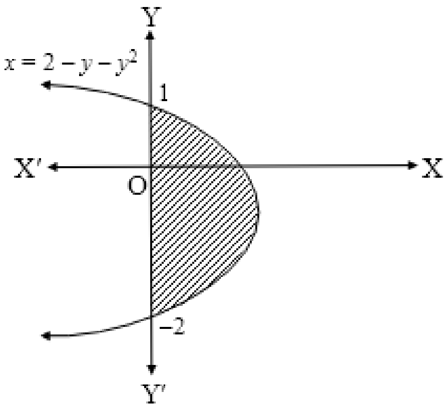
$\Rightarrow 2I = \sin t.e^t - \cos t.e^t + c_1$

$\Rightarrow I = \frac{1}{2}\ x\ [\sin(\log x) - \cos(\log x)] + c,$

where $c = \frac{c_1}{2}$

Q.106 The area bounded by the curve $x = 2 - y - y^2$ and the Y-axis is

Correct option: (C)



Putting $x = 0$ in the given equation, we get

$y = 1$ or $y = -2$

$\therefore$ Required Area $= \int_{-2}^{1} x\, dy = \int_{-2}^{1} (2 - y - y^2) dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= 2 - \frac{1}{2} - \frac{1}{3} - \left[-4 - 2 + \frac{8}{3}\right]$

$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$

$= 8 - 3 - \frac{1}{2}$

$= \frac{9}{2}$ sq.units

Q.107 $\int_{-2}^{1} [x+1] dx =$ **, (Where [x] is greatest integer function not greater than x)**

Correct option: (B)

Let $I = \int_{-2}^{1} [x+1] dx$

$= \int_{-2}^{1} ([x]+1) dx$

$= \int_{-2}^{1} [x] dx + \int_{-2}^{1} 1\ dx$

$=$

$\int_{-2}^{-1} [x] dx + \int_{-1}^{0} [x]\ dx + \int_{0}^{1} [x]\ dx + \int_{-2}^{1} 1\ dx$

$=$

$\int_{-2}^{-1} (-2) dx + \int_{-1}^{0} (-1)\ dx + \int_{0}^{1} (0)\ dx + \int_{-2}^{1} dx$

$= -2 \int_{-2}^{-1} dx - 1 \int_{-1}^{0} dx + \int_{-2}^{1} dx$

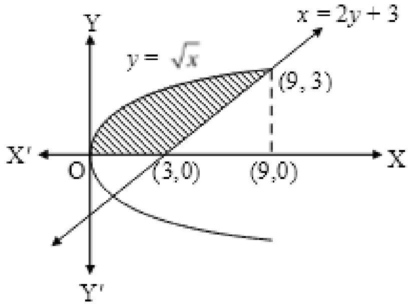
$= -2(-1 + 2) - 1(0 + 1) + (1 + 2)$

$= -2 - 1 + 3$

$\therefore\ I = 0$

Q.108 The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and X-axis in the 1st quadrant is

Correct option: (A)



Required area $= \int_{0}^{9} \sqrt{x}\, dx - \int_{3}^{9} \left(\frac{x-3}{2}\right) dx$

$= \left[\frac{2x^{3/2}}{3}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9$

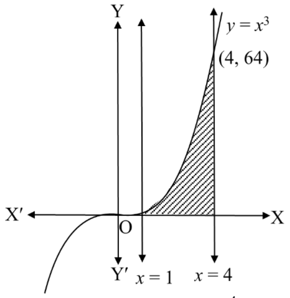
$= \frac{2}{3}(27 - 0) - \frac{1}{2}(36 - 18)$

$= 9$ sq.units

Q.109 The area bounded by the curve $y = x^3$, the X -axis and the lines $x = 1$ and $x = 4$ is

KUNAL ACADEMY

Correct option: (B)



Required area = $\int_1^4 x^3\, dx$

$= \left[\frac{x^4}{4}\right]_1^4 = 64 - \frac{1}{4} = \frac{255}{4}$ sq. units

Q.110 **If $I_n = \int_0^{\pi/4} tan^n\theta\, d\theta$, then $I_8 + I_6$ equals**

Correct option: (D) $\frac{1}{7}$

$I_8 + I_6 = \int_0^{\frac{\pi}{4}} (\tan^8\theta + \tan^6\theta)\, d\theta$

$= \int_0^{\frac{\pi}{4}} \tan^6\theta \sec^2\theta\, d\theta$

Put tan θ = t ⇒ $\sec^2\theta\, d\theta = dt$

When θ = 0, t = 0 and when θ = $\frac{\pi}{4}$, t = 1

$\therefore I_8 + I_6 = \int_0^1 t^6\, dt = \left[\frac{t^7}{7}\right]_0^1 = \frac{1}{7}$

Q.111 $\int_{\frac{1}{2}}^{2} \frac{1}{x}\text{cosec}^{101}\left(x - \frac{1}{x}\right)dx =$

Correct option: (A)

Let $I = \int_{\frac{1}{2}}^{2} \frac{1}{x}\cos ec^{101}\left(x - \frac{1}{x}\right)dx$ …(i)

Put $x = \frac{1}{t} \Rightarrow dx = \frac{-1}{t^2}dt$

$\therefore\ I = \int_{2}^{\frac{1}{2}} t \cos ec^{101}\left(\frac{1}{t} - t\right)\left(\frac{-1}{t^2}\right)dt$

$= \int_{2}^{\frac{1}{2}} \frac{-1}{t}\left[-\cos ec^{101}\left(t - \frac{1}{t}\right)\right]dt$

… [∵ cosec (– θ) = – cosec θ]

$= \int_{2}^{\frac{1}{2}} \frac{1}{t}\cos ec^{101}\left(t - \frac{1}{t}\right)dt$

$= -\int_{\frac{1}{2}}^{2} \frac{1}{t}\cos ec^{101}\left(t - \frac{1}{t}\right)dt$ …

$\left[\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right]$

$\therefore\ I = -\int_{\frac{1}{2}}^{2} \frac{1}{x}\cos ec^{101}\left(x - \frac{1}{x}\right)dx$

…(ii) $\left[\because \int_a^b f(x)dx = \int_a^b f(t)dt\right]$

Adding (i) and (ii), we get

2 I = 0

⇒ I = 0

Q.112 $\int x^n$**log x dx =**

Correct option: (D)

$\int x^n \log x\, dx = \log x.\ \frac{x^{n+1}}{n+1} - \int \frac{1}{x} \cdot \frac{x^{n+1}}{n+1}\, dx$

$= \frac{x^{n+1}}{n+1}\log x - \frac{x^{n+1}}{(n+1)^2} + c$

$= \frac{x^{n+1}}{n+1}\left(\log x - \frac{1}{n+1}\right) + c$

Q.113 **The area bounded by the curve $x = 2 - y - y^2$ and the Y-axis is**

Correct option: (C)

Given equation of curve

$x = 2 - y - y^2$ …(i)

Y-axis ⇒ $x = 0$

Put $x = 0$, in equation (i), we get

$0 = 2 - y - y^2$

⇒ $y^2 + y - 2 = 0$

⇒ $(y + 2)(y - 1) = 0$

⇒ $y + 2 = 0$ or $y = 1$

⇒ $y = -2$ or $y = 1$

Required area = $\int_{y=a}^{b} f(y)dy$

$= \int_{-2}^{1} (2 - y - y^2)dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{4}{2} + \frac{8}{3}\right)$

$= \left(\frac{12 - 3 - 2}{6}\right) - \left(\frac{-24 - 12 + 16}{6}\right)$

KUNAL ACADEMY

$= \frac{7}{6} - \frac{(-20)}{6}$

$= \frac{27}{6} = \frac{9}{2}$ sq. units.

Q.114 Area bounded by the curve $y = 3x^2e^{x^3}$, X-axis and the ordinates $x = 0$, $x = 2$ is

Correct option: (D)

Required area = $\int_0^2 y\,dx = \int_0^2 3x^2 e^{x^3}\,dx$

Put $x^3 = t \Rightarrow 3x^2dx = dt$

$\therefore x = 0 \Rightarrow t = 0$ and $x = 2 \Rightarrow t = 8$

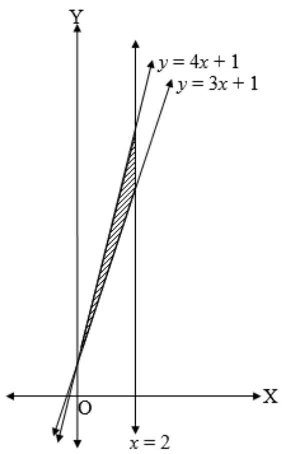
$\therefore$ Required area = $\int_0^8 e^t dt$

$= \left[e^t\right]_0^8$

$= \left(e^8 - 1\right)$ sq. units

Q.115 The area of the region bounded by curves $y = 3x+1, y = 4x+1$ and $x = 2$ is

Correct option: (B)



Required area = $\int_0^2 [4x + 1 - (3x+1)]dx$

$= \int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2$ = 2 sq. units

Q.116 $\int \frac{(5\sin\theta - 2)\cos\theta}{(5 - \cos^2\theta - 4\sin\theta)} d\theta =$

Correct option: (B)

Let I = $\int \frac{(5\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$

$= \int \frac{(5\sin\theta - 2)\cos\theta}{5 - \left(1 - \sin^2\theta\right) - 4\sin\theta} d\theta$

$= \int \frac{(5\sin\theta - 2)\cos\theta}{4 + \sin^2\theta - 4\sin\theta} d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$\therefore$ I = $\int \frac{5t - 2}{t^2 - 4t + 4} dt$

$= \int \frac{5t-2}{(t-2)^2} dt$

Let $\frac{5t-2}{(t-2)^2} = \frac{A}{t-2} + \frac{B}{(t-2)^2}$

On solving we get, A = 5 and B = 8

$\therefore$ I = $\int \frac{5}{t-2} dt + \int \frac{8}{(t-2)^2} dt$

$= 5 \log|t-2| - \frac{8}{t-2} + c$

$= 5 \log(\sin\theta - 2) - \frac{8}{\sin\theta - 2} + c$

Q.117 $\int e^x \left[\log\left(1 + \tan^2 x\right) + 2\tan x\right] dx =$

Correct option: (D)

Let I = $\int e^x \left[\log\left(1 + \tan^2 x\right) + 2\tan x\right] dx$

$= \int e^x \left[\log\left(\sec^2 x\right) + 2\tan x\right] dx$

$= \int e^x [2 log sec x + 2\tan x] dx$

$= 2\int e^x [log sec x + \tan x] dx$ ...

$\left[\frac{d}{dx}[\log(sec x)] = \tan x\right]$

$= 2e^x \log[\sec x] + c$

Q.118 The area of the region bounded by the parabola $x^2 = y$ and the line $y = x$ is

Correct option: (B)

Given curves are $y = x^2$ and $y = x$.

On solving, we get $x = 0$, $x = 1$

$\therefore$ Required area = $\int_0^1 (x - x^2)dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

KUNAL ACADEMY

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. units

Alternate Method:

The area bounded by $x^2 = 4ay$ and the line $y = mx$ is $\frac{8a^2m^3}{3}$ sq. units.

Here, $a = \frac{1}{4}$ and $m = 1$

$\therefore$ Required area $= \frac{8}{3} \times \frac{1}{16} \times 1$

$= \frac{1}{6}$ sq. units

Q.119 $\int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx =$

Correct option: (D)

Let $I = \int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx =$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right) dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

Put $\sin x - \cos x = t$

$\Rightarrow (\sin x + \cos x)\, dx = dt$

$\therefore \quad I = \sqrt{2} \int_{-1}^{0} \frac{dt}{\sqrt{1 - t^2}}$

$= \sqrt{2} \left[\sin^{-1} t\right]_{-1}^{0}$

$= \sqrt{2}\, [\sin^{-1}(0) - \sin^{-1}(-1)]$

$= \sqrt{2} \cdot \frac{\pi}{2} = \frac{\pi}{\sqrt{2}}$

Q.120 Let f be a differentiable function such that f(1) = 2 and $f'(x) = f(x)$, for all $x \in$ R. If $h(x) = f(f(x))$, then $h'(1)$ is equal to

Correct option: (B)

Given: $f'(x) = f(x)$ for all $x \in R$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating on both sides, we get

$\log|f(x)| = x + c$

$\Rightarrow f(x) = e^{x + c}$

$\Rightarrow f(x) = e^x \cdot e^c$

$\Rightarrow f(x) = e^x \cdot c_1 \quad \ldots(i) [e^c = c_1]$

As f(1) = 2

$\therefore \quad c_1 \cdot e = 2$

$\Rightarrow c_1 = \frac{2}{e}$

Equation (i) becomes

$f(x) = e^x \cdot \frac{2}{e}$

Now, $h(x) = f(f(x))$

$\therefore \quad h'(x) = f'(f(x)) \times f'(x)$

$\therefore \quad h'(1) = f'(f(1)) \times f'(1)$

$\Rightarrow h'(1) = f'(2) \times f'(1)$

$\Rightarrow h'(1) = e^2 \times \frac{2}{e} \times 2$

$\Rightarrow h'(1) = 4e$

KUNAL ACADEMY