Gravitation **Marks: 30**

ANSWER KEY

Physics

Q.1 B	Q.2 C	Q.3 A	Q.4 B	Q.5 B	Q.6 A	Q.7 B	Q.8 A
Q.9 C	Q.10 D	Q.11 A	Q.12 A	Q.13 B	Q.14 D	Q.15 B	Q.16 B
Q.17 D	Q.18 D	Q.19 A	Q.20 C	Q.21 A	Q.22 A	Q.23 C	Q.24 B
Q.25 B	Q.26 A	Q.27 D	Q.28 C	Q.29 C	Q.30 C		

Physics

Q.1 The difference in the acceleration due to gravity at the pole and equator is (g = acceleration due to gravity, R = radius of earth, θ = latitude, ω = angular velocity, cos 0° = 1, cos 90° = 0)

Correct option: (B)

Acceleration due to gravity at a place of latitude θ due to rotation of earth is

$g' = g - R\omega^2\cos^2\theta$

At equator, θ = 0°, cos 0 = 1

$\therefore \quad g' = g_e = g - R\omega^2$

At poles, θ = 90°, cos 90 = 0

$\therefore \quad g' = g_p = g$

$\therefore \quad g_p - g_e = g - (g - R\omega^2) = R\omega^2$

Q.2 Given mass of the moon is 1/81 of the mass of the earth and corresponding radius is 1/4 of the radius of earth. If escape velocity on the earth's surface is 11.2 km/s, the value of same on the surface of the moon is

Correct option: (C)

On earth, $v_e = \sqrt{\frac{2GM}{R}} = 11.2$ km/s

On moon, $v_m = \sqrt{\frac{2GM \times 4}{81 \times R}} = \frac{2}{9}\sqrt{\frac{2GM}{R}}$

$= \frac{2}{9} \times 11.2 = 2.5$ km/s

Q.3 Two planets have density in the ratio 2 : 3 and radii in the ratio 1 : 2. The ratio of acceleration due to gravity at their surface is

Correct option: (A) 1 : 3

$\frac{\rho_1}{\rho_2} = \frac{2}{3}, \frac{R_1}{R_2} = \frac{1}{2}$

$g \propto \rho R \Rightarrow g_1 \propto \rho_1 R_1$ and $g_2 \propto \rho_2 R_2$

$\therefore \frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} \times \frac{R_1}{R_2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

Q.4 Gravitational force between two objects separated by 40 cm is 2.5×10^{-9} N. If total mass of the two objects is 5.0 kg, then the mass of the objects in kg, are

Correct option: (B)

r = 40 cm = 40×10^{-2} m = 4×10^{-1} m, total mass = 5 kg

Let m and (5 – m) be the two masses

$F = \frac{G\, m_1\, m_2}{r^2}$

$\therefore 2.5 \times 10^{-9} = \frac{6.67 \times 10^{-11} \times m \times (5-m)}{(4\times10^{-1})^2}$

$\therefore 2.5 \times 10^{-9} = 6.67 \times \frac{m(5-m)}{16} \times 10^{-9}$

$\therefore 2.5 = \frac{40}{6} \times \frac{m(5-m)}{16}$

$\therefore m^2 - 5m + 6 = 0$

$\therefore (m - 2)(m - 3) = 0$

$\therefore m = 3$ or $m = 2$

Q.5 A satellite is moving very close to a planet of density 8×10^3 kg m^{-3}. If G = 6.67×10^{-11} Nm^2 kg^{-2}, then the time period of the satellite is nearly

Correct option: (B)

$T = \sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{3 \times 3.14}{6.67 \times 10^{-11} \times 8 \times 10^3}}$ s ≈

4200 s

Q.6 The ratio of energy required to raise a satellite to a height 'h' above the earth's surface to that required to put it into the orbit at the same height is (R = radius of earth)

Correct option: (A)

The formula for the energy required to raise a satellite to height h is

$$\mathbf{E}_1 = \Delta\mathbf{U} = \frac{\text{mgh}}{1 + \frac{\mathbf{h}}{\mathbf{R}}} = \frac{\text{mghR}}{\mathbf{R} + \mathbf{h}}$$

The formula for the energy required to set the satellite in orbit is

$$\mathbf{E}_2 = \frac{-\text{GMm}}{2(\mathbf{R} + \mathbf{h})} + \frac{\text{GMm}}{\mathbf{R}}$$

$$= \text{mgR}\left[1 - \frac{1}{2\left(1 + \frac{h}{R}\right)}\right] \quad \ldots(\because GM = gR^2)$$

$$\therefore \quad E_2 = \frac{\text{mgR}\left(\frac{2h}{R} + 1\right)}{2\left(1 + \frac{h}{R}\right)}$$

$$\therefore \frac{E_1}{E_2} = \frac{\text{mgh}}{1+\frac{h}{R}} \times \frac{2\left(1+\frac{h}{R}\right)}{\text{mgR}}$$

$$= \frac{2h}{R} \quad \ldots\left(\because h << R \Rightarrow 1+\frac{2h}{R} \approx 0\right)$$

Q.7 A revolving body of mass 'm' kg starts falling from a point '2R' above earth's surface. Its gain in kinetic energy when it has fallen to a point 'R' above the earth's surface is
(R = Radius of earth, M = Mass of earth)
(G = Universal gravitational constant)
Correct option: (B)

P. E. at h = 2R,

E_1 = (P. E.)i = $\dfrac{-GMm}{R+2R} = \dfrac{-GMm}{3R}$ …(i)

At h = R, when body is fallen, it will have P. E as well as K. E.

$\therefore$ E_2 = $(P. E.)_f$ + K. E.

$= \dfrac{-GMm}{R+R} + K.E.$ …(ii)

Applying law of conservation of energy.

$$\frac{-GMm}{3R} = \frac{-GMm}{2R} + K.E.$$

$$K.E. = \frac{-GMm}{3R} + \frac{GMm}{2R} = \frac{GMm}{6R}$$

Q.8 The mass of a spherical planet is 4 times the mass of the earth, but its radius (R) is same as that of the earth. How much work is done in lifting a body of mass 5 kg through a distance of 2 m on the planet? ($g = 10\ ms^{-2}$)
Correct option: (A)

We know,

G = $\dfrac{GM}{R^2}$

$\therefore$ g′ = $\dfrac{GM'}{R^2} = \dfrac{G \times 4M}{R^2}$ = 4 g

Work done W = Fh = mgh

$\therefore$ W = mg′h

= 4mgh = 4 × 5 × 10 × 2 = 400 J

Q.9 If the distance between the earth and sun becomes $1/4^{th}$, then its period of revolution around the sun will become
Correct option: (C)

$r_2 = \frac{1}{4} r_1$

$T \propto r^{\frac{3}{2}} \Rightarrow T_1 \propto r_1^{\frac{3}{2}}$ and $T_2 \propto r_2^{\frac{3}{2}}$

$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}} \Rightarrow T_2 = T_1\left(\frac{1}{4}\right)^{\frac{3}{2}}$

$\therefore T_2 = 365 \times 24 \times \frac{1}{\sqrt{64}} = \frac{365\times24}{8} = 1095$ hrs.

Q.10 Energy required to move a body of mass m from an orbit of radius 2R to 3R is
Correct option: (D)

Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right)$$

$$= \frac{GMm}{6R}$$

Q.11 The depth at which acceleration due to gravity becomes $\frac{g}{n}$ is (R = radius of earth, g = acceleration due to gravity) (n = integer)
Correct option: (A)

The gravitational acceleration at depth is given as

$$g_d = g\left[1 - \frac{d}{R}\right]$$

Given $g_d = \dfrac{g}{n}$

$$\therefore \frac{g}{n} = g\left[1 - \frac{d}{R}\right]$$

$$\frac{d}{R} = 1 - \frac{1}{n}$$

$$d = \left[\frac{n-1}{n}\right]R$$

Q.12 What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?
Correct option: (A)

Orbital energy, $E_0 = \dfrac{-GMm}{2(R+h)}$

$\therefore$ $E_0 = \dfrac{-GMm}{2(R+2R)} = \dfrac{-GMm}{6R}$ ….($\because$ h = 2R)

Energy at surface E = $\dfrac{-GMm}{R}$

$\therefore$ Min. energy required = $E_0 - E$

$$= \frac{-GMm}{6R} - \left(\frac{-GMm}{R}\right)$$

$= \frac{5\,GMm}{6R}$

Q.13 A satellite revolves around the earth in an elliptical orbit. Its speed

Correct option: (B)

According to the principle of **conservation of angular momentum** for a satellite orbiting a central body like the Earth, the angular momentum L is conserved. The angular momentum can be expressed as $L = mvr$ where m is the mass of the satellite, v is its speed, and r is its distance from the Earth. Since L and m are constant, the product vr must remain constant. This implies that speed v is *inversely proportional* to the distance r ($v \propto 1/r$). Therefore, when the satellite is closest to the Earth (i.e., r is minimum), its speed v will be **greatest**.

Q.14 A body of mass 'm' is raised to a height '10 R' from the surface of earth, where 'R' is the radius of earth. The increase in potential energy is

(G = universal constant of gravitation, M = mass of earth and g = acceleration due to gravity)

Correct option: (D)

When a particle of mass m is taken from the Earth's surface to a height h = nR, then the change in P.E. can be calculated as, $\Delta U = mgR\left(\frac{n}{n+1}\right)$

n = 10

$$\therefore\ \Delta U = mgR\left(\frac{10}{10+1}\right) = \frac{10mgR}{11}$$

As, $gR = \frac{GM}{R}$

$$\therefore\ \Delta U = \frac{10GMm}{11R}$$

Q.15 Radius of earth is equal to 6 × 10^6 m. Acceleration due to gravity is equal to 9.8 m/s^2. Gravitational constant G is equal to 6.67 × 10^{-11} Nm^2 kg^{-2}. Then mass of the earth is

Correct option: (B)

$$g = \frac{GM}{R^2}$$

$$\therefore M = \frac{gR^2}{G} = \frac{9.8 \times (6 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$\therefore M = \frac{9.8 \times 36}{6.67} \times 10^{23} = 52.89 \times 10^{23}\ \text{kg}$$

$$\therefore M \approx 5.3 \times 10^{24}\ \text{kg}$$

Q.16 The orbital speed of Jupiter is

Correct option: (B)

$$v_c = \sqrt{\frac{GM_S}{r}}$$

Orbital speed of all planets depends upon the mass of Sun and the separation. So,

$$v_c \propto \frac{1}{\sqrt{r}}$$

Since Jupiter is having more orbital radius in comparison to earth, so orbital speed of Jupiter is less than that of earth.

Q.17 If two identical satellites are at R and 7R away from earth surface, the wrong statement is

(R = Radius of earth)

Correct option: (D)

Orbital radius of satellites: $r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$$P.E_1 = \frac{-GMm}{r_1} \text{ and } P.E_2 = \frac{-GMm}{r_2}$$

$$K.E._1 = \frac{GMm}{2r_1} \text{ and } K.E_2 = \frac{GMm}{2r_2}$$

$$T.E_1 = \frac{-GMm}{2r_1} \text{ and } T.E_2 = \frac{-GMm}{2r_2}$$

$$\therefore \frac{P.E_1}{P.E_2} = \frac{K.E_1}{K.E_2} = \frac{T.E_1}{T.E_2} = 4$$

Q.18 Binding energy of a revolving satellite at height 'h' is 3.5×10^8 J. Its potential energy at that height is

Correct option: (D)

We know,

Potential energy = – 2(Binding energy)

$= -2 \times 3.5 \times 10^8\ \text{J} = -7.0 \times 10^8\ \text{J}$

Q.19 If the density of the earth is doubled by keeping its radius constant, then the acceleration due gravity on its surface will be

Correct option: (A)

We know,

$$g = \frac{GM}{R^2} \text{ and } M = \frac{4}{3}\pi R^3\rho$$

Keeping the radius constant and doubling the density, the mass M will be doubled, and hence g will be doubled.

$\therefore$ $g = 2 \times 9.8 = 19.6\ m/s^2$

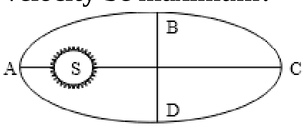
Q.20 The value of acceleration due to gravity at a depth of 1600 km is equal to

Correct option: (C)

$$g_d = g\left(1 - \frac{d}{R}\right) = 9.8\left(1 - \frac{1600}{6400}\right)$$

$$g_d = 9.8 \times \frac{3}{4} = 7.35\ ms^{-2}$$

Q.21 The earth rotates about the sun in an elliptical orbit. At which point will its velocity be maximum?



Correct option: (A)

From Kepler's second law of planetary motion, the velocity of a planet is maximum when its distance from sun is the least.

Q.22 The diameters of two planets are in the ratio $\sqrt{3}$: 2 and their mean densities are in the ratio 1 : 3. The acceleration due to gravity on the planets will be in the ratio

Correct option: (A)

$\frac{\rho_1}{\rho_2} = 1 : 3$, $\frac{d_1}{d_2} = \sqrt{3} : 2$

$g = \frac{4}{3}G\pi R\rho$

$\therefore \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{3} \times \frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{3}}$

Q.23 A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a satellite orbiting a few hundred kilometres above the earth's surface (R_{Earth} = 6400 km) will approximately be

Correct option: (C)

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}} \Rightarrow T_2 = 24\left(\frac{6400}{36000}\right)^{\frac{3}{2}} \approx 2 \text{ hour.}$$

Q.24 The force of attraction between any two particles in the universe is directly proportional to

Correct option: (B)

The force of attraction between two objects with mass is described by Newton's Law of Universal Gravitation. This law states that the force of gravity is directly proportional to the product of the masses of the two objects and inversely proportional to the square of the distance between their centers. Mathematically, this can be represented as: $F = G\frac{m_1 m_2}{r^2}$. Where: F is the force of gravity, G is the gravitational constant, m_1 and m_2 are the masses of the two objects, r is the distance between the centers of the two objects. Therefore, the correct answer is product of masses of these two particles.

Q.25 Two satellites revolve around the earth in circular orbits of radius R and 3R respectively. The ratio of the minimum period of revolution of the satellites will be

Correct option: (B)

From Kepler's third law of planetary motion,

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$$

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{R}{3R}\right]^3 \quad \ldots(\because r_1 = R \text{ and } r_2 = 3R)$$

$$\left[\frac{T_1}{T_2}\right]^2 = \frac{1}{27}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{1}{27}} = \frac{1}{3\sqrt{3}}$$

Q.26 Consider a light planet revolving around a massive star in a circular orbit of radius 'r' with time period 'T'. If the gravitational force of attraction between the planet and the star is proportional to $r^{-\frac{7}{2}}$, then T^2 is proportional to

Correct option: (A)

For the planet to orbit around the star, the centripetal force must be provided by gravitational force. Hence, $F_G = F_a$.

$F_a \propto - r^{-7/2}$ ....(Given)

(–ve sign indicates force is towards the centre of orbit)

Hence, $a \propto - r^{-7/2}$

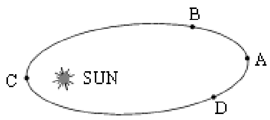
$\therefore -\omega^2 r \propto - r^{-7/2}$

$\therefore \omega^2 \propto r^{-9/2}$

$\therefore \frac{4\pi^2}{T^2} \propto r^{-9/2}$

$\Rightarrow T^2 \propto r^{9/2}$

Q.27 A planet is moving around the sun in an elliptical orbit at different positions A, B, C, D. The maximum rotational kinetic energy of a planet is at position

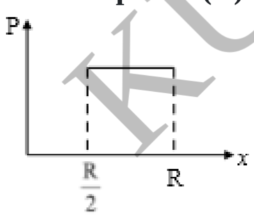


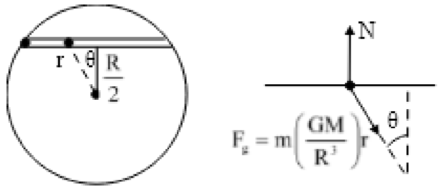
Correct option: (D)

From Kepler's second law of planetary motion, the velocity of a planet is maximum when its distance from Sun is the least.

Q.28 A tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{2}$ from the centre of earth. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on wall is depicted by the graph

Correct option: (C)





Pressing force = N

$$= \left(\frac{GMm}{R^3}\right) r \cos\theta$$

$$= \frac{GMm}{R^3} r \times \left(\frac{R/2}{r}\right)$$

$$= \frac{GMm}{2R^2} = \text{constant.}$$

Q.29 At a given place where acceleration due to gravity is 'g' m/s^2, a sphere of lead of density 'd' kg/m^3 is gently released in a column of liquid of density 'ρ' kg/m^3. If d > ρ, the sphere will

Correct option: (C) fall vertically with an acceleration g $\left(\frac{d-\rho}{d}\right)$.

Apparent weight = actual weight – upthrust force

$Vdg' = Vdg - V\rho g$

$\therefore g' = \left(\frac{d-\rho}{d}\right) g$

Q.30 If two satellites, identical in all aspects, are positioned at distances of 2R and 8R from the Earth's surface, the correct statement is as follows (R = Radius of earth)

Correct option: (C)

Orbital radius of satellites: $r_1 = R + 2R = 3R$

$r_2 = R + 8R = 9R$

$P.E_1 = \frac{-GMm}{r_1}$ and $P.E_2 = \frac{-GMm}{r_2}$

$K.E._1 = \frac{GMm}{2r_1}$ and $K.E_2 = \frac{GMm}{2r_2}$

$T.E_1 = \frac{-GMm}{2r_1}$ and $T.E_2 = \frac{-GMm}{2r_2}$

$\therefore \frac{P.E_1}{P.E_2} = \frac{K.E_1}{K.E_2} = \frac{T.E_1}{T.E_2} = 3$