Line and Plane and Vectors **Marks: 120**

ANSWER KEY

Maths

Q.1 A	Q.2 A	Q.3 A	Q.4 B	Q.5 D	Q.6 D	Q.7 B	Q.8 B
Q.9 C	Q.10 A	Q.11 D	Q.12 C	Q.13 C	Q.14 C	Q.15 A	Q.16 D
Q.17 A	Q.18 C	Q.19 C	Q.20 D	Q.21 B	Q.22 D	Q.23 B	Q.24 C
Q.25 A	Q.26 B	Q.27 A	Q.28 A	Q.29 A	Q.30 D	Q.31 B	Q.32 C
Q.33 A	Q.34 D	Q.35 A	Q.36 D	Q.37 B	Q.38 D	Q.39 A	Q.40 A
Q.41 D	Q.42 A	Q.43 D	Q.44 C	Q.45 B	Q.46 D	Q.47 C	Q.48 D
Q.49 D	Q.50 B	Q.51 A	Q.52 A	Q.53 C	Q.54 A	Q.55 D	Q.56 A
Q.57 B	Q.58 B	Q.59 A	Q.60 A				

Maths

Q.1 If the lines $\dfrac{x-2}{1}=\dfrac{y-4}{4}=\dfrac{z-6}{k}$ and $\dfrac{x+1}{3}=\dfrac{y+3}{5}=\dfrac{z+5}{7}$ are coplanar, then the value of k is

Correct option: (A)

The lines are coplanar

$$\therefore \begin{vmatrix} -1-2 & -3-4 & -5-6 \\ 1 & 4 & k \\ 3 & 5 & k \end{vmatrix} = 0$$

⇒ $-3(4k-5k)+7(k-3k)-11(-7)=0$

⇒ $k=7$

Q.2 If the vectors $\hat{i}+2\hat{j}+x\hat{k}$ and $y\hat{i}+6\hat{j}+4\hat{k}$ are collinear, then the values of *x* and *y* are respectively,

Correct option: (A)

Let $\bar{a}=\hat{i}+2\hat{j}+x\hat{k}$ and $\bar{b}=y\hat{i}+6\hat{j}+4\hat{k}$

As $\bar{a}$ and $\bar{b}$ are collinear vectors, we get

$\bar{a}=m\bar{b}$, where m is some real number

$\therefore\ \hat{i}+2\hat{j}+x\hat{k}=m\left(y\hat{i}+6\hat{j}+4\hat{k}\right)$

$\therefore\ \hat{i}+2\hat{j}+x\hat{k}=my\hat{i}+6m\hat{j}+4m\hat{k}$

$\therefore$ 1= my, 2 = 6m, x = 4m

⇒ m = $\dfrac{1}{3}$, y = 3, x = $\dfrac{4}{3}$

Q.3 The equation of line passing through (3, –1, 2) and perpendicular to the lines $\bar{r}$ = $\left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ and $\bar{r}$ = $\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ is

Correct option: (A)

Let a, b, c be the d.r.s. of the required line

d.r.s. of the given lines are 2, –2, 1 and 1, –2, 2.

$\therefore$ 2a – 2b + c = 0 …(i)

a – 2b + 2c = 0 …(ii)

$$\therefore\ \frac{a}{-4+2}=\frac{-b}{4-1}=\frac{c}{-4+2}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{-3}=\frac{c}{-2}$$

$\therefore$ Equation of the required line is

$$\frac{x-3}{-2}=\frac{y+1}{-3}=\frac{z-2}{-2}$$

$$\Rightarrow \frac{x-3}{2}=\frac{y+1}{3}=\frac{z-2}{2}$$

Q.4 If $\bar{a}=\dfrac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\bar{b}=\dfrac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$ then the value of $\left(2\bar{a}-\bar{b}\right)\cdot\left[\left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right]$=

Correct option: (B)

Here, $\bar{a}\cdot\bar{b}=0 and |\bar{a}|=\left|\bar{b}\right|=1$

$\therefore$ $\bar{a}$ and $\bar{b}$ are perpendicular unit vectors.

Now, $\left(2\bar{a}-\bar{b}\right)\cdot\left[\left(a\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right]$

$=\left[2\bar{a}-\bar{b}\bar{a}\times\bar{b}\bar{a}+2\bar{b}\right]$

$=-\left[\bar{a}\times\bar{b}2\bar{a}-\bar{b}\bar{a}+2\bar{b}\right]$

$=-\left(\bar{a}\times\bar{b}\right)\cdot\left\{\left(2\bar{a}-\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right\}$

$=-\left(\bar{a}\times\bar{b}\right).5\left(\bar{a}\times\bar{b}\right)$

$=-5\left|\bar{a}\times\bar{b}\right|^2$

$=-5\,|\bar{a}|^2\left|\bar{b}\right|^2$ …[$\because\bar{a}\perp\bar{b}$]

$=-5$

Q.5 If $\bar{a}$, $\bar{b}$, $\bar{c}$ are three vectors such that

$$\left|\bar{a}+\bar{b}+\bar{c}\right|=1,\bar{c}=\lambda\left(\bar{a}\times\bar{b}\right)\text{and}|\bar{a}|=\frac{1}{\sqrt{3}},\left|\bar{b}\right|=\frac{1}{\sqrt{2}},|\bar{c}|=\frac{1}{\sqrt{6}}$$

, then the angle between $\bar{a}$ and $\bar{b}$ is

Correct option: (D)

Let θ be the angle between $\bar{a}$ and $\bar{b}$

Since $\bar{c} = \lambda \left(\bar{a} \times \bar{b}\right)$

⇒ $\bar{c} \perp \bar{a}, \bar{c} \perp \bar{b}$

⇒ $\bar{c}.\bar{a} = \bar{c}.\bar{b} = 0$

Now,

$\left|\bar{a} + \bar{b} + \bar{c}\right| = 1$

⇒ $\left|\bar{a} + \bar{b} + \bar{c}\right|^2 = 1$

⇒

$|\bar{a}|^2 + \left|\bar{b}\right|^2 + |\bar{c}|^2 + 2\left(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}\right) = 1$

⇒ $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} + 2\left\{|\bar{a}|\left|\bar{b}\right| \cos\theta\right\} = 1$

⇒ cos θ = 0

⇒ θ = $\frac{\pi}{2}$

Q.6 The shortest distance between the lines $\bar{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ **and** $\bar{r} = (\mathrm{p}+1)\hat{i} + (2p-1)\hat{j} + (2\mathrm{p}+1)\hat{k}$ **is**

Correct option: (D)

Given lines can be written as

$\bar{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ and

$\bar{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + p\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

Shortest distance between lines $\bar{r}_1 = \bar{a}_1 + \lambda\bar{b}_1$ and $\bar{r}_2 = \bar{a}_2 + \lambda\bar{b}_2$ is

$$\left|\frac{(\bar{a}_2 - \bar{a}_1)\cdot\left(\bar{b}_1 \times \bar{b}_2\right)}{\left|\bar{b}_1 \times \bar{b}_2\right|}\right|$$

Here, $\bar{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \bar{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$,

$\bar{a}_2 = \hat{i} - \hat{j} + \hat{k}, \bar{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

Now, $\bar{a}_2 - \bar{a}_1$ = $\left(\hat{i} - \hat{j} + \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$

=$\hat{j} - 2\hat{k}$,

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & 2 \end{vmatrix}$$

= $\hat{i}(2+4) - \hat{j}(0) - 3\hat{k}$

= $6\hat{i} - 3\hat{k}$,

$\left|\bar{b}_1 \times \bar{b}_2\right|$ = $\sqrt{36+9}$ = $3\sqrt{5}$

∴ Shortest distance =

$$\left|\frac{\left(\hat{j} - 2\hat{k}\right)\cdot\left(6\hat{i} - 3\hat{k}\right)}{3\sqrt{5}}\right|$$

= $\frac{6}{3\sqrt{5}}$ = $\frac{2}{\sqrt{5}}$

Q.7 If the vectors $\bar{a}, \bar{b}, \bar{c}$ **are non-coplanar and *l*, m, n are distinct scalars such that**

$\left[l\bar{a} + m\bar{b} + n\bar{c} \quad l\bar{b} + m\bar{c} + n\bar{a} \quad l\bar{c} + m\bar{a} + n\bar{b}\right] = 0,$

then

Correct option: (B)

Given,

$\left[l\bar{a} + m\bar{b} + n\bar{c} \quad l\bar{b} + m\bar{c} + n\bar{a} \quad l\bar{c} + m\bar{a} + n\bar{b}\right] = 0$

⇒

$\left[l\bar{a} + m\bar{b} + n\bar{c} \;\; n\bar{a} + l\bar{b} + m\bar{c} \;\; m\bar{a} + n\bar{b} + l\bar{c}\right] = 0$

⇒ $\begin{vmatrix} l & m & n \\ n & l & m \\ m & n & l \end{vmatrix}\left[\bar{a}\;\bar{b}\;\bar{c}\right] = 0$

⇒ $\begin{vmatrix} l & m & n \\ n & l & m \\ m & n & l \end{vmatrix} = 0$ .... $\left[\because \;\left[\bar{a}\;\bar{b}\;\bar{c}\right] \neq 0\right]$

⇒ $l^3 + m^3 + n^3 - 3lmn = 0$

⇒ $(l + m + n)(l^2 + m^2 + n^2 - lm - mn - nl) = 0$

⇒ $l + m + n = 0$

Q.8 If $\bar{a} = 2\hat{i} + \hat{j} + \hat{k}$ **and** $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$**, then the magnitude of the vector** $\left(\bar{a} + \bar{b}\right)$ **is**

Correct option: (B)

$\bar{a}+\bar{b} = \left(2\hat{i}+\hat{j}+\hat{k}\right)+\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$= 3\hat{i}+3\hat{j}+4\hat{k}$

$\therefore |\bar{a}+\bar{b}| = \sqrt{3^2+3^2+4^2}$

$= \sqrt{9+9+16} = \sqrt{34}$

Q.9 **Equation of the plane, through the points $(-1,2,-2)$ and $(-1,3,2)$ and perpendicular to *y*z - plane, is**

Correct option: (C)

Equation of plane passing though (–1, 2, –2) and (–1, 3, 2) is

$$\frac{x+1}{(-1+1)} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

Above plane is perpendicular to *y*z – plane

$$\therefore \frac{y-3}{-1} = \frac{z-2}{-4}$$

⇒ 4(*y* – 3) = z – 2

⇒ 4*y* – 12 – z + 2 = 0

⇒ 4*y* – z = 10

Q.10 **The vector equation of the line passing through the points (1, –2, 5) and (– 2, 1, 3) is**

Correct option: (A)

Let $\bar{a} = -2\hat{i}+\hat{j}+3\hat{k}$ and $\bar{b} = \hat{i}-2\hat{j}+5\hat{k}$

$\therefore \bar{b}-\bar{a} = 3\hat{i}-3\hat{j}+2\hat{k}$

The vector equation of the line is

$\bar{r} = \bar{a}+\lambda\left(\bar{b}-\bar{a}\right)$

$\bar{r} = -2\hat{i}+\hat{j}+3\hat{k}+\lambda(3\hat{i}-3\hat{j}+2\hat{k})$

Q.11 **The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is**

Correct option: (D)

Here, $(x_1, y_1, z_1) = (3, 8, 3)$,

$(x_2, y_2, z_2) = (-3, -7, 6)$,

$(a_1, b_1, c_1) = (3, -1, 1)$

$(a_2, b_2, c_2) = (-3, 2, 4)$

∴ Shortest distance d is given as

d =

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(a_1c_2-a_2c_1)^2+(b_1c_2-b_2c_1)^2}}\right|$$

$$\therefore \text{ d} = \frac{\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(3)^2+(15)^2+(-6)^2}}$$

$= \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$ units

Q.12 **The value of a so that the volume of parallelopiped formed by vectors $\hat{i}$+ a $\hat{j}$ + $\hat{k}$, $\hat{j}$+ a $\hat{k}$, a $\hat{i}$ + $\hat{k}$ becomes minimum is**

Correct option: (C)

Volume of the parallelopiped formed by vectors is

i.e., $V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 - a + a^3$

$\therefore \frac{dV}{da} = -1 + 3a^2$, $\frac{d^2V}{da^2} = 6a$

For max. or min. of V, $\frac{dV}{da} = 0$

$\therefore a^2 = \frac{1}{3}$

$\therefore a = \frac{1}{\sqrt{3}}$

$\frac{d^2V}{da^2} = 6a > 0$ for $a = \frac{1}{\sqrt{3}}$

∴ V is minimum for $a = \frac{1}{\sqrt{3}}$

Q.13 **Let $\bar{a} = \hat{i} + \hat{j}$ and $\bar{b} = 2\hat{i} - \hat{k}$, then the point of intersection of the lines $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$ and $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$ is**

Correct option: (C)

Let $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$\bar{r} \times \bar{a} = \bar{b} \times \bar{a} \Rightarrow (\bar{r} - \bar{b}) \times \bar{a} = 0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x-2 & y & z+1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow (-z-1)\hat{i} - (-z-1)\hat{j} + (x-y-2)\hat{k} = 0$

$\Rightarrow z = -1,\ x - y = 2$ …(i)

Now, $\bar{r} \times \bar{b} = \bar{a} \times \bar{b} \Rightarrow (\bar{r} - \bar{a}) \times \bar{b} = 0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x-1 & y-1 & z \\ 2 & 0 & -1 \end{vmatrix} = 0$$

$\Rightarrow (1-y)\hat{i} - (1-x-2z)\hat{j} + (2-2y)\hat{k} = 0$

$\Rightarrow y = 1, x + 2z = 1$ …(ii)

Solving (i) and (ii), we get

$x = 3, y = 1, z = -1$

Q.14 **The lines** $\dfrac{6x-6}{18} = \dfrac{y+1}{3} = \dfrac{z-1}{5}$ **and** $\dfrac{3x+6}{12} = \dfrac{y-1}{3} = \dfrac{z+1}{2}$ **are ...**

Correct option: (C)

The given lines are

$$\frac{6x-6}{18} = \frac{y+1}{3} = \frac{z-1}{5}$$

$$\Rightarrow \frac{x-1}{3} = \frac{y+1}{3} = \frac{z-1}{5}$$

and $\dfrac{3x+6}{12} = \dfrac{y-1}{3} = \dfrac{z+1}{2}$

$$\Rightarrow \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{2}$$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -3 & 2 & -2 \\ 3 & 3 & 5 \\ 4 & 3 & 2 \end{vmatrix}$$

$= -3(-9) - 2(-14) - 2(-3)$

$= 27 + 28 + 6$

$= 61 \neq 0$

$\therefore$ The given lines are not intersecting.

Q.15 **If three points A, B and C have co-ordinates (1, *x*, 3), (3, 4, 7) and (*y*, –2, –5) respectively. They are collinear, then *x*, *y* =**

Correct option: (A)

Here $\bar{a} = \hat{i} + x\hat{j} + 3\hat{k}, \bar{b} = 3\hat{i} + 4\hat{j} + 7\hat{k}$,

and $\bar{c} = y\hat{i} - 2\hat{j} - 5\hat{k}$

$\therefore \overline{AB} = \lambda\overline{BC}$

$\Rightarrow 2\hat{i} + (4-x)\hat{j} + 4\hat{k} =$

$\lambda\left[(y-3)\hat{i} - 6\hat{j} - 12\hat{k}\right]$

On comparing, we get

$4 = -12\lambda \Rightarrow \lambda = \dfrac{-1}{3}$,

$4 - x = -6\lambda \Rightarrow x = 2$, and

$2 = \lambda(y-3) \Rightarrow -6 = y - 3 \Rightarrow y = -3$

Q.16 **If origin is the centroid of the triangle and position vector of vertices of the triangle are** $p\bar{i} + \bar{j} + 3\bar{k}$**,** $-2\bar{i} + q\bar{j} - 5\bar{k}$ **and** $4\bar{i} + 7\bar{j} + r\bar{k}$**, then p, q, r are _______.**

Correct option: (D)

As origin is the centroid of the triangle

$0\bar{i} + 0\bar{j} + 0\bar{k} =$

$$\frac{(p\bar{i} + \bar{j} + 3\bar{k}) + (-2\bar{i} + q\bar{j} - 5\bar{k}) + (4\bar{i} + 7\bar{j} + r\bar{k})}{3}$$

$\therefore$ Equating the coefficients of $\bar{i}, \bar{j}, \bar{k}$ on both the sides,

$0 = \dfrac{p-2+4}{3}$, $0 = \dfrac{1+q+7}{3}$, $0 = \dfrac{3-5+r}{3}$

$\therefore p = -2, q = -8, r = 2$.

Q.17 **The vector equation of the line passing through** $\hat{i} - \hat{j} + 3\hat{k}$ **and parallel to** $3\hat{i} + 2\hat{j} - 5\hat{k}$ **is**

Correct option: (A)

Vector equation of line passing through $\bar{a}$ and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

$\therefore \bar{r} = (\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 5\hat{k})$

Q.18 **If** $\bar{a}, \bar{b}, \bar{c}$ **are position vectors of points A, B, C respectively, with** $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$**, then the ratio in which point C divides segment AB is**

Correct option: (C)

$2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$

$$\Rightarrow \bar{c} = \frac{2\bar{a} + 3\bar{b}}{5}$$

$$\Rightarrow \bar{c} = \frac{3\bar{b} + 2\bar{a}}{3+2}$$

$\therefore$ Point C divides segment AB internally in the ratio 3 : 2.

Q.19 If $|\bar{a}|$= 3, $|\bar{b}|$= 4, $|\bar{a}-\bar{b}|$ = 5, then $|\bar{a}+\bar{b}|$ =

Correct option: (C)

$$\left|\bar{a}-\bar{b}\right|^2 = |\bar{a}|^2 + \left|\bar{b}\right|^2 - 2\bar{a}\cdot\bar{b}$$

$\Rightarrow (5)^2 = (3)^2 + (4)^2 - 2\bar{a}\cdot\bar{b}$

$\Rightarrow \bar{a}\cdot\bar{b} = 0$

Now, $\left|\bar{a}+\bar{b}\right|^2 = \left|\bar{a}-\bar{b}\right|^2 + 4\bar{a}\cdot\bar{b}$

$\Rightarrow \left|\bar{a}+\bar{b}\right|^2 = \left|\bar{a}-\bar{b}\right|^2$

$\Rightarrow \left|\bar{a}+\bar{b}\right| = 5$

Q.20 $\bar{a}\cdot[(\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})]$ is equal to

Correct option: (D)

$\bar{a}\cdot[(\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})]$

$= \bar{a}\cdot(\bar{b}\times\bar{a}+\bar{b}\times\bar{b}+\bar{b}\times\bar{c}) + \bar{a}\cdot(\bar{c}\times\bar{a}+\bar{c}\times\bar{b}+\bar{c}\times\bar{c})$

$= [\bar{a}\ \bar{b}\ \bar{a}]+[\bar{a}\ \bar{b}\ \bar{b}]+[\bar{a}\ \bar{b}\ \bar{c}]+[\bar{a}\ \bar{c}\ \bar{a}]+[\bar{a}\ \bar{c}\ \bar{b}]+[\bar{a}\ \bar{c}\ \bar{c}]$

$= \bar{0}+\bar{0}+[\bar{a}\ \bar{b}\ \bar{c}]+\bar{0}-[\bar{a}\ \bar{b}\ \bar{c}]+\bar{0}=\bar{0}$

Q.21 A line L_1 passes through the point, whose p. v. (position vector) $3\hat{i}$, is parallel to the vector $-\hat{i}+\hat{j}+\hat{k}$.

Another line L_2 passes through the point having p.v. $\hat{i}+\hat{j}$ is parallel to vector $\hat{i}+\hat{k}$, then the point of intersection of lines L_1 and L_2 has p.v.

Correct option: (B)

Equation of line L_1 is $\bar{r}$ =

$3\hat{i} + \lambda\left(-\hat{i}+\hat{j}+\hat{k}\right)$ … (i)

Equation of line L_2 is $\overline{r'} = \hat{i}+\hat{j}+\lambda'\left(\hat{i}+\hat{k}\right)$

The point of intersection of L_1 and L_2 will satisfy $\bar{r} = \overline{r'}$

$\Rightarrow 3\hat{i} + \lambda\left(-\hat{i}+\hat{j}+\hat{k}\right) =$

$\hat{i}+\hat{j}+\lambda'\left(\hat{i}+\hat{k}\right)$

$\Rightarrow (3-\lambda)\hat{i}+\lambda\hat{j}+\lambda\hat{k} =$

$\left(1+\lambda'\right)\hat{i}+\hat{j}+\lambda'\hat{k}$

$\Rightarrow 3 - \lambda = 1 + \lambda'$ and $\lambda = 1$

$\Rightarrow \lambda = 1$ and $\lambda' = 1$

Substituting the value of λ in (i), we get the point of intersection.

$\therefore$ The point of intersection of lines L_1 and L_2 has p.v. $2\hat{i}+\hat{j}+\hat{k}$.

Q.22 The value of k such that the line $\dfrac{x-4}{1} = \dfrac{y-2}{1} = \dfrac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$ is

Correct option: (D)

line $\dfrac{x-4}{1} = \dfrac{y-2}{1} = \dfrac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$.

$\therefore$ Point (4, 2, k) lies on the plane $2x - 4y + z = 7$

$\therefore 2(4) - 4(2) + k = 7$

$\Rightarrow k = 7$

Q.23 If two vertices of a triangle are A(3, 1, 4) and B(– 4, 5, –3) and the centroid of the triangle is G(–1, 2, 1), then the third vertex C of the triangle is

Correct option: (B)

Let $\bar{a}, \bar{b}, \bar{c}$ and $\bar{g}$ be the position vectors of A, B, C and G respectively.

$\bar{a} = 3\hat{i}+1\hat{j}+4\hat{k}$,

$\bar{b} = -4\hat{i}+5\hat{j}-3\hat{k}$,

$\bar{g} = -\hat{i}+2\hat{j}+\hat{k}$,

G is centroid of ΔABC.

$\therefore\ \bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

$3\bar{g} = \bar{a}+\bar{b}+\bar{c}$

$3\left(-\hat{i}+2\hat{j}+\hat{k}\right) =$

$3\hat{i}+\hat{j}+4\hat{k}-4\hat{i}+5\hat{j}-3\hat{k}+\bar{c}$

$\therefore\ \bar{c} =$

$-3\hat{i}+6\hat{j}+3\hat{k}-3\hat{i}-\hat{j}-4\hat{k}+4\hat{i}-5\hat{j}+3\hat{k}$

$= -2\hat{i} + 0\hat{j} + 2\hat{k}$

∴ Third vertex C ≡ (–2, 0, 2)

Q.24 The vector of magnitude 6 units and perpendicular to vectors $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is

Correct option: (C)

Let the required vector be $\bar{r}$ = . $x\hat{i} + y\hat{j} + z\hat{k}$.

Then, $|\bar{r}| = 6$

$\Rightarrow x^2 + y^2 + z^2 = 36$ ….(i)

Now, $\bar{r}$ is perpendicular to vectors

$\bar{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\bar{b} = \hat{i} - 2\hat{j} + \hat{k}$.

$$\therefore \bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{vmatrix} = -5\left(\hat{i} + \hat{j} + \hat{k}\right)$$

∴ $x = y = z$

Let $x = y = z = \lambda$

From (i), we get

$3\lambda^2 = 36$

∴ $\lambda = 2\sqrt{3}$

∴ Required vector is $2\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Q.25 The plane passing through the point (5, 1, 2) perpendicular to the line $2(x - 2) = y - 4 = z - 5$ will meet the line at the point

Correct option: (A)

The required plane is perpendicular to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$ (say)

the d.r.s of normal to the plane are proportional to 1, 2, 2

∴ Equation of the plane is

$x + 2y + 2z + d = 0$ ...(i)

Since it passes through the point (5, 1, 2), we have

(5) + 2(1) + 2(2) + d = 0

⇒ d = – 11

∴ The equation (i) becomes $x + 2y + 2z - 11 = 0$

Any general point on the given line is given by

λ + 2, 2λ + 4, 2λ + 5.

This point lies in the required plane

∴ λ + 2 + 2(2λ + 4) + 2(2λ + 5) – 11 = 0

⇒ λ + 2 + 4λ + 8 + 4λ + 10 – 11 = 0

⇒ 9λ + 9 = 0 ⇒ λ = – 1

∴ The point of intersection is

[(–1) + 2, 2(–1) + 4, 2(–1)+5]

⇒ (1, 2, 3)

Q.26 Two adjacent sides of a parallelogram ABCD are given by $\overline{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overline{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then cos α =

Correct option: (B)

Let θ be the angle between $\overline{AB}$ and $\overline{AD}$.

$$\therefore \cos\theta = \frac{\overline{AB} \cdot \overline{AD}}{\left|\overline{AB}\right|\left|\overline{AD}\right|}$$

$$= \frac{\left(2\hat{i} + 10\hat{j} + 11\hat{k}\right) \cdot \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)}{\sqrt{4 + 100 + 121}\sqrt{1 + 4 + 4}}$$

$$= \frac{-2 + 20 + 22}{\sqrt{225}\sqrt{9}}$$

$$= \frac{40}{45}$$

$$= \frac{8}{9}$$

$$\therefore \sin\theta = \sqrt{1 - \left(\frac{8}{9}\right)^2} = \frac{\sqrt{17}}{9}$$

α is the angle of rotation of AD.

∴ The angle between side AB and AD

= 90° …[Given]

⇒ α + θ = 90°

⇒ α = 90° – θ

⇒ cos α = cos (90° – θ)

⇒ cos α = sin θ

$\Rightarrow \cos\alpha = \frac{\sqrt{17}}{9}$

Q.27 The values of x for which the angle between the vectors $\bar{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\bar{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, are

Correct option: (A)

Let θ be the angle between vectors $\bar{a}$ and $\bar{b}$.

We have $\bar{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$and

$\bar{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$

$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta$

∴ $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} < 0$ …[∵ θ is obtuse]

∴ $\bar{a} \cdot \bar{b} < 0$

⇒ $(2x^2)(7) + (4x)(-2) + (1)(x) < 0$

⇒ $14x^2 - 7x < 0$

⇒ $7x(2x - 1) < 0$

⇒ $x(2x - 1) < 0$

⇒ $0 < x < \dfrac{1}{2}$

Q.28 If the line passing through the points (a, 1, 6) and (3, 4, b) crosses the yz – plane at the point $\left(0, \dfrac{17}{2}, \dfrac{-13}{2}\right)$, then

Correct option: (A)

The equation of the line passing through (a, 1, 6) and (3, 4, b) is

$$\frac{x-a}{3-a} = \frac{y-1}{3} = \frac{z-6}{b-6}$$

It crosses the yz-plane at $\left(0, \dfrac{17}{2}, -\dfrac{13}{2}\right)$. This means that this point lies on the line

∴ $\dfrac{-a}{3-a} = \dfrac{\frac{17}{2} - 1}{3} = \dfrac{-\frac{13}{2} - 6}{b-6}$

⇒ 2a = 5(a – 3) and b – 6 = –5

⇒ a = 5 and b = 1

Q.29 The XZ plane divides the line segment joining the points (3, 2, b) and (a, –4, 3) in the ratio

Correct option: (A) 1 : 2

Let the XZ plane divides the line segment joining the given points in the ratio k : 1 at the point P (x, y, z).

∴ $x = \frac{ka + 3}{k + 1}$, $y = \frac{-4k + 2}{k + 1}$

z = $\frac{3k + b}{k + 1}$

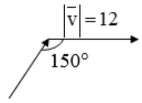
Since P (x, y, z) lie on the XZ plane, its y co-ordinate will be zero.

∴ $0 = \frac{-4k + 2}{k + 1}$

⇒ –4k + 2 = 0

⇒ k = $\frac{1}{2}$

∴ k : 1 = 1 : 2

Q.30 If $|\bar{u}|$ = 8  **then $|\bar{u} \times \bar{v}|$ is**

Correct option: (D)

$|\bar{u} \times \bar{v}| = |\bar{u}||\bar{v}|\sin(150°)$

$= 8 \times 12 \times \dfrac{1}{2}$

= 48

Q.31 If $|\bar{a}| = 2, |\bar{b}| = 3$ and $\bar{a}, \bar{b}$ are mutually perpendicular vectors, then the area of the triangle whose vertices are 0, a + 2b, a – 2b is

Correct option: (B)

Let position vectors of A, B, C be

0, a + 2b, a – 2b

Area of ΔABC = $\dfrac{1}{2}\left|\vec{AB} \times \vec{AC}\right|$

$= \dfrac{1}{2}\left|\left(\bar{a} + 2\bar{b}\right)\left(\bar{a} - 2\bar{b}\right)\right|$

$= \dfrac{1}{2}\left|\bar{a} \times \bar{a} - \bar{a} \times 2\bar{b} + 2\bar{b} \times \bar{a} - 2\bar{b} \times \bar{b}\right|$

$= \dfrac{1}{2}\left|2\bar{b} \times \bar{a} + 2\bar{b} \times \bar{a}\right|$

$= \dfrac{1}{2} \times 4\left|\bar{b} \times \bar{a}\right|$

= 2 × 2 × 3 = 12 sq. units

Q.32 The value of x, if $x(\hat{i} + \hat{j} + \hat{k})$is a unit vector, is

Correct option: (C)

$x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

∴ $\left|x(\hat{i} + \hat{j} + \hat{k})\right| = 1$

⇒ $|x|\sqrt{3} = 1$

⇒ $|x| = \dfrac{1}{\sqrt{3}}$

⇒ $x = \pm\dfrac{1}{\sqrt{3}}$

Q.33 The equation of plane passing through (1, 2,–1) and containing the lines whose direction ratios are 2, 1, 3 and 4, 1, 2

Correct option: (A)

Let $(x_1, y_1, z_1) = (1, 2, -1)$,

$a_1, b_1, c_1 = 2, 1, 3$ and $a_2, b_2, c_2 = 4, 1, 2$

∴ the equation of required plane is

$$\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 1 & 3 \\ 4 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(-2) + (y-2)(10) + (z+1)(-2) = 0$

$\Rightarrow -2x + 2 + 10y - 20 - 2z - 2 = 0$

$\Rightarrow x - 5y + z + 10 = 0$

Q.34 Let $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + \hat{j}$ and $\bar{c}$ **be a vector such that** $|\bar{c} - \bar{a}| = 4, \left|\left(\bar{a} \times \bar{b}\right) \times \bar{c}\right| = 3$ **, and the angle between** $\bar{c}$ and $\bar{a} \times \bar{b}$ is $\frac{\pi}{6}$ **, then** $\bar{a}.\bar{c}$ **is equal to**

Correct option: (D)

$\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + \hat{j}$

$\therefore \quad |\bar{a}| = \sqrt{4+1+4} = 3$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$\therefore \quad \left|\bar{a} \times \bar{b}\right| = \sqrt{4+4+1} = 3$

Angle between $\bar{c}$ and $\bar{a} \times \bar{b}$ is $\frac{\pi}{6}$ …[Given]

$$\therefore \quad \sin\frac{\pi}{6} = \frac{\left|\left(\bar{a} \times \bar{b}\right) \times \bar{c}\right|}{\left|\bar{a} \times \bar{b}\right||\bar{c}|}$$

$\frac{1}{2} = \frac{3}{3 \times |\bar{c}|} \Rightarrow |\bar{c}| = 2$

Now, $|\bar{c} - \bar{a}| = 4$ …[Given]

$\Rightarrow |\bar{c}|^2 + |\bar{a}|^2 - 2\bar{a}.\bar{c} = 16$

$\Rightarrow 4 + 9 - 2\, a \cdot c = 16$

$\Rightarrow a \cdot c = \frac{-3}{2}$

Q.35 Find the equation of the perpendicular drawn from the point (2, 4, –1) to the line

$$x + 5 = \frac{1}{4}(y + 3) = -\frac{1}{9}(z - 6)$$

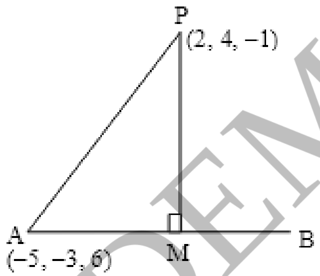
and obtain the co-ordinates of the foot of this perpendicular

Correct option: (A)

Any point on the line

$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$ is given by

$M \equiv (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$.



The d.r.s. of PM are

$\lambda - 7, 4\lambda - 7, -9\lambda + 7$

Since, PM is perpendicular to AM,

$\therefore 1(\lambda - 7) + 4(4\lambda - 7) - 9(-9\lambda + 7) = 0$

$\Rightarrow 98\lambda - 98 = 0 \Rightarrow \lambda = 1$

$\therefore M = (-4, 1, -3)$

Now, Equation of perpendicular passing through P(2, 4, –1) and M(–4, 1, –3) is

$$\frac{x-2}{-4-2} = \frac{y-4}{1-4} = \frac{z+1}{-3+1}$$

$$\Rightarrow \frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

Q.36 The co-ordinates of the point where the line joining the points $(2, -3, 1)$ **and** $(3, -4, -5)$ **and intersects the plane** $2x + y + z = 7$ **are**

Correct option: (D)

The equation of the line passing through the points (2, –3, 1) and (3, –4, –5) is

$$\frac{x-2}{3-2} = \frac{y+3}{-4+3} = \frac{z-1}{-5-1}$$

$$\Rightarrow \frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = t(\text{say})$$

$\Rightarrow x = t + 2, y = -t - 3, z = -6t + 1$

Substituting the values of x, y, z in the given equation of plane, we get

$2(t + 2) + (-t - 3) + (-6t + 1) = 7$

$\Rightarrow 2 - 5t = 7$

$\Rightarrow 5t = -5$

$\Rightarrow t = -1$

$\therefore\ x = -1 + 2 = 1,$

$y = -(-1) - 3 = -2$

$z = -6(-1) + 1 = 7$

$\therefore\ (x, y, z) = (1, -2, 7)$

Q.37 If $\bar{a}$ and $\bar{b}$ be two unit vectors and θ is the angle between them, then $\left|\bar{a}-\bar{b}\right|$ is equal to

Correct option: (B)

$$\left|\bar{a}-\bar{b}\right|^2 = \left(\bar{a}-\bar{b}\right)\cdot\left(\bar{a}-\bar{b}\right)$$

$$= \bar{a}\cdot\bar{a} - \bar{a}\cdot\bar{b} - b\cdot\bar{a} + \bar{b}\cdot\bar{b}$$

$$= 1 - 2\,\bar{a}\cdot\bar{b} + 1$$

$$\ldots.\left[\because |\bar{a}| = \left|\bar{b}\right| = 1\right]$$

$$= 2 - 2.1.1.\cos\theta = 2(1 - \cos\theta)$$

$$= 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$= 4\sin^2\frac{\theta}{2}$$

$$\therefore \left|\bar{a}-\bar{b}\right| = 2\sin\frac{\theta}{2}$$

Q.38 The direction cosines of normal to the plane

$$\bar{r}\,.\left(2\hat{i}-3\hat{j}+\hat{k}\right)+9=0$$

Correct option: (D)

Given equation of plane is

$$\bar{r}\,.\left(2\hat{i}-3\hat{j}+\hat{k}\right)+9=0$$

$$\Rightarrow \bar{r}\,.\left(2\hat{i}-3\hat{j}+\hat{k}\right)=-9 \ \ldots\text{(i)}$$

$$\overline{n} = 2\hat{i}-3\hat{j}+\hat{k}$$

$$\therefore \hat{n} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{4+9+1}} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}$$

∴ The d.c.s. of normal to the plane are

$$\frac{2}{\sqrt{14}},\ \frac{-3}{\sqrt{14}},\ \frac{1}{\sqrt{14}}$$

Q.39 The value of α, so that the volume of the parallelopiped formed by $\hat{i}+\alpha\hat{j}+\hat{k}$, $\hat{j}+\alpha\hat{k}$ and $\alpha\hat{i}+\hat{k}$ becomes maximum, is

Correct option: (A)

Volume of parallelopiped is $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\therefore\ V = \begin{vmatrix}1 & \alpha & 1\\ 0 & 1 & \alpha\\ \alpha & 0 & 1\end{vmatrix}$$

$$= 1 - \alpha(-\alpha^2) - \alpha$$

$$= 1 + \alpha^3 - \alpha$$

Differentiating w.r.t. α, we get

$$\frac{dV}{d\alpha} = 3\alpha^2 - 1$$

$$\therefore \frac{d^2V}{d\alpha^2} = 6\alpha$$

Let $\frac{dV}{d\alpha} = 0$

$\therefore\ 3\alpha^2 - 1 = 0$

$\therefore\ \alpha = \pm\frac{1}{\sqrt{3}}$

at $\alpha = \frac{-1}{\sqrt{3}}$,

$$\frac{d^2V}{d\alpha^2} = \frac{-6}{\sqrt{3}} < 0$$

$\therefore$ V is maximum at $\alpha = \frac{-1}{\sqrt{3}}$.

Q.40 Let $\bar{a} = 3\hat{i}+2\hat{j}+x\hat{k}$ and $\bar{b} = \hat{i}-\hat{j}+\hat{k}$, for some real *x*. Then $\left|\bar{a}\times\bar{b}\right|$ = r is possible, if

Correct option: (A)

$$\bar{a}\times\bar{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$$

$$= \hat{i}(2+x) - \hat{j}(3-x) + \hat{k}(-3-2)$$

$= (x+2)\hat{i} + (x-3)\hat{j} - 5\hat{k}$

$\therefore \ |\bar{a} \times \bar{b}| = \sqrt{(x+2)^2 + (x-3)^2 + (-5)^2}$

$= \sqrt{2x^2 - 2x + 38}$

$= \sqrt{2(x^2 - x + 19)}$

$= \sqrt{2\left(x^2 - x + \frac{1}{4} + \frac{75}{4}\right)}$

$= \sqrt{2\left[\left(x - \frac{1}{2}\right)^2\right] + \frac{75}{2}}$ = r …[Given]

Now, $2\left(x - \frac{1}{2}\right)^2 \geq 0$

⇒ r ≥ $\sqrt{\frac{75}{2}}$ i.e. r ≥ $5\sqrt{\frac{3}{2}}$

Q.41 Let $\bar{a} = -\hat{i} - \hat{k}$, $\bar{b} = -\hat{i} + \hat{j}$ and $\bar{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\bar{r}$ is a vector such that $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$ and $\bar{r}.\bar{a} = 0$, then the value of $\bar{r}.\bar{b}$ is

Correct option: (D)

Given, $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$

⇒ $(\bar{r} - \bar{c}) \times \bar{b} = \bar{0}$

∴ $\bar{r} - \bar{c}$ is parallel to $\bar{b}$

⇒ $\bar{r} - \bar{c} = \lambda\bar{b}$ for some scalar λ

⇒ $\bar{r} = \bar{c} + \lambda\bar{b}$ …(i)

⇒ $\bar{r}.\bar{a} = \bar{c}.\bar{a} + \lambda\left(\bar{b} \cdot \bar{a}\right)$

⇒ $0 = \bar{c}.\bar{a} + \lambda\left(\bar{b} \cdot \bar{a}\right)$ ….$[\because \ \bar{r} \cdot \bar{a} = 0\,(given)]$

⇒ $\lambda = -\frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}$

Substituting the value of λ in (i), we get

$\bar{r} = \bar{c} - \frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\bar{b}$

⇒ $\bar{r}.\bar{b} = \bar{c}.\bar{b} - \frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}(\bar{b}.\bar{b})$

⇒ $\bar{r}.\bar{b} = 1 - \frac{(-4)}{1} \times 2 = 9$

Q.42 The equation of the plane passing through A(–2, 2, 2), B(2, –2, –2) and perpendicular to $x + 2y - 3z = 7$ is

Correct option: (A)

Equation of any plane passing through (–2, 2, 2) is

a(x + 2) + b(y – 2) + c(z – 2) = 0 ...(i)

Also, plane (i) passes through (2, –2, –2).

∴ a(4) + b(–4) + c(–4) = 0

⇒ a – b – c = 0 ...(ii)

Plane (i) is perpendicular to $x + 2y - 3z = 7$.

∴ a + 2b – 3c = 0 ...(iii)

From (ii) and (iii), we get

a = 5, b = 2, c = 3

Substituting the values of a, b, c in (i), we get

$5x + 2y + 3z = 0$

Q.43 The intercept made by the plane $2x + y - z = 5$ on X-axis is

Correct option: (D)

$2x + y - z = 5$

⇒ $\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$

⇒ $\frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{5} + \frac{z}{-5} = 1$

∴ The intercept made by the given plane on X-axis is $\frac{5}{2}$.

Q.44 If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, then $\lambda + 1 =$

Correct option: (C)

The d.r.s of line and plane are 1, 2, 2 and $2, -1, \sqrt{\lambda}$

∴ sin θ =

$$\frac{2(1) + (-1)(2) + \left(\sqrt{\lambda}\right)(2)}{\sqrt{1^2 + 2^2 + 2^2} \cdot \sqrt{2^2 + (-1)^2 + \left(\sqrt{\lambda}\right)^2}}$$

⇒ $\frac{1}{3} = \frac{2\sqrt{\lambda}}{3 \cdot \sqrt{5 + \lambda}}$

⇒ $\sqrt{5 + \lambda} = 2\sqrt{\lambda}$

⇒ $\lambda = \frac{5}{3}$

⇒ $\lambda + 1 = \frac{8}{3}$

Q.45 Equation of the plane passing through points (1, – 1, 3) and (2, 3, – 4) and parallel to X-axis is

Correct option: (B)

The plane passes through (1, –1, 3) and (2, 3 –4)

The points satisfies the equation of plane in option [B]

∴ option [B] is correct answer.

Alternate method:

Let ax + by + cz + d = 0 be the equation of the required plane.

Since, the plane is parallel to X-axis,

∴ a = 0

The points (1, –1, 3) and (2, 3, – 4) lie in the plane,

∴ – b + 3c + d = 0, and …(i)

3b – 4c + d = 0 …(ii)

Solving the equations (i) and (ii), we get

$$\frac{b}{3-(-4)} = \frac{c}{3+1} = \frac{d}{4-9}$$

$$\Rightarrow \frac{b}{7} = \frac{c}{4} = \frac{d}{-5}$$

∴ Equation of the required plane is 7y + 4z – 5 = 0

Q.46 For A(1, –2, 4), B(5, –1, 7), C(3, 6, –2), D(4, 5, –1), the projection of $\overline{AB}$ on $\overline{CD}$ is _______.

Correct option: (D)

$\overline{AB} = 4\hat{i} + \hat{j} + 3\hat{k}, \overline{CD} = \hat{i} - \hat{j} + \hat{k}$

Projection of $\overline{AB}$ on $\overline{CD}$ = $\left(\frac{\overline{AB}\cdot\overline{CD}}{\left|\overline{CD}\right|}\right) \cdot \hat{c}$

$\ldots\left[\hat{c} \text{ is unit vector along } \overline{CD}\right]$

$$= \left(\frac{4-1+3}{\sqrt{3}}\right)\left(\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}\right)$$

$$= 2\hat{i} - 2\hat{j} + 2\hat{k}$$

Q.47 If $\bar{a}$, $\bar{b}$, $\bar{c}$ are position vectors of points A, B, C respectively, with $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then the ratio in which point C divides segment AB is

Correct option: (C)

$$2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$$

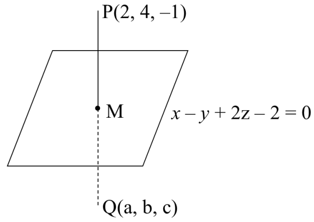
$$\Rightarrow \bar{c} = \frac{2\bar{a} + 3\bar{b}}{5}$$

$$\Rightarrow \bar{c} = \frac{3\bar{b} + 2\bar{a}}{3+2}$$

∴ Point C divides segment AB internally in the ratio 3:2.

Q.48 The mirror image of P(2, 4, –1) in the plane $x - y$ + 2z – 2 = 0 is (a, b, c), then the value of a + b + c is

Correct option: (D)



The d.r.s. of the normal to the plane are 1, –1, 2.

∴ The equation of line PM is

$$\frac{x-2}{1} = \frac{y-4}{-1} = \frac{z+1}{2} = \lambda(\text{say})$$

⇒ x = λ + 2, y = –λ + 4, z = 2λ – 1

Let M ≡ (λ + 2, –λ + 4, 2λ – 1)

∴ Equation of plane becomes

1(λ + 2) –1(–λ + 4) + 2(2λ – 1) – 2 = 0

⇒ λ = 1

∴ M ≡ (3, 3, 1)

Since M is the mid-point of PQ.

∴ $\frac{2+a}{2} = 3, \frac{4+b}{2} = 3, \frac{-1+c}{2} = 1$

⇒ a = 4, b = 2, c = 3

⇒ a + b + c = 9

Q.49 If $\bar{a} + 2\bar{b} + 3\bar{c} = 0$ and $\left(\bar{a}\times\bar{b}\right) + \left(\bar{b}\times\bar{c}\right) + (\bar{c}\times\bar{a}) = \lambda\left(\bar{b}\times\bar{c}\right)$, then λ has the value

Correct option: (D)

$\bar{a} + 2\bar{b} + 3\bar{c} = 0 \Rightarrow \bar{a} = -2\bar{b} - 3\bar{c}$ …(i)

$$\left(\bar{a}\times\bar{b}\right) + \left(\bar{b}\times\bar{c}\right) + (\bar{c}\times\bar{a}) = \lambda\left(\bar{b}\times\bar{c}\right)$$

$\Rightarrow (\lambda-1)\left(\bar{b}\times\bar{c}\right) = \left(\bar{a}\times\bar{b}\right) + (\bar{c}\times\bar{a})$ …(ii)

$= \left(\bar{a}\times\bar{b}\right) - (\bar{a}\times\bar{c})$ $\ldots\left[\because \bar{a}\times\bar{b} = -\left(\bar{b}\times\bar{a}\right)\right]$

$= \bar{a}\times\left(\bar{b}-\bar{c}\right)$

$= \left(-2\bar{b}-3\bar{c}\right)\times\left(\bar{b}-\bar{c}\right)$ …[From (i)]

$= 2\left(\bar{b} \times \bar{c}\right) - 3\left(\bar{c} \times \bar{b}\right)$

$= 2\left(\bar{b} \times \bar{c}\right) + 3\left(\bar{b} \times \bar{c}\right)$

$= 5\left(\bar{b} \times \bar{c}\right)$

From (ii), we get

$\lambda - 1 = 5$

$\Rightarrow \lambda = 6$

Q.50 The vectors $\overline{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overline{BC} = -\hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is

Correct option: (B)

Let $\overline{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\bar{b} = -\hat{i} - 2\hat{k}$

∴ The diagonals d_1 and d_2 are $\overline{a} + \bar{b}$ and $\overline{a} - \bar{b}$ respectively.

$d_1 = (3\hat{i} - 2\hat{j} + 2\hat{k}) + (-\hat{i} - 2\hat{k})$

$= 2\hat{i} - 2\hat{j}$

$d_2 = 4\hat{i} - 2\hat{j} + 4\hat{k}$

$\therefore \cos\theta = \frac{\overline{d_1}.\overline{d_2}}{\left|\overline{d_1}\right|\left|\overline{d_2}\right|} = \frac{(2\hat{i} - 2\hat{j}).(4\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{8}.\sqrt{36}}$

$= \frac{12}{12\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45°$

Q.51 The value of k for which the planes $3x - 6y - 2z = 7$ and $2x + y - kz = 5$ are perpendicular to each other is

Correct option: (A)

For perpendicular planes,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (3)(2) + (-6)(1) + (-2)(-k) = 0$

$\Rightarrow 6 - 6 + 2k = 0$

$\Rightarrow k = 0$

Q.52 If (1, 2, –3) is the foot of the perpendicular drawn from origin on a plane, then equation of that plane is

Correct option: (A)

The plane passes through (1, 2, –3)

This point satisfies the equation of plane in option [A]

Also, it has d.r.s. 1, 2, –3.

∴ option [A] is correct answer.

Alternate method:

Let M (1, 2, –3) be the foot of perpendicular from the origin O (0, 0, 0) to the plane D. r. s of normal are

1, 2, –3

∴ the equation of the required plane is

$1(x - 1) + 2(y - 2) - 3(z + 3) = 0$

$\Rightarrow x + 2y - 3z = 14$

Q.53 The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

Correct option: (C)

The equation of given lines are

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$a_1a_2 + b_1b_2 + c_1c_2 = 3(2) + 2(-12) + (-6)(-3)$

$= 0$

∴ Lines are perpendicular

$\therefore \theta = 90°$

Q.54 The distance of the point (5, 3, – 1) from the plane passing through points (2, 1, 0), (3, –2, 4) and (1, – 3, 3) is

Correct option: (A)

Equation of the plane passing through (2, 1, 0), (3, –2, 4) and (1, –3, 3) is

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & 4-0 \\ 1-2 & -3-1 & 3-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & 4 \\ -1 & -4 & 3 \end{vmatrix} = 0$$

$\Rightarrow (x - 2)(7) - (y - 1)(7) + z(-7) = 0$

$\Rightarrow x - 2 - y + 1 - z = 0$

$\Rightarrow x - y - z - 1 = 0$

∴ The distance of this plane from (5, 3 –1) is

$d = \left|\frac{5 - 3 + 1 - 1}{\sqrt{1 + 1 + 1}}\right| = \frac{2}{\sqrt{3}}$ units

Q.55 Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\bar{c} = \hat{a} + 2\hat{b}$ and $\bar{d} = 5\hat{a} + 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is

Correct option: (D)

Let θ be the angle between $\hat{a}$ and $\hat{b}$.

Since $\bar{c} = \hat{a} + 2\hat{b}$ and $\overline{d} = 5\hat{a} + 4\hat{b}$ are perpendicular to each other.

$\therefore\ \bar{c} \cdot \overline{d} = 0$

$\Rightarrow \left(\hat{a} + 2\hat{b}\right) \cdot \left(5\hat{a} + 4\hat{b}\right) = 0$

$\Rightarrow 5(\hat{a} \cdot \hat{a}) + 14\left(\hat{a} \cdot \hat{b}\right) + 8\left(\hat{b} \cdot \hat{b}\right) = 0$

$\Rightarrow 5|\hat{a}|^2 + 14|\hat{a}|\left|\hat{b}\right| \cos\theta + 8\left|\hat{b}\right|^2 = 0$

⇒ 5 + 14cosθ + 8 = 0

⇒ cos θ = $-\frac{13}{14}$

⇒ θ = $\cos^{-1}\left(-\frac{13}{14}\right)$

Q.56 If the Cartesian equation of a line is 6*x* – 2 = 3*y* + 1 = 2z – 2, then the vector equation of the line is

Correct option: (A)

Given Cartesian equation of the line is

6*x* – 2 = 3*y* + 1 = 2z – 2

⇒ $6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right)$ = 2(z – 1)

$\Rightarrow \frac{x - \frac{1}{3}}{\frac{1}{6}} = \frac{y + \frac{1}{3}}{\frac{1}{3}} = \frac{z - 1}{\frac{1}{2}}$

$\Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z - 1}{3}$

∴ The given line passes through $\left(\frac{1}{3}, \frac{-1}{3}, 1\right)$

and has direction ratios proportional to 1, 2, 3.

∴ Vector equation is

$\bar{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Q.57 Let the line $\frac{x-1}{4} = \frac{y+2}{3} = \frac{z-4}{2}$ lie in the plane 3*x* + 2*y* – αz + β = 0. Then, (α, β) equals

Correct option: (B)

Point (1, –2, 4) lies in the plane 3*x* + 2*y* – αz + β = 0

∴ 3(1) +2(–2) – α(4) + β = 0

⇒ 4α – β = –1 …(i)

Also, the d.r.s of the normal are perpendicular to the given plane.

∴ 3(4) +2(3) – α(2) = 0

⇒ 12 + 6 – 2 α = 0

⇒ α = 9

Substituting value of α in equation (i), we get

β = 37

Q.58 The acute angle between the line $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$ and the plane $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right)$= 5

Correct option: (B)

Comparing the equations of line and plane with $\bar{r} = \bar{a} + \lambda\bar{b}$ and $\bar{r} \cdot \bar{n} = p$, we get $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{n} = 2\hat{i} - \hat{j} + \hat{k}$

∴ The angle between the line and plane is given by

sin θ = $\left|\frac{\bar{b} \cdot \bar{n}}{\left|\bar{b}\right||\bar{n}|}\right|$

$= \left|\frac{(1)(2) + (1)(-1) + (1)(1)}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + (-1)^2 + (1)^2}}\right| =$

$\frac{2}{3\sqrt{2}}$

θ = $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Q.59 Let θ be the angle between the lines AB and AC, where A, B and C are the three points with co-ordinates (1, 2, –1), (2, 0, 3), (3, –1, 2) respectively, then $\sqrt{462}$ cosθ is equal to

Correct option: (A)

Given, A ≡ (1, 2, –1), B ≡ (2, 0, 3), C ≡ (3, –1, 2)

The d.r.s of AB = 1, –2, 4 and d.r.s of AC = 2, –3, 3

∴ cosθ = $\left|\frac{1(2) + (-2)(-3) + 4(3)}{\sqrt{1+4+16}\sqrt{4+9+9}}\right|$

⇒ cosθ = $\frac{2+6+12}{\sqrt{21}\sqrt{22}} = \frac{20}{\sqrt{462}}$

⇒ $\sqrt{462}$ cosθ = 20

Q.60 **If $\overline{a}, \bar{b}, \bar{c}$ are three coplanar vectors such that $|\overline{a}| = 1, \left|\bar{b}\right| = 2, \bar{b} \cdot \bar{c} = 8$ and the angle between $\bar{b}$ and $\bar{c}$ is 45° then the value of $\left|\overline{a} \times \left(\bar{b} \times \bar{c}\right)\right|$ is**

Correct option: (A)

$\bar{b} \cdot \bar{c} = 8$

$\Rightarrow \left|\bar{b}\right||\bar{c}| \cos 45° = 8$

$\Rightarrow (2)\ |\bar{c}|\left(\frac{1}{\sqrt{2}}\right) = 8$

$\Rightarrow |\bar{c}| = 4\sqrt{2}$

Since $\overline{a}, \bar{b}, \bar{c}$ are coplanar.

$\Rightarrow \overline{a} \cdot \left(\bar{b} \times \bar{c}\right) = 0$

$\Rightarrow \overline{a} \perp \left(\bar{b} \times \bar{c}\right)$

$\left|\overline{a} \times \left(\bar{b} \times \bar{c}\right)\right|$

$= |\overline{a}|\left|\bar{b} \times \bar{c}\right| \sin 90°$

$= (1) \left|\bar{b} \times \bar{c}\right|(1)$

$= \left|\bar{b}\right||\bar{c}|\sin 45°$

$= 2\left(4\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= 8$