Probability and Binomial **Marks: 180**

ANSWER KEY

Maths

Q.1 D	Q.2 D	Q.3 A	Q.4 D	Q.5 B	Q.6 A	Q.7 C	Q.8 C
Q.9 B	Q.10 D	Q.11 A	Q.12 A	Q.13 C	Q.14 A	Q.15 C	Q.16 A
Q.17 D	Q.18 B	Q.19 B	Q.20 B	Q.21 A	Q.22 B	Q.23 C	Q.24 B
Q.25 D	Q.26 C	Q.27 A	Q.28 B	Q.29 C	Q.30 B	Q.31 A	Q.32 A
Q.33 B	Q.34 D	Q.35 D	Q.36 C	Q.37 C	Q.38 C	Q.39 B	Q.40 B
Q.41 A	Q.42 D	Q.43 B	Q.44 B	Q.45 B	Q.46 B	Q.47 C	Q.48 B
Q.49 C	Q.50 D	Q.51 A	Q.52 A	Q.53 D	Q.54 A	Q.55 B	Q.56 B
Q.57 A	Q.58 B	Q.59 C	Q.60 C	Q.61 B	Q.62 C	Q.63 D	Q.64 A
Q.65 C	Q.66 B	Q.67 A	Q.68 A	Q.69 A	Q.70 A	Q.71 B	Q.72 B
Q.73 B	Q.74 A	Q.75 A	Q.76 A	Q.77 B	Q.78 D	Q.79 C	Q.80 D
Q.81 B	Q.82 D	Q.83 A	Q.84 B	Q.85 B	Q.86 A	Q.87 C	Q.88 B
Q.89 D	Q.90 A						

Maths

Q.1 A and B are independent events with P(A) = $\frac{1}{4}$ and P(A ∪ B) = 2P(B) – P(A), then P(B) is

Correct option: (D)

P(A ∪ B) = 2P(B) – P(A)

∴ P(A) + P(B) – P(A ∩ B) = 2P(B) – P(A)

∴ P(A) + P(B) – P(A) · P(B) = 2P(B) – P(A) …

[∵A and B are independent events]

∴ P(B) + P(A)·P(B) = 2P(A)

∴ P(B) = $\frac{2P(A)}{(1+P(A))}$ = $\frac{2\times\frac{1}{4}}{\left(1+\frac{1}{4}\right)}$ = $\frac{2}{5}$

Q.2 The probability that a person wins a prize on a lottery ticket is $\frac{1}{4}$. If he purchases 5 lottery tickets at random, then the probability that he wins at least one prize is

Correct option: (D)

Given, p = $\frac{1}{4}$

∴ q = 1 – p = $\frac{3}{4}$

Also, n = 5

∴ P(person wins atleast one prize)

= 1 – P(person wins no prize)

= 1 – P(X = 0)

= 1 – $^5C_0p^0q^5$

= 1 – $^5C_0\left(\frac{1}{4}\right)^0\cdot\left(\frac{3}{4}\right)^5$

= 1 – $\frac{243}{1024}$

= $\frac{781}{1024}$

Q.3 The probability distribution of a discrete r.v. X is

X = x	0	1	2	3	4
P(X = x)	k	2k	4k	2k	k

Then value of P(X ≤ 2) is

Correct option: (A)

Since $\sum_{x=0}^{4} P(X = x)$= 1,

k + 2k + 4k + 2k + k = 1

⇒ 10k = 1

⇒ k = $\frac{1}{10}$

Now, P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2)

= $\frac{1}{10}+\frac{2}{10}+\frac{4}{10}$

= $\frac{7}{10}$

Q.4 The odds against J solving a problem are 2 : 3 and the odds in favour of K solving the problem are 4 : 3. If both try to solve the problem independently, then the probability that at least one of them will solve a problem is _______.

Correct option: (D)

From the data,

P(J) = $\frac{3}{2+3}=\frac{3}{5}$, P(J′) = $\frac{2}{3+3}=\frac{2}{5}$

P(K) = $\frac{4}{4+3}=\frac{4}{7}$, P(K′) = $\frac{3}{4+3}=\frac{3}{7}$

Required probability

= P(At least one solve a problem)

= 1 – P (None of them solve a problem)

= 1 – P(J′ ∩ K′) = 1 – P(J′) · P(K′)

= 1 – $\left[\frac{2}{5}\times\frac{3}{7}\right]=\frac{29}{35}$.

Q.5 If X ~ B (4, p) and 2 P (X = 3) = 3 P (X = 2), then value of p is

Correct option: (B)

2 P(X = 3) = 3 P(X = 2)

⇒ 2. $^4C_3p^3q$ = 3. $^4C_2p^2q^2$

⇒ 2(4p) = 3(6q)

⇒ 4p = 9q

⇒ 4p = 9(1 – p)

$\Rightarrow 13p = 9$

$\Rightarrow p = \frac{9}{13}$

Q.6 If 6P(X) = 8P(Y) = 14P(X ∩ Y) = 1, then the P$\left(\frac{X'}{Y}\right)$ = …….

Correct option: (A)

$$P\left(\frac{\overline{X}}{Y}\right) = \frac{P\left(\overline{X} \cap Y\right)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)} = \frac{\frac{1}{8} - \frac{1}{14}}{\frac{1}{8}} = \frac{3}{7}$$

Q.7 The p.d.f. of a random variable X is given by

$f(x) = \frac{K}{\sqrt{x}} \quad , \quad if\ 0 \le x \le 4$

$\quad = 0 \quad , \quad \text{otherwise}$

Then P(1 < X < 4) =

Correct option: (C)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_0^4 \frac{K}{\sqrt{x}}\, dx = 1$$

$$\Rightarrow K \left[2\sqrt{x}\right]_0^4 = 1$$

$$\Rightarrow 4K = 1$$

$$\Rightarrow K = \frac{1}{4}$$

$$\therefore P(1 < X < 4) = \int_1^4 f(x)\ d\ x$$

$$= 2K\left[\sqrt{x}\right]_1^4 = 2 \times \frac{1}{4}(2-1) = \frac{1}{2}$$

Q.8 If the error involved in making a certain measurement is continuous random variable X with probability density function

f(x) = k(4 – x^2) if –2 ≤ x ≤ 2

= 0, otherwise

Then, P[–1 < x < 1] =

Correct option: (C)

Since f(x) is the p.d.f of X.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_{-2}^{2} k(4 - x^2)\, dx = 1$$

$$\Rightarrow k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\Rightarrow k\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\Rightarrow k\left(\frac{32}{3}\right) = 1$$

$$\Rightarrow k = \frac{3}{32}$$

$$\therefore P[-1 < x < 1] = \int_{-1}^{1} k\left(4 - x^2\right)\, dx$$

$$= 2\int_0^1 k\left(4 - x^2\right) dx$$

$$= \frac{6}{32}\left[4x - \frac{x^3}{3}\right]_0^1 = \frac{11}{16}$$

Q.9 A die is thrown four times. The probability of getting perfect square in at least one throw is

Correct option: (B)

Here, p = probability of getting perfect square in any throw = P(1 or 4) = $\frac{2}{6} = \frac{1}{3}$

$$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$$

Also, n = 4

P(getting perfect square in at least one throw)

= 1 – P(not getting perfect square in any throw)

$\Rightarrow P(X \ge 1) = 1 - P(X = 0)$

$$= 1 - {}^4C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4$$

$$= 1 - \left(\frac{2}{3}\right)^4 = \frac{65}{81}$$

Q.10 The p.m.f. of a random variable X is given by

$$P[X=x]=\frac{\binom{5}{x}}{2^5}$$, **if x = 0, 1, 2, 3, 4, 5**

= 0, otherwise

Then which of the following is not correct?

Correct option: (D)

P(X ≤ 1) = P(X = 0) + P(X = 1)

$$= \frac{^5C_0}{2^5} + \frac{^5C_1}{2^5} = \frac{6}{2^5}$$

P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2)

$$= \frac{^5C_0}{2^5} + \frac{^5C_1}{2^5} + \frac{^5C_2}{2^5} = \frac{16}{2^5}$$

P(X ≥ 3) = P(X = 3) + P(X = 4) + P(X = 5)

$$= \frac{^5C_3}{2^5} + \frac{^5C_4}{2^5} + \frac{^5C_5}{2^5} = \frac{16}{2^5}$$

P(X ≤ 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

$$= \frac{^5C_0}{2^5} + \frac{^5C_1}{2^5} + \frac{^5C_2}{2^5} + \frac{^5C_3}{2^5} = \frac{26}{2^5}$$

P(X ≥ 4) = P(X = 4) + P(X = 5)

$$= \frac{^5C_4}{2^5} + \frac{^5C_5}{2^5} = \frac{6}{2^5}$$

∴ P(X ≤ 2) > P(X ≥ 3) is not true.

Q.11 A party of 23 persons take their seats at a round table. The odds against two persons sitting together are

Correct option: (A)

Required probability $= \frac{(21)!\,2!}{(22)!} = \frac{1}{11} = \frac{1}{1+10}$

∴ Odds against = 10 : 1.

Q.12 For a random variable X, V(X) = 4 and E(X^2) = 13, the value of E(X) is

Correct option: (A)

We know that,

V (X) = E (X^2) – [E (X)]^2

∴ 4 = 13 – [E (X)]^2

∴ [E (X)]^2 = 13 – 4 = 9

∴ E (X) = 3

Q.13 A random variable X assumes values 1, 2, 3, …., n with equal probabilities, if var(X) = E(X), then n is

Correct option: (C)

X = 1, 2, 3, ….n

P(X) = $\frac{1}{n}$

$$E(X) = \sum_{i=1}^{n} x_i p_i = \frac{(1+2+3+\ldots+n)}{n}$$

$$= \frac{n(n+1)}{2n}$$

$$E(X) = \frac{n+1}{2}$$

$$Var(X) = \sum_{i=1}^{n} x_i^2 p_i - [E(X)]^2$$

$$= \frac{1^2+2^2+3^2+\ldots+n^2}{n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2} \times \left(\frac{n-1}{6}\right)$$

$$= \frac{n^2-1}{12}$$

Var(X) = E(X) …[Given]

$$\Rightarrow \frac{n^2-1}{12} = \frac{n+1}{2}$$

$$\Rightarrow \frac{n-1}{12} = \frac{1}{2}$$

⇒ n – 1 = 6

⇒ n = 7

Q.14 Probability of guessing correctly at least 7 out of 10 answers in a "True" or "False" test is

Correct option: (A)

P(answer is correct) = p = $\frac{1}{2}$

∴ q = 1 – $\frac{1}{2}$ = $\frac{1}{2}$

Also, n = 10

∴ P(at least 7 answers are correct) = P(X ≥ 7)

= P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2$

$+ {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$

$= ({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}) \frac{1}{2^{10}}$

$= \frac{176}{1024}$

$= \frac{11}{64}$

Q.15 A box contains 8 nails and 16 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, then the probability that it is rusted or is a nail is

Correct option: (C)

n(S) = 8 + 16 = 24

Total rusted items = 4 + 8 = 12

Unrusted nails = 4

∴ Required probability = $\frac{12+4}{24}$

$= \frac{16}{24} = \frac{2}{3}$

Q.16 Five persons are chosen at random from a group containing 4 men, 2 women and 4 children. The chance that exactly two of them will be children is

Correct option: (A)

Five persons can be selected from a group of 10 persons in ${}^{10}C_5$ ways.

A: Event of selecting 2 children.

$n(A) = {}^4C_2 \cdot {}^6C_3$

∴ $P(A) = \frac{{}^4C_2 \cdot {}^6C_3}{{}^{10}C_5} = \frac{10}{21}$

Q.17 The p.m.f. of a random variable X is

$P(X = x) = \frac{1}{2^5}\binom{5}{x}$, $x = 0, 1, 2, 3, 4, 5$

= 0, otherwise,

then

Correct option: (D)

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{{}^5C_0}{2^5} + \frac{{}^5C_1}{2^5} + \frac{{}^5C_2}{2^5} = \frac{16}{2^5}$

$P(X \ge 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$= \frac{{}^5C_3}{2^5} + \frac{{}^5C_4}{2^5} + \frac{{}^5C_5}{2^5} = \frac{16}{2^5}$

∴ $P(X \le 2) = P(X \ge 3)$

Q.18 Four defective oranges are accidentally mixed with sixteen good ones. Three oranges are drawn from the mixed lot. The probability distribution of defective oranges is

Correct option: (B)

Let X denote the random variable

X = 0, 1, 2, 3

$P(X = 0) = \frac{{}^{16}C_3}{{}^{20}C_3} = \frac{28}{57}$

$P(X = 1) = \frac{{}^{16}C_2 \cdot {}^4C_1}{{}^{20}C_3} = \frac{8}{19}$

$P(X = 2) = \frac{{}^{16}C_1 \cdot {}^4C_2}{{}^{20}C_3} = \frac{8}{95}$

$P(X = 3) = \frac{{}^4C_3}{{}^{20}C_3} = \frac{1}{285}$

Q.19 If X is a binomial variable with range {0, 1, 2, 3, 4} and $P(X = 3) = 3P(X = 4)$ then the parameter ' p ' of the binomial distribution is

Correct option: (B)

Here, n = 4

P(X = 3) = 3P(X = 4)

∴ ${}^4C_3 p^3 q = 3\ {}^4C_4 p^4 q^0$

∴ $4p^3q = 3p^4$

∴ 4(1 – p) = 3p …[p + q = 1]

∴ 4 – 4p = 3p

∴ 4 = 7p

∴ $p = \frac{4}{7}$

Q.20 A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected

Correct option: (B) $\frac{2}{7}$

The probability of husband is not selected = $1 - \frac{1}{7} = \frac{6}{7}$

The probability that wife is not selected = $1 - \frac{1}{5} = \frac{4}{5}$

The probability that only husband selected = $\frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

The probability that only wife selected = $\frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$

Hence, required probability = $\frac{6}{35} + \frac{4}{35} = \frac{10}{35}$
$= \frac{2}{7}$

Q.21 If P(A) = 0.4, P(B) = *x*, P(A ∪ B) = 0.7 and the events A and B are mutually exclusive, then *x* =

Correct option: (A)

Since events are mutually exclusive, therefore

P(A ∩ B) = 0 i.e., P(A ∪ B) = P(A) + P(B)

⇒ 0.7 = 0.4 + x

⇒ $x = \frac{3}{10}$

Q.22 A fair coin is tossed 99 times. If X is the number of times head occur then $P[X = r]$ is maximum when $r =$

Correct option: (B)

P(getting head) = p = $\frac{1}{2}$

∴ q = 1 – $\frac{1}{2} = \frac{1}{2}$

Here, n = 99

∴ P(X = r) = $99C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{99-r}$

$= 99C_r \left(\frac{1}{2}\right)^{99}$

If n is odd, the maximum value of ^nC_r occurs at

r = $\frac{n-1}{2}$ and r = $\frac{n+1}{2}$

∴ r = $\frac{99-1}{2}$ and r = $\frac{99+1}{2}$

∴ r = 49 and r = 50

Q.23 If a random variable X follows the Binomial distribution B(33, p) such that 3P(X = 0) = P(X = 1), then the variance of X is

Correct option: (C)

Given that,

3P (X = 0) = P (X = 1)

⇒ $3 \times {}^{33}C_0\, p^0\, q^{33} = {}^{33}C_1\, p^1 q^{32}$

⇒ $3 \times 1 \times 1 \times q^{33} = 33 \times p \times q^{32}$

⇒ 3q = 33p

⇒ p = $\frac{1}{11}q$

Now, p + q = 1

⇒ $\frac{1}{11}q + q = 1$

⇒ 12q = 11

⇒ q = $\frac{11}{12}$

⇒ p = $\frac{1}{12}$

Now, variance = n p q = $33 \times \frac{1}{12} \times \frac{11}{12} = \frac{121}{48}$

Q.24 In a town 40% of the people have brown hair, 25% have brown eyes and 15% have both. If a person selected at random from the town has brown hair, the probability that he has brown eyes is,

Correct option: (B)

X: Brown hair

⇒ P(X) = $\frac{40}{100}$

Y: Brown eyes

⇒ P(Y) = $\frac{25}{100}$

∴ P(X ∩ Y) = $\frac{15}{100}$

∴ P(Y/X) = $\frac{P(X \cap Y)}{P(X)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{3}{8}$

Q.25 Two balls are selected from two black and two red balls. The probability that the two balls will have no black balls is

Correct option: (D)

n(S) = $^4C_2$

P(no black ball) = P(red ball)

$= \frac{^2C_2}{^4C_2} = \frac{1}{6}$

Q.26 The p.m.f. of a r.v. X is given by

X = x	0	1	2	3
P(X = x)	q^3	$3q^2p$	$3qp^2$	p^3

If p + q = 1, then E(X) =

Correct option: (C)

The given probability mass function (p.m.f.) for the random variable X is:

$X = x$	**0**	**1**	**2**	**3**
$P(X = x)$	q^3	$3q^2p$	$3qp^2$	p^3

We are also given that $p + q = 1$. We need to find $E(X)$.

Method 1: Using the definition of Expectation

The expectation of a discrete random variable X is given by $E(X) = \sum x_i P(X = x_i)$.

Substituting the given values:

$E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3)$

$$E(X) = 0 \cdot (q^3) + 1 \cdot (3q^2p) + 2 \cdot (3qp^2) + 3 \cdot (p^3)$$

$$E(X) = 3q^2p + 6qp^2 + 3p^3$$

Factor out $3p$ from the expression:

$$E(X) = 3p(q^2 + 2qp + p^2)$$

Recognize the term in the parenthesis as a perfect square identity: $(q + p)^2 = q^2 + 2qp + p^2$.

$$E(X) = 3p(q + p)^2$$

Given that $p + q = 1$, substitute this value:

$$E(X) = 3p(1)^2$$

$$E(X) = 3p$$

Method 2: Recognizing the Binomial Distribution

The given probabilities correspond to the terms of a binomial expansion $(q + p)^3$:

- $P(X = 0) = q^3 = \binom{3}{0} p^0 q^3$
- $P(X = 1) = 3q^2p = \binom{3}{1} p^1 q^2$
- $P(X = 2) = 3qp^2 = \binom{3}{2} p^2 q^1$
- $P(X = 3) = p^3 = \binom{3}{3} p^3 q^0$

This indicates that X follows a Binomial distribution $B(n, p)$ with parameters $n = 3$ and p.

For a Binomial distribution $X \sim B(n, p)$, the expected value is given by $E(X) = np$.

Substituting $n = 3$ and p:

$$E(X) = 3p$$

Both methods yield the same result.

The final answer is $\mathbf{3p}$.

Q.27 A quadratic equation $ax^2 + bx + c = 0$, with distinct coefficients is formed. If a, b, c are chosen from the numbers 2, 3, 5, then the probability that the equation has real root is [KEAM 2018]

Correct option: (A) $\frac{1}{3}$

The quadratic equation $ax^2 + bx + c = 0$ has real roots when, $\Delta = b^2 - 4ac \geq 0$

Since a, b, c are chosen from the numbers 2, 3, 5. 6 different equations having distinct coefficients can be formed. Of these, only two equations having b = 5 will have real roots.

$\therefore$ Required probability = $\frac{2}{6} = \frac{1}{3}$

Q.28 If X and Y are two events such that X ⊆ Y, then $P\left(\frac{Y}{X}\right)$ =

Correct option: (B)

Since, X ⊆ Y ⇒ X ∩ Y = Y ∩ X = X

Hence, $P\left(\frac{Y}{X}\right) = \frac{P(Y \cap X)}{P(X)} = \frac{P(X)}{P(X)} = 1$

Q.29 The corners of regular tetrahedrons are numbered 1, 2, 3, 4. Three tetrahedrons are tossed. The probability that the sum of upward corners will be 5 is

Correct option: (C)

Required combinations are {(2, 2, 1), (1, 2, 2), (2, 1, 2), (1, 3, 1), (3, 1, 1), (1, 1, 3)}

$\therefore$ Required probability = $\frac{6}{4^3} = \frac{6}{64} = \frac{3}{32}$

Q.30 The probability of success for the Binomial distribution satisfying the relation, 4P(X = 4) = P(X = 2) and having the parameter n = 6, is

Correct option: (B)

4P(X = 4) = P(X = 2)

$\Rightarrow 4.\ {}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$

$\Rightarrow 4p^2 = q^2$

$\Rightarrow 4p^2 = (1 - p)^2$

$\Rightarrow 3p^2 + 2p - 1 = 0$

$\Rightarrow (3p - 1)(p + 1) = 0$

$\Rightarrow p = \frac{1}{3}$ …[$\because 0 < p < 1$]

Q.31 A coin is tossed twice in succession. Let X represent the number of tails in two tosses, then the probability distribution of X is given by

Correct option: (A)

X can take values 0, 1 and 2

P(X = 0) = Probability of getting no tail = $\frac{1}{4}$

P(X = 1) = Probability of getting one tail = $\frac{1}{2}$

P(X = 2) = Probability of getting two tails = $\frac{1}{4}$

Q.32 Two cards are drawn successively with replacement from well shuffled pack of 52 cards, then the probability distribution of number of queens is

Correct option: (A)

Let X denote the number of queens.

∴ Possible values of X are 0,1, 2.

P(queen) = $\frac{4}{52} = \frac{1}{13}$

P(not a queen) = $\frac{48}{52} = \frac{12}{13}$

P(X = 0) $= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

P(X = 1) $= \left(\frac{1}{13} \times \frac{12}{13}\right) + \left(\frac{12}{13} \times \frac{1}{13}\right)$

$= \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$

P(X = 2) $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

The probability distribution of X is

X = x	0	1	2
P[X = x]	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Q.33 If r.v. X ~ B$\left(n = 5, p = \frac{1}{3}\right)$, then P (2 < X < 4) =

Correct option: (B)

Here, n = 5, p = $\frac{1}{3}$

and q = $1 - \frac{1}{3} = \frac{2}{3}$

∴ P(2 < X < 4) = P (X = 3)

$= {}^5C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243}$

Q.34 If three numbers are drawn at random successively without replacement from a set S = {1, 2, …, 10}, then the probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

Correct option: (D)

n(S) = ${}^{10}C_3$

X: event that minimum of chosen number is 3

Y: event that maximum of chosen number is 7.

P(X) = $\frac{{}^7C_2}{{}^{10}C_3}$, P(Y) = $\frac{{}^6C_2}{{}^{10}C_3}$, P(X ∩ Y) = $\frac{{}^3C_1}{{}^{10}C_3}$

P(X ∪ Y) = P(X) + P(Y) – P(X ∩ Y)

$= \frac{{}^7C_2}{{}^{10}C_3} + \frac{{}^6C_2}{{}^{10}C_3} - \frac{{}^3C_1}{{}^{10}C_3} = \frac{11}{40}$

Q.35 Let a random variable x have a Binomial distribution with mean 8 and variance 4. If $P(x \leqslant 2) = \frac{k}{2^{16}}$, then k is equal to

Correct option: (D)

Let X ~ B(n, p)

According to the given conditions,

mean = np = 8 and variance = npq = 4

⇒ p = q = $\frac{1}{2}$ and n = 16

P(X ≤ 2) = $\frac{K}{2^{16}}$

⇒ P(X = 0) + P(X = 1) + P(X = 2) = $\frac{K}{2^{16}}$

∴

$${}^{16}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{15} + {}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14} = \frac{K}{2^{16}}$$

∴ $\frac{1 + 16 + 120}{2^{16}} = \frac{K}{2^{16}}$

∴ K = 137

Q.36 Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Then mean of number of tens is

Correct option: (C)

Probability of getting ten = $\frac{4}{52} = \frac{1}{13}$

∴ Probability of getting a card without ten = $\frac{12}{13}$

Let random variable X denotes the number of tens.

∴ Possible values of X are 0, 1, 2

Consider following probability distribution table.

X = x	0	1	2
P(X = x)	$\frac{12}{13} \times \frac{12}{13}$	$\frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13}$	$\frac{1}{13} \times \frac{1}{13}$

∴ Required mean

$$= 0 + 1 \times \left(\frac{12}{13 \times 13} + \frac{12}{13 \times 13}\right) + 2 \times \left(\frac{1}{13} \times \frac{1}{13}\right)$$

$$= \frac{24}{169} + \frac{2}{169}$$

$$= \frac{26}{169} = \frac{2}{13}$$

Q.37 The p.d.f. of a continuous r.v.X is given by

f(x) = $\frac{x}{8}$, 0 < x < 4

= 0, otherwise, then P(X ≤ 2) is

Correct option: (C)

$$P(X \le 2) = \int_0^2 \frac{x}{8} dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{1}{8} \times 2 = \frac{1}{4}$$

Q.38 Given X ~ B(n, p). If E(X) = 6, Var(X) = 4.2, then what is the number of trials?

Correct option: (C)

E(X) = np = 6

Var(X) = npq = 4.2

$$\therefore \frac{npq}{np} = \frac{4.2}{6}$$

$$\Rightarrow q = \frac{7}{10}$$

$$\Rightarrow p = 1 - \frac{7}{10} = \frac{3}{10}$$

Now, np = 6

$$\Rightarrow n \times \frac{3}{10} = 6$$

⇒ n = 20

Q.39 Given: X ~ B(n, p). If E(X) = 6, Var(X) = 4.2, then the number of trials are

Correct option: (B)

Given, E(X) = 6 and Var(X) = 4.2

∴ np = 6 and npq = 4.2

∴ $\frac{np}{npq} = \frac{6}{4.2}$ ⇒ q = 0.7

∴ p = 1 – 0.7 = 0.3

∴ np = 6 ⇒ n = $\frac{6}{0.3}$ = 20

Q.40 The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true?

Correct option: (B)

P(X ∪ Y ∪ Z) = $\frac{3}{4}$

⇒ P(X) + P(Y) + P(Z) – P(X ∩ Y) – P(Y ∩ Z) – P(X ∩ Z) + P(X ∩ Y ∩ Z) = $\frac{3}{4}$ …(i)

P(X ∩ Y) + P(Y ∩ Z) + P(X ∩ Z) – 2P(X ∩ Y ∩ Z) = $\frac{1}{2}$ …(ii)

and

P(X ∩ Y) + P(Y ∩ Z) + P(X ∩ Z) – 3P(X ∩ Y ∩ Z) = $\frac{2}{5}$ …(iii)

From (ii) and (iii), we get

P(X ∩ Y ∩ Z) = $\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$ …(iv)

⇒ P(X) P(Y) P(Z) = $\frac{1}{10}$ ⇒ pmc = $\frac{1}{10}$

From (i), (ii) and (iv), we have

P(X) + P(Y) + P(Z) – $\left(\frac{1}{2} + \frac{2}{10}\right) + \frac{1}{10} = \frac{3}{4}$

⇒ p + m + c = $\frac{27}{20}$

Q.41 Bag I contains 4 white and 3 black balls. Bag II contains 6 white and 5 black balls. One ball is drawn at random from one of the two bags and it is found to be black. The probability that it was drawn from bag I is ________.

Correct option: (A)

Event A: Bag I is selected.

Event B: Bag II is selected.

Event C: Black ball is drawn.

∴ Required probability,

$$P(A/C) = \frac{P(A) \cdot P(C/A)}{P(A)P(C/A) + P(B) \cdot P(C/B)}$$

$$P(A/C) = \frac{\left(\frac{1}{2} \times \frac{3}{7}\right)}{\left(\frac{1}{2} \times \frac{3}{7}\right) + \left(\frac{1}{2} \times \frac{5}{11}\right)} = \frac{33}{68}$$

Q.42 The raw data $x_1, x_2, \ldots\ldots, x_n$ is an A.P. with common difference d and first term 0. $\overline{x}$ and σ^2 are mean and variance of x_i, i = 1, 2, ……. n, then σ^2 is

Correct option: (D)

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

$$= \frac{\frac{n}{2}[2x_1 + (n-1)d]}{n}$$

$$= \frac{(n-1)d}{2} \quad \ldots[\because x_1 = 0]$$

$$\sum x_i^2 = x_1^2 + x_2^2 + \ldots + x_n^2$$

$= 0 + d^2 + (2d)^2 + \ldots + [(n-1)d]^2$

$= d^2[1 + 2^2 + 3^2 + \ldots + (n-1)^2]$

$$= d^2\left[\frac{n(n-1)(2n-1)}{6}\right]$$

$$\therefore \sigma^2 = \frac{1}{n}\sum x_i^2 - (\overline{x})^2$$

$$= \frac{d^2(n-1)(2n-1)}{6} - \left[\frac{(n-1)d}{2}\right]^2$$

$$= \frac{d^2(n-1)}{2}\left(\frac{2n-1}{3} - \frac{n-1}{2}\right)$$

$$= \frac{d^2(n-1)}{2}\left(\frac{n+1}{6}\right)$$

$$= \frac{(n^2-1)d^2}{12}$$

Q.43 If X ~ B(21, p) and E(X) = 7, then the variance of X is

Correct option: (B)

Here, n = 21 and E(X) = np = 7

$\therefore p = \frac{E(X)}{n} = \frac{7}{21} = \frac{1}{3}$

$\therefore q = 1 - p = \frac{2}{3}$

Variance = npq = $\frac{14}{3}$

Q.44 A man takes a step forward with probability 0.4 and backwards with probability 0.6. The probability that at the end of eleven steps, he is one step away from the starting point is

Correct option: (B)

Let a step forward be a success and the step backward be a failure.

$\therefore$ Probability of success = p = 0.4, and Probability of failure = q = 0.6

Now, in 11 steps

number of successes = 6, number of failure = 5

OR

number of successes = 5, number of failures = 6

$\therefore$ Required probability = ${}^{11}C_6\, p^6q^5 + {}^{11}C_5\, p^5q^6$

$= \frac{11!}{6!5!}\, p^6q^5 + \frac{11!}{5!6!}\, p^5q^6$

$= {}^{11}C_6\, p^5q^5(p + q)$

$= {}^{11}C_6\, (0.4)^5\, (0.6)^5(1)$

$= {}^{11}C_6\, (0.4)^5(0.6)^5$

$= {}^{11}C_6\, (0.24)^5$

Q.45 Two cards are drawn from a pack of well shuffled 52 playing cards one by one without replacement. Then the probability that both cards are queens is

Correct option: (B)

Probability of first card to be a queen = $\frac{4}{52}$ and

probability of also second to be a queen = $\frac{3}{51}$

Hence, required probability = $\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

Q.46 The following is p.d.f. of continuous random variable X

$$f(x) = \begin{cases} \frac{x}{8} & , \text{ if } 0 < x < 4 \\ 0 & , \text{ otherwise} \end{cases}$$

Then $F(0.5), F(1.7)$ and $F(5)$ is respectively

Correct option: (B)

$$F(x) = \int_0^x \frac{x}{8}\,dx = \frac{1}{8}\left[\frac{x^2}{2}\right]_0^x = \frac{1}{16}x^2$$

$F(0.5) = \frac{1}{16}\,(0.5)^2 = \frac{0.25}{16} = 0.015625$

$F(1.7) = \frac{1}{16}(1.7)^2 = \frac{2.89}{16} = 0.18062$

$F(5) = 1 \ldots \left[\begin{array}{l} f(x) = 0, if x \notin (0,4) \\ \therefore F(x) = 1 for x \geq 4 \end{array}\right]$

Q.47 In a binomial distribution the probability of getting a success is $\frac{1}{4}$ and standard deviation is 3, then its mean is

Correct option: (C)

Probability of getting a success, p = $\frac{1}{4}$

$\therefore$ q = 1 – $\frac{1}{4} = \frac{3}{4}$

Given that Standard deviation = 3

Standard deviation = $\sqrt{Variance}$

⇒ Variance = 9

⇒ npq = 9 ⇒ $n\frac{1}{4}\,\frac{3}{4}$ = 9 ⇒ n = 48

Mean = np = 48 × $\frac{1}{4}$ = 12

Q.48 If μ and σ^2 are mean and variance of a random variable X whose p.m.f. is given by

P(X = *x*) = $\binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}$, *x* = 0, 1, 2, 3, …., 6, then the value 2μ + 12σ^2 =

Correct option: (B)

P(X = *x*) = $\binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}$

Here, n = 6, p = $\frac{1}{3}$ and q = $\frac{2}{3}$

μ = np = $6\left(\frac{1}{3}\right)$ = 2

σ^2 = npq = $6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{3}$

$\therefore$ 2μ + 12σ^2 = 2(2) + 12$\left(\frac{4}{3}\right)$

= 4 + 16 = 20

Q.49 The p.m.f. of a r.v. X is P(*x*) =

$$\begin{cases} \frac{x}{(n+1)(2n+1)}, & x = 1, 2, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

Then, E(X) =

Correct option: (C)

X	1	2	3	…	n
P(X)	$\frac{1}{(n+1)(2n+1)}$	$\frac{2}{(n+1)(2n+1)}$	$\frac{3}{(n+1)(2n+1)}$	…	$\frac{n}{(n+1)(2n+1)}$

E(X) = $\sum x_i \cdot P(x_i)$

$= 1 \times \frac{1}{(n+1)(2n+1)} + 2 \times \frac{2}{(n+1)(2n+1)} + 3 \times \frac{3}{(n+1)(2n+1)} + \ldots + n \times \frac{n}{(n+1)(2n+1)}$

$= \frac{1}{(n+1)(2n+1)}$ (1 + 4 + 9 + … + n^2)

$= \frac{1}{(n+1)(2n+1)}$ ($1^2 + 2^2 + 3^2 + \ldots + n^2$)

$= \frac{1}{(n+1)(2n+1)} \times \frac{n(n+1)(2n+1)}{6}$

$= \frac{n}{6}$

Q.50 From a group of 5 boys and 3 girls, three persons are chosen at random. Find the probability that there are more girls than boys

Correct option: (D)

Three persons can be chosen out of 8 in ${}^8C_3$ = 56 ways.

The number of girls is more than that of the boys if either 3 girls are chosen or two girls and one boy is chosen. This can be done in ${}^3C_3 + {}^3C_2 \times {}^5C_1$ ways = 1 + 3 × 5 = 16 ways.

$\therefore$ Required probability = $\frac{16}{56} = \frac{2}{7}$

Q.51 An organization consists of 25 members including 4 doctors. A committee of 4 is to be formed at random. The probability that the committee contains at least 3 doctors is

Correct option: (A) $\frac{17}{2530}$

A committee of 4 can be formed in ${}^{25}C_4$ ways

O: Event that the committee contains at least 3 doctors

$\therefore$ n(O) = ${}^4C_3 . {}^{21}C_1 + {}^4C_4$ = 85

$\therefore$ P(O) = $\frac{85}{{}^{25}C_4} = \frac{85}{12650} = \frac{17}{2530}$

Q.52 A die is thrown. Let X be the event that the number obtained is greater than 3. Let Y be the event that the number obtained is less than 5. Then, P(X ∪ Y) is

Correct option: (A)

Here, X = {4, 5, 6}

⇒ P(X) = $\frac{3}{6} = \frac{1}{2}$

and Y = {4, 3, 2, 1}

⇒ P(Y) = $\frac{4}{6} = \frac{2}{3}$

$\therefore$ X ∩ Y = {4}

⇒ P(X ∩ Y) = $\frac{1}{6}$

$\therefore$ P(X ∪ Y) = $\frac{1}{2} + \frac{2}{3} - \frac{1}{6}$ = 1

Q.53 If the sum of the mean and the variance of a Binomial distribution for 5 trials is 1.8, then the value of p is

Correct option: (D)

Given, E(X) + V(X) = 1.8

⇒ np + npq = 1.8

⇒ np(1 + q) = 1.8

⇒ n(1 – q) (1 + q) = 1.8

⇒ 1 – q^2 = 0.36 …[∵ n = 5]

⇒ q = 0.8

$\therefore$ p = 1 – 0.8 = 0.2

Q.54 The value of k for which the function f(x) = $\begin{cases} ke^{-3x}, & x > 0 \\ 0, & otherwise \end{cases}$ is a p.d.f.?

Correct option: (A)

f(x) is a p.d.f. if

$\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\therefore \int_{0}^{\infty} ke^{-3x}\, dx = \frac{k}{-3}\left[e^{-3x}\right]_0^{\infty} = 1$

$\therefore \frac{k}{-3}\left[e^{-\infty} - e^{0}\right] = \frac{k}{-3}(0-1) = \frac{k}{3} = 1$

$\therefore$ k = 3

Q.55 If X ~ B$\left(8, \frac{1}{2}\right)$, then P(|X – 4| ≤ 2) =

Correct option: (B)

X ~ B$\left(8, \frac{1}{2}\right)$

Here, n = 8, p = $\frac{1}{2}$

$\therefore$ q = 1 – $\frac{1}{2} = \frac{1}{2}$

|X – 4| ≤ 2 ⇒ X = 2, 3, 4, 5, 6

$\therefore$ P(|X – 4| ≤ 2)

= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)

= $^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 + {}^8C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 + {}^8C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 + {}^8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2$

= $\frac{1}{2^8}\left({}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6\right)$

= $\frac{1}{256}$ (28 + 56 + 70 + 56 + 28)

= $\frac{1}{256}$ (238) = $\frac{119}{128}$

Q.56 The c.d.f. of a discretc r.v. x is

x	1	2	3	4	5	6
F(x)	0.18	0.43	0.54	0.68	0.89	1.00

Then P (1 < x < 5) =

Correct option: (B)

P(x = 2) = F(2) – F(1) = 0.43 – 0.18 = 0.25

P(x = 3) = F(3) – F(2) = 0.54 – 0.43 = 0.11

P(x = 4) = F(4) – F(3) = 0.68 – 0.54 = 0.14

$\therefore$ P(1 < x < 5) = P(x = 2) + P(x = 3) + P(x = 4)

= 0.25 + 0.11 + 0.14

= 0.50

Q.57 Let A and B are independent events with $P(B) = \frac{2}{5}, P(A \cup B) = \frac{11}{20}$, then P (A′| B) is root of the equation

Correct option: (A)

Here, P (B) = $\frac{2}{5}$, P (A ∪ B) = $\frac{11}{20}$

We know that,

P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

⇒ P (A ∪ B) = P (A) + P (B) – P (A) . P (B) …[A and B are independent events]

⇒ $\frac{11}{20}$ = P (A) + $\frac{2}{5}$ – P (A) . $\frac{2}{5}$

⇒ $\frac{11}{20} - \frac{2}{5}$ = P (A) $\left(1 - \frac{2}{5}\right)$

⇒ $\frac{3}{20} = \frac{3}{5}$ P (A)

⇒ P (A) = $\frac{1}{4}$ ⇒ P (A′) = $\frac{3}{4}$

Now,

P (A′/B) = $\frac{P(A' \cap B)}{P(B)}$

= $\frac{P(A').P(B)}{P(B)}$ …[A′ and B are independent events]

P (A′/ B) = $\frac{3}{4}$

⇒ x = $\frac{3}{4}$ is the root of 4x^2 – 7x + 3 = 0.

Q.58 Probability that a person will develop immunity after vaccination is 0.8. If 8

people are given the vaccine then probability that all develop immunity is

Correct option: (B)

Probability that person will develop immunity (p) = 0.8

q = 1 – p = 0.2

∴ Required probability = $^8C_8 (0.8)^8 (0.2)^0$

$= (0.8)^8$

Q.59 If for a discrete random variable X, $E(X^2) = 1$, Var(X) = 0.96, then E(X) =

Correct option: (C)

$Var(X) = E(X^2) – [E(X)]^2$

$\Rightarrow 0.96 = 1 – [E(X)]^2$

$\Rightarrow [E(X)]^2 = 0.04$

$\Rightarrow E(X) = \sqrt{0.04} = 0.2$

Q.60 A boy tosses fair coin 3 times. If he gets ₹ 2*x* for *x* heads then his expected gain equals to ₹ ______

Correct option: (C)

$y = 2x$

x	0	1	2	3
y	0	2	4	6
P(y)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

∴ Expected gain = $\sum y_i P(y_i)$

$= 0\left(\frac{1}{8}\right) + 2\left(\frac{3}{8}\right) + 4\left(\frac{3}{8}\right) + 6\left(\frac{1}{8}\right)$

$= 3$

Q.61 In order to get a head at least once probability ≥ 0.9, the minimum number of time a unbiased coin needs to be tossed is

Correct option: (B)

Let coin is tossed ‘n’ number of times.

Probability of getting head is $p = \frac{1}{2}$

$\therefore q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X \geq 1) > \frac{9}{10}$

$\therefore 1 - P(X < 1) > \frac{9}{10}$

$\therefore 1 - \left(\frac{1}{2}\right)^n > \frac{9}{10}$

$\therefore \left(\frac{1}{2}\right)^n < \frac{1}{10}$

$\therefore 10 < 2^n$

∴ n = 4 is the minimum value.

Q.62 The p.m.f. of a r.v. X is

$$P(x) = \begin{cases} kx^2; & x = 1, 2, 3, 4 \\ 0; & otherwise \end{cases}$$

Then, E(X) =

Correct option: (C)

X = x	1	2	3	4
P(X = x)	k	4k	9k	16k

Since, P(1) + P(2) + P(3) + P(4) = 1

∴ k + 4k + 9k + 16k = 1

⇒ 30k = 1

$\Rightarrow k = \frac{1}{30}$

$\therefore E(X) = \sum x_i . P(x_i)$

$= 1.\frac{1}{30} + 2.\frac{4}{30} + 3.\frac{9}{30} + 4.\frac{16}{30}$

$= \frac{100}{30}$

$= \frac{10}{3}$

Q.63 The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. The probability that out of 5 workmen, 3 or more will contract the disease is

Correct option: (D)

P(workmen suffering from disease) = 10% = 0.1

∴ q = 1 – p = 1 – 0.1 = 0.9

Also, n = 5

∴ Required probability

$= P(X \geq 3)$

$= P(X = 3) + P(X = 4) + P(X = 5)$

$= {}^5C_3(0.1)^3 (0.9)^2 + {}^5C_4(0.1)^4 (0.9)^1 + {}^5C_5 (0.1)^5(0.9)^0$

= 0.0081 + 0.00045 + 0.00001

= 0.00856

Q.64 A fair coin is tossed a fixed number of times. If the probability of getting 7

heads is equal to that of getting 9 heads, then the probability of getting 3 heads is

Correct option: (A) $\frac{35}{2^{12}}$

Let the coin be tossed n times.

Probability of getting head is p = $\frac{1}{2}$

$\therefore$ q = 1 – $\frac{1}{2} = \frac{1}{2}$

Then, P(7 heads) = ${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_7\left(\frac{1}{2}\right)^n$

and P(9 heads) = ${}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9} = {}^nC_9\left(\frac{1}{2}\right)^n$

According to the given condition,

P(7 heads) = P(9 heads)

$\therefore {}^nC_7 = {}^nC_9$

$\therefore$ n = 16

$\therefore$ P (3 heads) = ${}^{16}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{16-3}$

$= {}^{16}C_3\left(\frac{1}{2}\right)^{16} = \frac{35}{2^{12}}$

Q.65 Two cards are drawn successively with replacement from a well shuffled pack of 52 cards, then mean of number of queens is

Correct option: (C)

Total number of cards = 52

Total number of queens = 4

Probability of getting a queen

P(queen) = $\frac{4}{52} = \frac{1}{13}$

Probability of not getting a queen

P(non queen) = $\frac{48}{52} = \frac{12}{13}$

Let X be a random variable such that

X = number of queens in 2 draws

Case I: No queens are drawn (X = 0)

P(X = 0) = P(non queen) × P(non queen)

$= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

Case II: One queen is drawn (X = 1)

P(X = 1) = P(non queen and queen) or P(queen and non queen)

$= \frac{12}{13} \times \frac{1}{13} + \frac{1}{13} \times \frac{12}{13} = \frac{24}{169}$

Case III: Two queens are drawn (X = 2)

P(X = 2) = P(queen) × P(queen)

$= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

$\therefore$ Required Mean is

E(X) = $\sum x \cdot P(x) = 0 \times \frac{144}{169} + 1 \times \frac{24}{169} + 2 \times \frac{1}{169}$

$= \frac{26}{169}$

E(X) = $\frac{2}{13}$

Q.66 The probability mass function of random variable X is given by

P[X = r] = $\begin{cases} \frac{{}^nC_r}{32}, \text{n, r} \in \mathbb{N} \\ 0, \text{otherwise} \end{cases}$**, then P[X ≤ 2]**

=

Correct option: (B)

Since $\sum_{x=0}^{n} P(X = x) = 1$

$\therefore \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 \ldots + \ldots + {}^nC_n}{32} = 1$

$\Rightarrow 2^n = 32$

$\Rightarrow$ n = 5

P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2)

$= \frac{{}^5C_0}{32} + \frac{{}^5C_1}{32} + \frac{{}^5C_2}{32} = \frac{1}{2}$

Q.67 If the mean and variance of a binomial distribution are 4 and 2 respectively, then probability of getting 2 heads is

Correct option: (A)

Mean = np = 4

Variance = npq = 2

$\therefore \frac{\text{npq}}{\text{np}} = \frac{2}{4}$

$\Rightarrow$ q = $\frac{1}{2}$

$\Rightarrow$ p = 1 – $\frac{1}{2} = \frac{1}{2}$

Now, np = 4

$\Rightarrow$ n$\left(\frac{1}{2}\right)$ = 4

$\Rightarrow$ n = 8

$\therefore$ Required probability = P(X = 2)

$= {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6$

$= 28\left(\frac{1}{2^8}\right) = \frac{28}{256}$

Q.68 X is a continuous random variable with probability density function

f(*x*) = $\begin{cases} \frac{x}{6} + k; & 0 \leq x \leq 3 \\ 0 \;\; ; & otherwise \end{cases}$

The value of k is equal to

Correct option: (A)

Since, f(*x*) is the p.d.f. of X.

$\therefore \int_{-\infty}^{\infty} \mathrm{f}(x)\, \mathrm{d}x = 1$

$\Rightarrow \int_0^3 \left(\frac{x}{6} + k\right) \mathrm{d}x = 1$

$\Rightarrow \left[\frac{x^2}{12} + kx\right]_0^3 = 1 \Rightarrow \frac{3}{4} + 3k = 1$

$\Rightarrow 3k = \frac{1}{4} \Rightarrow k = \frac{1}{12}$

Q.69 Two coins are tossed. What is the probability of getting 2 heads or 2 tails?

Correct option: (A) $\frac{1}{2}$

Here, n(S) = 2 × 2 = 4

X: Event of getting 2 heads or 2 tails

∴ X = {(H H), (T T)}

⇒ n(X) = 2

⇒ P(X) = $\frac{2}{4} = \frac{1}{2}$

Q.70 A binomial random variable X satisfies 9·p (X = 4) = p (X = 2) when n = 6. Then p is equal to

Correct option: (A)

9·p (X = 4) = p (X = 2) and n = 6

$\Rightarrow 9 \times {}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$

$\Rightarrow 9p^2 = q^2$

⇒ 3p = q ...[∵ 0 < p, q < 1]

⇒ 3p = 1– p

$\Rightarrow p = \frac{1}{4}$

Q.71 Three urns respectively contain 2 white and 3 black, 3 white and 2 black and 1 white and 4 black balls. If one ball is drawn from each urn, then the probability that the selection contains 1 black and 2 white balls is

Correct option: (B)

Case I: $B_1W_2W_3$

P(black ball from urn 1) = $\frac{3}{5}$

P(white ball from urn 2) = $\frac{3}{5}$

P(white ball from urn 3) = $\frac{1}{5}$

$P(B_1W_2W_3) = \frac{3}{5} \times \frac{3}{5} \times \frac{1}{5} = \frac{9}{125}$

Case II: $W_1B_2W_3$

P(white ball from urn 1) = $\frac{2}{5}$

P(black ball from urn 2) = $\frac{2}{5}$

P(white ball from urn 3) = $\frac{1}{5}$

$\therefore\; P(W_1B_2W_3) = \frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} = \frac{4}{125}$

Case III: $W_1W_2B_3$

P(white ball from urn 1) = $\frac{2}{5}$

P(white ball from urn 2) = $\frac{3}{5}$

P(black ball from urn 3) = $\frac{4}{5}$

$\therefore\; P(W_1W_2B_3) = \frac{2}{5} \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{125}$

∴ Required probability =

$\frac{9}{125} + \frac{4}{125} + \frac{24}{125} = \frac{37}{125}$

Q.72 If the probability function of a random variable X is defined by P(X = k) = a $\left(\frac{k+1}{2^k}\right)$ **for k = 0, 1, 2, 3, 4, 5, then the probability that X takes a prime value is**

Correct option: (B) $\frac{23}{60}$

X = k	0	1	2	3	4	5
P(X = k)	a	a	$\frac{3a}{4}$	$\frac{4a}{8}$	$\frac{5a}{16}$	$\frac{6a}{32}$

Since $\sum_{k=0}^{5} P$ (X = k) = 1,

a + a + $\frac{3a}{4} + \frac{4a}{8} + \frac{5a}{16} + \frac{6a}{32}$ = 1

$\Rightarrow \frac{15}{4}a = 1 \Rightarrow a = \frac{4}{15}$

Now, P(X = prime value)

= P(X = 2) + P(X = 3) + P(X = 5)

$= \frac{3a}{4} + \frac{4a}{8} + \frac{6a}{32}$

$= \frac{23a}{16}$

$= \frac{23}{16} \times \frac{4}{15} = \frac{23}{60}$

Q.73 A random variable X has p.m.f. $P(X = x) = \frac{^4C_x}{2^4}, x = 0, 1, 2, 3, 4$ **and μ and** σ^2 **are mean and variance respectively of random variable X , then**

Correct option: (B)

P (X = 0) = $\frac{4C_0}{2^4} = \frac{1}{16}$

P (X = 1) = $\frac{4C_1}{2^4} = \frac{4}{16}$

P (X = 2) = $\frac{4C_2}{2^4} = \frac{6}{16}$

P (X = 3) = $\frac{4C_3}{2^4} = \frac{4}{16}$

P (X = 4) = $\frac{4C_4}{2^4} = \frac{1}{16}$

μ = E (X) = $\sum x_i\, p(x_i)$

= 0 $\left(\frac{1}{16}\right) + 1\left(\frac{4}{16}\right) + 2\left(\frac{6}{16}\right) + 3\left(\frac{4}{16}\right) + 4\left(\frac{1}{16}\right)$

$= \frac{32}{16} = 2$

σ^2 = Var(x) = $E(X)^2 - [E(x)]^2$

= $0^2 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2^2 \times \frac{6}{16} + 3^2 \times \frac{4}{16} + 4^2 \times \frac{1}{16} - (2)^2$

= 5 – 4 = 1

Q.74 Following is the probability distribution of smart phones sold in a shop per day

Number of smart phones	0	1	2	3	4	5
Probability	k	0.3	0.15	0.15	0.1	2k

then E(X) =

Correct option: (A)

Since $\sum_{x=0}^{5} P(X = x) = 1,$

k + 0.3 + 0.15 + 0.15 + 0.1 + 2k = 1

⇒ 3k + 0.7 = 1

⇒ k = 0.1

∴ E(X) = $\sum x_i \cdot P(x_i)$

= 0 × 0.1 + 1 × 0.3 + 2 × 0.15 + 3 × 0.15 + 4 × 0.1 + 5 × 0.2

= 2.45

Q.75 One purse contains 4 silver and 3 copper coins and another purse contains 5 silver and 4 copper coins. One coin is drawn at random from each purse. The probability that one is silver and the other is copper coin is $\frac{30+k}{62+k}$**. Then k = ________.**

Correct option: (A)

Required probability = P(Silver coin from first purse and copper coin from second purse) + P(Copper coin from first purse and silver coin from second purse)

$= \left(\frac{4}{7} \times \frac{4}{9}\right) + \left(\frac{3}{7} \times \frac{5}{9}\right) = \frac{31}{63}$

$= \frac{30+k}{62+k} = \frac{31}{63}$ ∴ k = 1

Q.76 If X and Y are two events such that P(X ∪ Y) + P(X ∩ Y) = $\frac{7}{8}$ **and P(X) = 2P(Y), then P(X) =**

Correct option: (A)

Since, we have

P(X∪Y) + P(X ∩ Y) = P(X) + P(Y)

= P(X) + $\frac{P(X)}{2}$

$\Rightarrow \frac{7}{8} = \frac{3P(X)}{2}$

⇒ P(X) = $\frac{7}{12}$

Q.77 If the p.d.f. of a r.v. X is

f(x) = $\begin{cases} \frac{x+2}{18}, & -2 < x < 4 \\ 0, & otherwise \end{cases}$

then P(|X| < 1) =

Correct option: (B)

P(|X| < 1) = P(–1 < X < 1)

$$= \int_{-1}^{1} \left(\frac{x+2}{18}\right) dx$$

$$= \frac{1}{18} \int_{-1}^{1} (x+2)\, dx$$

$$= \frac{1}{18} \left[\frac{x^2}{2} + 2x\right]_{-1}^{1}$$

$$= \frac{1}{18}\left(\frac{5}{2} + \frac{3}{2}\right) = \frac{4}{18} = \frac{2}{9}$$

Q.78 Two dice are thrown together. The probability that sum of the numbers is divisible by 2 or 3 is

Correct option: (D)

Two dice are thrown.

n(S) = 36

A: sum of numbers on two dice is divisible by 2 or 3.

A = {(1, 1), (1, 2), (1, 3), (1, 5), (2, 1), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (3, 6), (4, 2), (4, 4), (4, 5), (4, 6), (5, 1), (5, 3), (5, 4), (5, 5), (6, 2), (6, 3), (6, 4), (6, 6)}

∴ n(A) = 24

Hence, required probability = $\frac{24}{36} = \frac{2}{3}$

Q.79 For a binomial distribution,

Correct option: (C)

For a binomial distribution, the mean is equal to np and the variance is np(1-p), where n is the number of trials and p is the probability of success. Therefore, the variance will always be less than or equal to the mean, as p(1-p) is always less than or equal to 1. Hence, the correct option is Mean > variance.

Q.80 A die is thrown ten times. If getting an even number is considered as a success, then the probability of four successes is

Correct option: (D)

Probability of getting an even number is

p = $\frac{3}{6} = \frac{1}{2}$

∴ q = 1–$\frac{1}{2} = \frac{1}{2}$

Also, n =10

∴ Required probability = P (X =4)

$= {}^{10}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = {}^{10}C_4\left(\frac{1}{2}\right)^{10}$

$= {}^{10}C_4\left(\frac{1}{2}\right)^{10}$ ....[∵ ${}^nC_r = {}^nC_{n-r}$]

Q.81 The probability that a leap year selected at random will contain 53 Sundays is

Correct option: (B)

In a leap year, there are 366 days in which 52 weeks and two days. The combination of 2 days may be: Sun – Mon, Mon – Tue, Tue – Wed, Wed – Thu, Thu – Fri, Fri – Sat, Sat – Sun.

∴ P(53 Sun) = $\frac{2}{7}$

Q.82 Two unbiased dice are thrown. Then the probability that neither a doublet nor a total of 10 will appear is

Correct option: (D)

n(S) = 36

Let A be event represents the occurrence of either a doublet or a total of 10.

∴ A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (6, 4), (4, 6)}

∴ n(A) = 8 ⇒ P(A) = $\frac{8}{36} = \frac{2}{9}$

Hence, required probability = 1 – P(A)

$= 1 - \frac{2}{9} = \frac{7}{9}$

Q.83 If a random variable X has the following probability distribution of X

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

Then $P(X \geq 6) =$

Correct option: (A)

Since $\sum_{x=0}^{7} P(X = x) = 1$,

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow (k + 1)(10k - 1) = 0$

$\Rightarrow k = \frac{1}{10}$ ...[∵ k ≥ 0, k + 1 ≠ 0]

$P(X \geq 6)$ = 2k^2 + 7k^2 + k

= 9k^2 + k

$= 9\left(\frac{1}{10}\right)^2 + \frac{1}{10} = \frac{19}{100}$

Q.84 A random variable X has the following probability distribution:

X = x	1	2	3	4
P (X = x)	0.1	0.2	0.3	0.4

The mean and standard deviation of X are respectively

Correct option: (B)

Mean = E(X) = $\sum x_i . P(x_i)$

= 1(0.1) + 2(0.2) + 3(0.3) + 4(0.4) = 3

Var (X) = E(X^2) – [E(X)]^2

= 1^2(0.1) + 2^2(0.2) + 3^2(0.3) + 4^2(0.4) – (3)^2

= 0.1 + 0.8 + 2.7 + 6.4 – 9 = 10 – 9 = 1

∴ S.D. = 1

Q.85 A random variable X has the following probability distribution

X:	0	1	2	3	4
P(X):	k	2k	4k	2k	k

then the value of
$P(1 \leqslant X < 4 | X \leqslant 2) =$

Correct option: (B)

P(1 ≤ X < 4/ X ≤ 2)

$= \frac{P(1 \leq X \leq 2)}{P(X \leq 2)}$ …

$\left[P(A/B) = \frac{P(A \cap B)}{P(B)}\right]$

$= \frac{P(X=1)+P(X=2)}{P(X=0)+P(X=1)+P(X=2)}$

$= \frac{6k}{7k} = \frac{6}{7}$

Q.86 The probability that a student is not a swimmer is $\frac{1}{5}$. The probability that out of 5 students selected at random 4 are swimmers is

Correct option: (A)

Here, q = $\frac{1}{5}$

∴ p = 1 – $\frac{1}{5}$ = $\frac{4}{5}$

Also, n = 5

∴ Required probability = P (X = 4)

$= {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right) = 5\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right) = \left(\frac{4}{5}\right)^4$

Q.87 If two balanced dice are tossed once, the probability of the event, that the sum of the integers coming on the upper sides of the two dice is 9, is

Correct option: (C)

Required probability = $\frac{4}{36} = \frac{1}{9}$

Q.88 The mean and variance of a binomial distribution are 4 and 3 respectively, then the probability of getting exactly six successes in this distribution is

Correct option: (B)

In Binomial distribution, Variance = npq and
Mean = np, Variance = 3 = npq,
Mean = 4 = np

Now, q = $\frac{3}{4}$, p = $\frac{1}{4}$ and n = 16

Probability of success = ${}^{16}C_6 \left(\frac{3}{4}\right)^{10} \left(\frac{1}{4}\right)^6$

Q.89 Two dice are thrown. The number of sample points in the sample space when six does not appear on either dice is

Correct option: (D)

It six does not appear on either dice then, there are only five possible outcomes associated with one dice, the number of sample points is 5 × 5.

Q.90 A pair of dice is thrown 4 times. If getting a doublet is considered a success, then find the probability distribution of number of successes.

Correct option: (A)

When a pair of dice is thrown, the sample space has 36 equally likely outcomes, out of which six are doublets – (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

If p = probability of getting a doublet, then p = $\frac{6}{36} = \frac{1}{6}$, So q = $1-\frac{1}{6} = \frac{5}{6}$. Here n = 4

Thus, we have a binomial distribution with $p = \frac{1}{6}$, $q = \frac{5}{6}$ and n = 4.

If X denotes the number of doublets obtained, then X takes the values 0, 1, 2, 3, 4.

$P(0) = {}^4C_0 q^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$,

$P(1) = {}^4C_1 pq^3 = 4 \cdot \frac{1}{6}\left(\frac{5}{6}\right)^3 = \frac{500}{1296}$,

$P(2) = {}^4C_2 p^2q^2 = 6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{150}{1296}$,

$P(3) = {}^4C_3 p^3 q = 4\left(\frac{1}{6}\right)^3 \frac{5}{6} = \frac{20}{1296}$ and

$P(4) = {}^4C_4 p^4 = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

∴ The required probability distribution is

X = x	0	1	2	3	4
P(X = x)	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$