Straight Lines and Pair of straight lines **Marks: 160**

ANSWER KEY

Maths

Q.1 D	Q.2 A	Q.3 D	Q.4 D	Q.5 D	Q.6 D	Q.7 B	Q.8 B
Q.9 C	Q.10 D	Q.11 B	Q.12 A	Q.13 D	Q.14 A	Q.15 B	Q.16 B
Q.17 D	Q.18 A	Q.19 D	Q.20 B	Q.21 D	Q.22 D	Q.23 D	Q.24 A
Q.25 B	Q.26 B	Q.27 C	Q.28 A	Q.29 D	Q.30 C	Q.31 B	Q.32 D
Q.33 A	Q.34 B	Q.35 B	Q.36 A	Q.37 C	Q.38 C	Q.39 B	Q.40 A
Q.41 B	Q.42 A	Q.43 A	Q.44 D	Q.45 D	Q.46 D	Q.47 C	Q.48 D
Q.49 B	Q.50 A	Q.51 B	Q.52 C	Q.53 D	Q.54 B	Q.55 C	Q.56 C
Q.57 C	Q.58 D	Q.59 B	Q.60 C	Q.61 A	Q.62 C	Q.63 B	Q.64 C
Q.65 C	Q.66 B	Q.67 B	Q.68 D	Q.69 C	Q.70 D	Q.71 D	Q.72 C
Q.73 A	Q.74 B	Q.75 C	Q.76 D	Q.77 C	Q.78 B	Q.79 A	Q.80 D

Maths

Q.1 The equation of a line passing through the point (7, –4) and perpendicular to the line passing through the points (2, 3) and (1, –2) is

Correct option: (D)

Slope of line through (2, 3) and (1, –2) is $\frac{-2-3}{1-2} = 5$

Slope of required line will be $-\frac{1}{5}$

Equation of line passing through (7, –4) and having slope $-\frac{1}{5}$ is

$(y + 4) = -\frac{1}{5}(x - 7) \Rightarrow 5y + 20 = -x + 7$

i.e., $x + 5y + 13 = 0$

Q.2 Angle between straight lines given by $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ is:

Correct option: (A)

Given equation of pair of lines is

$3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$

∴ a = –3, b = 3

Now, a + b = –3 + 3 = 0,

∴ The lines are perpendicular to each other.

Q.3 The points A(–a, –b), B(0, 0), C(a, b) and D(a^2, ab) are

Correct option: (D)

$AB = \sqrt{(0+a)^2 + (0+b)^2} = \sqrt{a^2+b^2}$

$BC = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

$CD = \sqrt{(a^2-a)^2 + (\text{ab}-b)^2}$

$= \sqrt{[a(a-1)]^2 + [b(a-1)]^2}$

$= \sqrt{a^2(a-1)^2 + b^2(a-1)^2}$

$= \sqrt{(a^2+b^2)(a-1)^2}$

$= (a-1)\sqrt{a^2+b^2}$

$AD = \sqrt{(a^2+a) + (\text{ab}+\text{b})^2}$

$= (a+1)\sqrt{a^2+b^2}$

Now, AB + BC + CD

$= \sqrt{a^2+b^2} + \sqrt{a^2+b^2} + (a-1)\sqrt{a^2+b^2}$

$= (a+1)\sqrt{a^2+b^2}$

= AD

Hence, the points are collinear.

Q.4 The number of integral values of p in the domain [–5, 5], such that the equation $2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$ represents pair of lines, are

Correct option: (D)

Given equation of pair of lines is

$2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$

Comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

a = 2, h = 2, b = –p

If the given equation represents a pair of straight lines, then

$h^2 \geq ab$

$\Rightarrow 4 \geq -2p$

$\Rightarrow 2 \geq -p$

$\Rightarrow p \geq -2$

∴ Possible values of p from domain [–5, 5] are –2, –1, 0, 1, 2, 3, 4, 5.

∴ Number of integral values of p = 8

Q.5 If line $qx - py + r = 0$ is perpendicular to one of the lines represented by $ax^2 + 2hxy + by^2 = 0$, then

Correct option: (D)

If one of the lines $ax^2 + 2hxy + by^2 = 0$ is perpendicular to $lx + my + n = 0$, then $al^2 + 2hlm + bm^2 = 0$.

∴ If one of the lines $ax^2 + 2hxy + by^2 = 0$ is perpendicular to $qx - py + r = 0$, then $aq^2 - 2hqp + bp^2 = 0$

Q.6 Equation of pair of straight lines drawn through (1, 1) and perpendicular to the pair of lines $3x^2 - 7xy + 2y^2 = 0$ is

Correct option: (D)

The joint equation of the lines through the point (x_1, y_1) and at right angles to the lines $ax^2 + 2hxy + by^2 = 0$ is

$b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$

∴ joint equation of pair of lines drawn through (1, 1) and perpendicular to the pair of lines $3x^2 - 7xy + 2y^2 = 0$ is

$2(x - 1)^2 + 7(x - 1)(y - 1) + 3(y - 1)^2 = 0$

Q.7 The equation $xy + a^2 = a(x + y)$ represents

Correct option: (B)

Given equation of pair of lines is

$xy + a^2 = ax + ay$

i.e. $ax + ay - xy - a^2 = 0$

∴ $A = 0, B = 0, C = -a^2, F = \frac{a}{2}, G = \frac{a}{2}, H = -\frac{1}{2}$

Now, $ABC + 2FGH - AF^2 - BG^2 - CH^2$

$= 0 - 2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{-1}{2}\right) - (a^2)\left(\frac{-1}{2}\right)^2 = 0$

∴ the given equation represents a pair of straight lines.

Q.8 What is the equation of the locus of a point which moves such that 4 times its distance from the X-axis is the square of its distance from the origin?

Correct option: (B)

Let the point be (x_1, y_1).

According to the given condition,

$4|y_1| = (x_1^2 + y_1^2)$

$\Rightarrow x_1^2 + y_1^2 - 4|y_1| = 0$

∴ equation of the locus is

$x^2 + y^2 - 4|y| = 0$

Q.9 The joint equation of two lines through the origin each making an angle of 30° with the Y-axis is

Correct option: (C)

Slopes of the required lines are $m_1 = \sqrt{3}, m_2 = -\sqrt{3}$

∴ Required lines are $\left(y - \sqrt{3}x\right)\left(y + \sqrt{3}x\right) = 0$

$\Rightarrow 3x^2 - y^2 = 0$

Q.10 Which pair of points lie on the same side of $3x - 8y - 7 = 0$?

Correct option: (D)

Consider option [D],

$L_{(-1, -1)} = 3(-1) - 8(-1) - 7 < 0$

and $L_{(3, 7)} = 3 \times 3 - 8 \times 7 - 7 < 0$

Hence, (– 1, – 1) and (3, 7) lie on the same side of line.

Q.11 The joint equation of the lines passing through the origin and having slopes 3 and $-\frac{1}{3}$ is

Correct option: (B)

Given, $m_1 = 3, m_2 = -\frac{1}{3}$

The joint equation of the pair of lines having slopes m_1, m_2 and passing through the origin is

$y^2 - (m_1 + m_2)xy + m_1 m_2 x^2 = 0$

∴ $y^2 - \left(\frac{8}{3}\right)xy - x^2 = 0$

∴ $3x^2 + 8xy - 3y^2 = 0$

Q.12 If the line $ay^2 + bxy + ex + dy = 0$ represents a pair of lines then

Correct option: (A)

Given equation of pair of lines is

$ay^2 + bxy + ex + dy = 0$

∴ $A = 0, B = a, C = 0, F = \frac{d}{2}, G = \frac{e}{2}, H = \frac{b}{2}$

$$\therefore \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & \frac{b}{2} & \frac{e}{2} \\ \frac{b}{2} & a & \frac{d}{2} \\ \frac{e}{2} & \frac{d}{2} & 0 \end{vmatrix} = 0$$

$$\Rightarrow 0\left(0 - \frac{d^2}{4}\right) - \frac{b}{2}\left(0 - \frac{de}{4}\right) + \frac{e}{2}\left(\frac{bd}{4} - \frac{ae}{2}\right) = 0$$

$$\Rightarrow \frac{bde}{8} + \frac{bde}{8} - \frac{ae^2}{4} = 0$$

$\Rightarrow bde - ae^2 = 0$

⇒ e(bd – ae) = 0

⇒ bd – ae = 0 or e = 0

Q.13 The auxiliary equation of the lines passing through the origin and having slopes $\sqrt{3}$ + 1 and $\sqrt{3}$ – 1 is

Correct option: (D)

Joint equation of pair of lines having slopes m_1, m_2 and passing through the origin is $y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$

∴ equation of the lines passing through the origin and having slopes $\sqrt{3}$ + 1 and $\sqrt{3}$ – 1 is

$$y^2 - \left[\left(\sqrt{3}+1\right)+\left(\sqrt{3}-1\right)\right] xy + \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) x^2 = 0$$

∴ $y^2 - 2\sqrt{3}xy + 2x^2 = 0$

Dividing both sides by x^2 and substituting m = $\frac{y}{x}$, we get required auxiliary equation

i.e., $m^2 - 2\sqrt{3}\,m + 2 = 0$

Q.14 The acute angle included between the lines x sin θ – y cos θ = 5 and x sin α – y cos α + 11 = 0 is

Correct option: (A)

Slope of the line x sin θ – y cos θ = 5 is

$m = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Slope of the line x sin α – y cos α + 11 = 0 is

$m = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

Let β be the angle between the given lines

∴ $\tan\beta = \left|\frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha}\right| \Rightarrow \tan\beta = \tan(\theta - \alpha)$

β = |θ – α|

Q.15 The line $\frac{x}{4}+\frac{y}{3}=1$ meets the X-axis and Y-axis at points A and B respectively. A square ABCD is constructed on the segment AB away from the origin. The co-ordinates of the vertex of the square farthest from the origin are

Correct option: (B)

A ≡ (4, 0), B ≡ (0, 3)

∴ OA = 4, OB = 3

∴ Side of square (AB) = $\sqrt{4^2+3^2} = \sqrt{25} = 5$

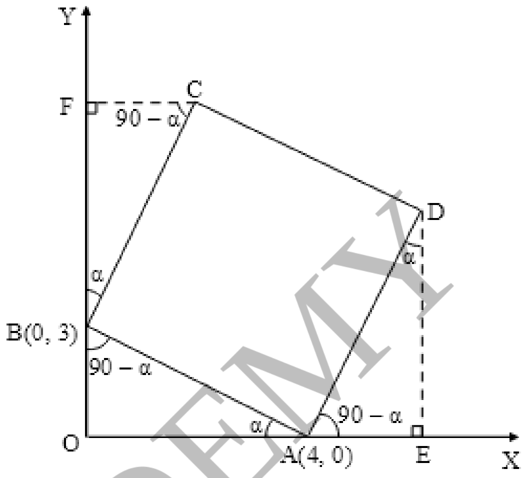
Let ∠OAB = α

∴∠OBA = 90 – α

From the geometry of figure,

∠BCF = 90 – α and ∠ADE = α

Also sin α = $\frac{3}{5}$, cos α = $\frac{4}{5}$



Co-ordinates of C are

(FC, OF) ≡ (5 sin α, 3 + 5 cos α)

≡ $\left(5\times\frac{3}{5}\ ,\ 3+5\times\frac{4}{5}\right)$

≡ (3, 7)

Co-ordinates of D are

(OE, DE) ≡ (4 + AE, DE)

≡ (4 + 5 sin α, 5 cos α)

≡ (7, 4)

Now, OC = $\sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$

OD = $\sqrt{7^2+4^2} = \sqrt{49+16} = \sqrt{65}$

∴ D is farthest from the origin.

Q.16 If ax^2 + 6xy + by^2 – 10x + 10y – 6 = 0 represents two perpendicular lines, then |a| equals

Correct option: (B)

Given equation of pair of lines is

$ax^2 + 6xy + by^2 - 10x + 10y - 6 = 0$

A = a, B = b, C= –6, F = –5, G = 5, H = 3

The lines are perpendicular

∴ a + b = 0 ⇒ a = –b

Also these lines satisfy the condition

$ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$

⇒ $6a^2 + 2(-75) - 25a + 25a + 54 = 0$

⇒ $6a^2 - 96 = 0 \Rightarrow a^2 - 16 = 0 \Rightarrow a = \pm 4$

Q.17 The combined equation of the lines which pass through the origin and each of which makes an angle of 30° with the line 2x – y = 0 is

Correct option: (D)

Given line $2x - y = 0 \Rightarrow$ Slope = 2

Let the slope of required line be m

$\therefore \tan 30° = \left|\dfrac{m-2}{1+2m}\right|$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \left|\dfrac{m-2}{1+2m}\right|$

$\Rightarrow m^2 + 16m - 11 = 0$ …(i)

Since, the line passes through origin, its equation is

$y = mx \Rightarrow m = \dfrac{y}{x}$

Substituting the value of m in equation (i), we get

$\left(\dfrac{y}{x}\right)^2 + 16\left(\dfrac{y}{x}\right) - 11 = 0$

$\Rightarrow 11x^2 - 16xy - y^2 = 0$

Q.18 In a triangle ABC, A ≡ (3,1), B ≡ (2, –5). If centroid of the triangle lies on the locus $y = 3 + 2x^2$, then equation of locus of the third vertex C is

Correct option: (A)

Let (x, y) be the co-ordinate of point C and G (a,b) be the centroid.

Since, G lies on the locus $y = 3 + 2x^2$

$\therefore b = 3 + 2a^2$ ….(i)

Also, a =

$\dfrac{3+2+x}{3} = \dfrac{x+5}{3},\ b = \dfrac{1-5+y}{3} = \dfrac{y-4}{3}$

Substituting the values of a and b in (i), we get

$\dfrac{y-4}{3} = 3 + 2\left(\dfrac{x+5}{3}\right)^2$

$\Rightarrow 3y - 12 = 27 + 2x^2 + 20x + 50$

$\Rightarrow 2x^2 + 20x - 3y + 89 = 0$

Q.19 If the slopes of lines given by the equation $ax^2 + 2hxy + by^2 = 0$ are in the ratio 5 : 3, then the ratio h^2 : ab

Correct option: (D)

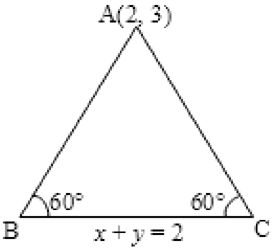
Given that $m_1 : m_2 = 5 : 3$

If the slopes of the lines given by $ax^2 + 2hxy + by^2 = 0$ are in the ratio m:n, then $(m + n)^2\, ab = 4mnh^2$.

$\therefore\ \dfrac{h^2}{ab} = \dfrac{(5+3)^2}{4\times5\times3} = \dfrac{64}{60} = \dfrac{16}{15}$

Q.20 The vertex of an equilateral triangle is (2, 3) and the side opposite to it is $x + y = 2$. The equations of the other sides of the triangle are

Correct option: (B)



Slope of $x + y = 2$ is $m = -1$

Let the slope of AC be m_1.

$\therefore \tan 60° = \left|\frac{-1-m_1}{1-m_1}\right|$

$\Rightarrow \sqrt{3} = \left|\frac{-1-m_1}{1-m_1}\right|$

$\Rightarrow -1 - m_1 = \pm\sqrt{3}\,(1 - m_1)$

$\Rightarrow -1 - m_1 = \sqrt{3} - \sqrt{3}\,m_1$ or $-1 - m_1 = -\sqrt{3} + \sqrt{3}\,m_1$

$\Rightarrow m_1 - \sqrt{3}m_1 = -\left(\sqrt{3}+1\right)$ or $m_1 + \sqrt{3}m_1 = \sqrt{3} - 1$

$\Rightarrow m_1 = \frac{-\left(\sqrt{3}+1\right)}{1-\sqrt{3}}$ or $m_1 = \frac{\sqrt{3}-1}{1+\sqrt{3}}$

$\Rightarrow m_1 = \frac{-\left(\sqrt{3}+1\right)^2}{1-3}$ or $m_1 = \frac{\left(\sqrt{3}-1\right)^2}{3-1}$

$\Rightarrow m_1 = 2 + \sqrt{3}$ or $m_1 = 2 - \sqrt{3}$

$\therefore$ The equations of other sides are

$y - 3 = \left(2+\sqrt{3}\right)(x - 2)$ and $y - 3 = \left(2-\sqrt{3}\right)(x - 2)$

$\Rightarrow \left(2+\sqrt{3}\right)x - y = 1 + 2\sqrt{3}$ and $\left(2-\sqrt{3}\right)x - y = 1 - 2\sqrt{3}$

Q.21 If $4ab = 3h^2$, then the ratio of slopes of the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ will be

Correct option: (D)

Here, $m_1 + m_2 = \dfrac{-2h}{b}$ ...(i)

and $m_1m_2 = \dfrac{a}{b}$

$\therefore (m_1 - m_2)^2 = (m_1+m_2)^2 - 4m_1m_2$

$= \left(\frac{-2h}{b}\right)^2 - 4\left(\frac{a}{b}\right)$

$= \dfrac{4h^2 - 4ab}{b^2}$

$= \dfrac{4h^2 - 3h^2}{b^2}$ …[∵ $4ab = 3h^2$ (given)]

$= \frac{h^2}{b^2}$

$\therefore m_1 - m_2 = \frac{h}{b}$ ...(ii)

By solving (i) and (ii), we get

$m_1 = \frac{-h}{2b}$ and $m_2 = \frac{-3h}{2b}$

$\therefore m_1 : m_2 = 1 : 3$

Q.22 If slope of one of the lines $ax^2 + 2hxy + by^2 = 0$ is 5 times that of the other, then $5h^2$ =

Correct option: (D)

Given equation of pair of lines is

$ax^2 + 2hxy + by^2 = 0$

Given that $m_1 = 5m_2$

$\therefore m_1 + m_2 = 5m_2 + m_2 = \frac{-2h}{b}$

$\Rightarrow m_2 = \frac{-h}{3b} \Rightarrow m_2^2 = \frac{h^2}{9b^2}$ ...(i)

$m_1m_2 = (5m_2)m_2 = \frac{a}{b}$

$\therefore m_2^2 = \frac{a}{5b}$ ...(ii)

$\therefore$ From (i) and (ii), we get

$5h^2 = 9ab$

Q.23 The joint equation of the pair of lines passing through A(1, 1) and which are parallel to the co-ordinate axes is

Correct option: (D)

Joint equation of co-ordinate axes is $xy = 0$.

Joint equation of the pair of lines passing through the point (x_1, y_1) and parallel to the lines given by $ax^2 + 2hxy + by^2 = 0$ is:

$a(x - x_1)^2 + 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$

$\therefore$ The joint equation of pair of lines passing through the point (1, 1) and parallel to the lines given by $xy = 0$ is:

$0(x-1)^2 + 1(x-1)(y-1) + 0(y-1)^2 = 0$

i.e., $xy - x - y + 1 = 0$

Q.24 If lines represented by equation $px^2 - qy^2 = 0$ are distinct then

Correct option: (A)

Given equation of pair of lines is

$px^2 - qy^2 = 0$

$\therefore a = p, b = -q, c = 0$

Since, the lines are real and distinct

$\therefore h^2 - ab > 0$

$\Rightarrow 0 - p(-q) > 0$

$\Rightarrow pq > 0$

Q.25 The joint equation of pair of lines having slopes 1 and 3 and passing through the origin is

Correct option: (B)

Joint equation of pair of lines having slopes m_1 and m_2 and passing through the origin is

$y^2 - (m_1 + m_2)xy + m_1m_2 x^2 = 0$

$\Rightarrow 3x^2 - 4xy + y^2 = 0$

Alternate method:

Equations of the lines are $y = x$ and $y = 3x$ respectively.

i.e. $y - x = 0$ and $y - 3x = 0$

$\therefore$ the combined equation of the pair of lines is

$(y - x)(y - 3x) = 0$

$\therefore y^2 - 3xy - xy + 3x^2 = 0 \Rightarrow 3x^2 - 4xy + y^2 = 0$

Q.26 The combined equation of lines, through the origin forming an equilateral triangle with the line $x + y + \sqrt{3} = 0$ is

Correct option: (B)

Let $y = mx$ be the equation of line.

Slope of the given line $y = -x - \sqrt{3}$ is -1

Since, the pair of straight lines and the given line form an equilateral traingle, angle between them is 60°.

$\therefore \tan\frac{\pi}{3} = \left|\frac{m+1}{1-m}\right| \Rightarrow \sqrt{3} = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow 3(1 - m)^2 = (1 + m)^2$

$\Rightarrow 3(1 + m^2 - 2m) = (1 + m^2 + 2m)$

$\therefore m^2 - 4m + 1 = 0$ ….(i)

The equation of line passing through origin is,

$y = mx \Rightarrow m = \frac{y}{x}$

Substituting the value of m in (i), we get

$\left(\frac{y}{x}\right)^2 - 4\left(\frac{y}{x}\right) + 1 = 0 \Rightarrow x^2 - 4xy + y^2 = 0$

Q.27 A point equidistant from the lines $24x + 7y + 25 = 0$, $12x + 5y + 13 = 0$ and $3x + 4y + 5 = 0$ is

Correct option: (C)

Lengths of perpendiculars from (0,0) on the given lines are each equal to 1.

Q.28 The distance of the line $3x + 2y = 5$ from the point (2, 2) measured parallel to the line $x - y = 1$ is

Correct option: (A)

The slope of line $x - y = 1$ is 1.

∴ It makes an angle of 45° with X-axis.

The equation of line passing through (2, 2) and making an angle of 45° is,

$\frac{x-2}{\cos 45^o} = \frac{y-2}{\sin 45^o} = r$

$\Rightarrow \frac{x-2}{1/\sqrt{2}} = \frac{y-2}{1/\sqrt{2}} = r$

∴ Co-ordinates of any point on this line are

$\left(2 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$

If this point lies on $3x + 2y = 5$, then

$3\left(2 + \frac{r}{\sqrt{2}}\right) + 2\left(2 + \frac{r}{\sqrt{2}}\right) = 5$

$\Rightarrow$ r = $\left|-\sqrt{2}\right|$

= $\sqrt{2}$

Q.29 The equation of the pair of lines through the point (2, 1) and perpendicular to the pair of lines $4xy + 2x + 6y + 3 = 0$ is

Correct option: (D)

Given equation of pair of lines is

$4xy + 2x + 6y + 3 = 0$

$\Rightarrow 2x(2y + 1) + 3(2y + 1) = 0$

$\Rightarrow (2y + 1)(2x + 3) = 0$

∴ Separate equations of lines are $2x + 3 = 0$ and $2y + 1 = 0$

i.e. $x = \frac{-3}{2}$ and $y = \frac{-1}{2}$

The equation of line passing through (2, 1) and perpendicular to $x = \frac{-3}{2}$ is $y = 1$ i.e. $y - 1 = 0$

The equation of line passing through (2, 1) and perpendicular to $y = \frac{-1}{2}$ is $x = 2$ i.e. $x - 2 = 0$

∴ Combined equation of pair of lines is $(x - 2)(y - 1) = 0$

$\Rightarrow xy - x - 2y + 2 = 0$

Q.30 The acute angle between the diagonals of a parallelogram whose vertices are A(2, –1), B(0, 2), C (2, 3) and $D(4, 0)$ is

Correct option: (C)

Here,

Slope of 1st diagonal (m_1) = $\frac{3+1}{2-2} = \infty$

Slope of 2nd diagonal (m_2) = $\frac{0-2}{4-0} = -\frac{1}{2}$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right| = \left|\frac{1}{\frac{1}{2}}\right|$

∴ $\tan\theta = 2$

∴ $\theta = \tan^{-1}2$

Q.31 Locus of a point having ordinate 5 is

Correct option: (B)

The locus of a point having an ordinate of 5 is a *horizontal line* that passes through all points with a y-coordinate of 5. This is represented by the equation *y = 5*.

Q.32 If P_1 and P_2 are perpendicular distances (in units) from point (2, –1) to the pair of lines $2x^2 - 5xy + 2y^2 = 0$, then the value of P_1P_2 is

Correct option: (D)

Given equation of pair of lines is

$2x^2 - 5xy + 2y^2 = 0$

∴ $2x^2 - 4xy - xy + 2y^2 = 0$

∴ $2x(x - 2y) - y(x - 2y) = 0$

∴ $(2x - y)(x - 2y) = 0$

∴ separate equations of the lines are

$2x - y = 0$ and $x - 2y = 0$

∴ Perpendicular distances of the above lines from (2, –1) are

$P_1 = \left|\frac{2(2) - (-1)}{\sqrt{(2)^2 + (-1)^2}}\right| = \left|\frac{5}{\sqrt{5}}\right|$ and

$P_2 = \left|\frac{2 - 2(-1)}{\sqrt{(1)^2 + (-2)^2}}\right| = \left|\frac{4}{\sqrt{5}}\right|$

∴ $P_1P_2 = \frac{5}{\sqrt{5}} \times \frac{4}{\sqrt{5}} = 4$

Q.33 If sum of the slopes of lines represented by $x^2 - 2xy\tan\theta - y^2 = 0$ is 4, then θ =

Correct option: (A)

Given equation of pair of lines is

$x^2 - 2xy\tan\theta - y^2 = 0$

$\therefore$ a = 1, h = –tan θ, b = –1

$\therefore$ $m_1 + m_2 = \dfrac{-2h}{b}$ = –2tan θ

$\therefore$ 4 = –2tanθ …[$\because m_1 + m_2 = 4$, Given]

$\therefore$ θ = $\tan^{-1}(-2)$

Q.34 Transforming to parallel axes through a point (p, q), the equation $x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 0$, then

Correct option: (B)

$2x^2 + 3xy + 4y^2 = 0$

i.e. $2X^2 + 3XY + 4Y^2 = 0$

Replacing X by x – p and Y by y – q, we get

$2(x - p)^2 + 3(x - p)(y - q) + 4(y - q)^2 = 0$

$\Rightarrow 2(x^2 - 2xp + p^2) + 3(xy - xq - py + pq) + 4(y^2 - 2qy + q^2) = 0$

$\Rightarrow 2x^2 + 3xy + 4y^2 - x(4p + 3q) - y(3p + 8q) + 2p^2 + 3pq + 4q^2 = 0$

Comparing the above equation with

$2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$, we get

4p + 3q = –1 ...(i)

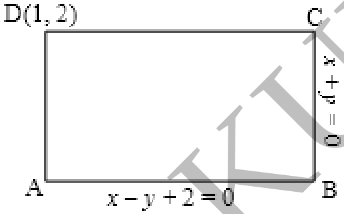
3p + 8q = –18 ...(ii)

On solving (i) and (ii), we get

p = 2, q = –3

Q.35 The area of rectangle whose one vertex is at point (1, 2) and two sides lie along the lines $x + y = 0$ and $x - y + 2 = 0$ is

Correct option: (B)



Distance of D from the line $x + y = 0$ is

length = $\left|\frac{1+2}{\sqrt{1+1}}\right| = \frac{3}{\sqrt{2}}$

Distance of D from the line $x - y + 2 = 0$ is

breadth = $\left|\frac{1-2+2}{\sqrt{1+1}}\right| = \frac{1}{\sqrt{2}}$

$\therefore$ Area of rectangle = $\frac{3}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{3}{2}$ sq. units

Q.36 The equation of a line joining the origin to the point (– 4, 5) is

Correct option: (A)

$$m = \frac{5-0}{-4-0} = \frac{5}{-4}$$

$\therefore$ the required equation is $5x + 4y = 0$.

Q.37 Difference of slopes of the lines represented by equation $x^2(\sec^2 q - \sin^2 q) - 2xy \tan q + y^2 \sin^2 q = 0$ is

Correct option: (C)

Given equation of pair of lines is

$x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$

$\therefore$ a = $\sec^2\theta - \sin^2\theta$, h = – tan θ, b = $\sin^2\theta$

Now, $m_1 + m_2 = \dfrac{2\tan\theta}{\sin^2\theta}$,

$$m_1 m_2 = \frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}$$

$\therefore m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4m_1m_2}$

$$= \sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)}$$

$$= \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - 4(\sec^2\theta\,\mathrm{cosec}^2\theta - 1)}$$

$$= \sqrt{4\sec^2\theta\,\mathrm{cosec}^2\theta - 4\sec^2\theta\,\mathrm{cosec}^2\theta + 4}$$

= 2

Q.38 If the acute angle between the lines $x^2 - 4hxy + 3y^2 = 0$ is 60° then h =

Correct option: (C)

Given equation of pair of lines is

$x^2 - 4hxy + 3y^2 = 0$

$\therefore$ A = 1, H = –2h, B = 3

Now, θ = 60°

$\Rightarrow \tan\theta = \sqrt{3}$

$$\therefore \tan\theta = \left|\frac{2\sqrt{H^2 - AB}}{A + B}\right|$$

$$\Rightarrow \sqrt{3} = \left|\frac{2\sqrt{4h^2 - 3}}{4}\right| \Rightarrow h = \pm\frac{\sqrt{15}}{2}$$

Q.39 The angle between the lines $x\cos\alpha_1 + y\sin\alpha_1 = p_1$ and $x\cos\alpha_2 + y\sin\alpha_2 = p_2$ is

Correct option: (B)

$$\theta = \tan^{-1}\left|\frac{-cot\alpha_1 + cot\alpha_2}{1 + cot\alpha_1 \cot\alpha_2}\right|$$

$$= \tan^{-1}\left|\frac{\tan\alpha_1 - tan\alpha_2}{1 + tan\alpha_2 \tan\alpha_1}\right| = \alpha_1 - \alpha_2$$

Q.40 If the sum of slopes of the lines given by $3x^2 + kxy - y^2 = 0$ is zero, then the value of k is

Correct option: (A)

Given equation of pair of lines is

$3x^2 + kxy - y^2 = 0$

$\therefore$ a = 3, h = $\frac{k}{2}$, b = –1

Now, $m_1 + m_2 = \frac{-2h}{b} = k$

According to the given condition,

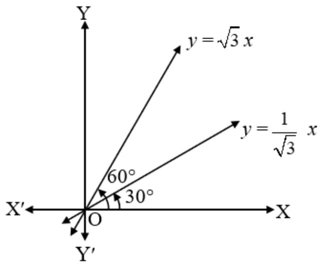
$m_1 + m_2 = 0$

$\Rightarrow$ k = 0

Q.41 The joint equation of the lines through the origin, trisecting angles in first and third quadrants is

Correct option: (B)

The lines trisecting the first and third quadrants are as shown in the figure.



$\therefore$ The joint equation of the lines is

$$\left(y - \frac{1}{\sqrt{3}}x\right)\left(y - \sqrt{3}x\right) = 0$$

$$\Rightarrow \left(\sqrt{3}y - x\right)\left(y - \sqrt{3}x\right) = 0$$

$$\Rightarrow \sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$$

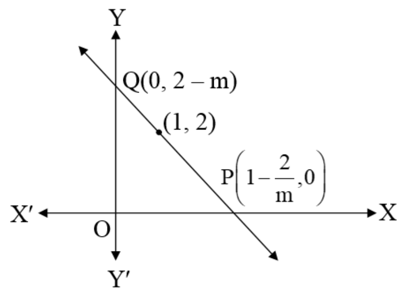
$$\Rightarrow \sqrt{3}\,(x^2 + y^2) - 4xy = 0$$

Q.42 A line is drawn through the point (1, 2) to meet the co-ordinate axes at P and Q such that it forms a ΔOPQ, where O is the origin. If the area of ΔOPQ is least, then the slope of the line PQ is

Correct option: (A)

The equation of line PQ passing through (1, 2) is

$y - 2 = m(x - 1)$



$$A(\Delta OPQ) = \frac{1}{2} \times OP \times OQ$$

$$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m) = \frac{1}{2}\left(4 - m - \frac{4}{m}\right)$$

$$\therefore\ A = 2 - \frac{m}{2} - \frac{2}{m}$$

$$\therefore\ \frac{dA}{dm} = -\frac{1}{2} + \frac{2}{m^2}$$

Now, $\frac{dA}{dm} = 0$

$$\Rightarrow -\frac{1}{2} + \frac{2}{m^2} = 0$$

$\Rightarrow m^2 = 4$

$\Rightarrow m = \pm 2$

$$\frac{d^2A}{dm^2} = -\frac{4}{m^3}$$

At m = 2,

$$\frac{d^2A}{dm^2} < 0$$

At m = –2,

$$\frac{d^2A}{dm^2} > 0$$

$\therefore$ Area of ΔOPQ will be least at m = –2

$\Rightarrow$ Slope of the line PQ = –2

Q.43 The length of perpendicular from the point (a cos α, a sin α) upon the straight line $y = x\tan\alpha + c$, c > 0 is

Correct option: (A)

Here, equation of line is $y = x\tan\alpha + c$, c > 0

Length of the perpendicular drawn on line from point (a cosα, a sinα) is

$$p = \left|\frac{a\cos\alpha\tan\alpha - a\sin\alpha + c}{\sqrt{1+\tan^2\alpha}}\right| = \frac{c}{\sec\alpha} = c\cos\alpha$$

Q.44 If the lines $(p - q)x^2 + 2(p + q)xy + (q - p)y^2 = 0$ are mutually perpendicular, then

Correct option: (D)

Given equation of pair of lines is

$(p - q)x^2 + 2(p + q)xy + (q - p)y^2 = 0$

$\therefore$ a = p – q, h = p + q, b = q – p

Since, the lines are mutually perpendicular

$\therefore$ a + b = 0

$\Rightarrow$ (p – q) + (q – p) = 0

The above equation is true for all values of p and q.

Q.45 The line $(K + 1)^2 x + Ky - 2K^2 - 2 = 0$ passes through a point regardless of the value of K. Which of the following is the equation of the line with slope 2 and passing through that point?

Correct option: (D)

$(K + 1)^2x + Ky - 2K^2 - 2 = 0$

$\therefore (K^2 + 2K + 1)x + Ky - 2K^2 - 2 = 0$

$\therefore K^2(x - 2) + K(2x + y) + (x - 2) = 0$

$\therefore (K^2 + 1)(x - 2) + K(2x + y) = 0$

$\therefore x - 2 = 0$ i.e. $x = 2$

and $2x + y = 0$

$\therefore 2(2) + y = 0$

$\therefore y = -4$

$\therefore$ The fixed point is (2, – 4)

$\because$ The required line has slope 2 and passes through the point (2, –4)

$\therefore$ Equation of line is

$y - (-4) = 2(x - 2)$

$\therefore y + 4 = 2x - 4$

$\therefore y = 2x - 8$

Q.46 The equation of pair of lines $y = px$ and $y = qx$ can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is

Correct option: (D)

Equation of angle bisector of two lines whose general equation is $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$\therefore$ Comparing given equation $x^2 - 4xy - 5y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get

a = 1, b = –5, h = –2

$\therefore$ Equation of angle bisectors is

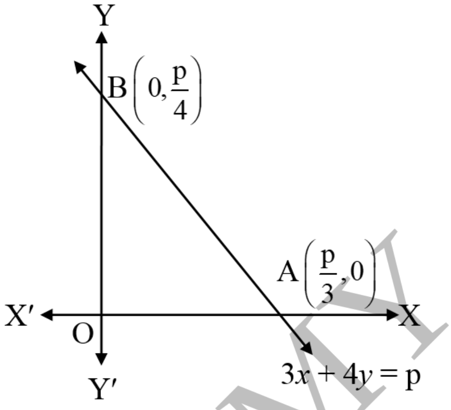
$$\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2}$$

$\Rightarrow x^2 - y^2 = -3xy$

$\Rightarrow x^2 + 3xy - y^2 = 0$

Q.47 If the line $3x + 4y = p$ makes a triangle of area 24 square units with the co-ordinate axes, then the value of p is

Correct option: (C)



Let the line $3x + 4y = p$ cuts the X and Y axes at points A and B respectively.

$3x + 4y = p$

$\therefore \frac{3x}{p} + \frac{4y}{p} = 1$

$\therefore \frac{x}{\frac{p}{3}} + \frac{y}{\frac{p}{4}} = 1$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where a = $\frac{p}{3}$ and b = $\frac{p}{4}$

$\therefore$ A ≡ (a, 0) = $\left(\frac{p}{3}, 0\right)$ and B ≡ (0, b) = $\left(0, \frac{p}{4}\right)$

$\therefore$ OA = $\frac{p}{3}$ and OB = $\frac{p}{4}$

Given, A (ΔOAB) = 24 sq. units

$\therefore \left|\frac{1}{2} \times OA \times OB\right| = 24$

$\therefore \left|\frac{1}{2} \times \frac{p}{3} \times \frac{p}{4}\right| = 24$

$\therefore p^2 = 576$

$\therefore p = \pm 24$

Q.48 The value(s) of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is parallel to Y-axis is / are

Correct option: (D)

Given equation of line is

$(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$

$\Rightarrow (4 - k^2)y = (k - 3)x + k^2 - 7k + 6$

$\therefore$ Slope (m) = $\frac{k-3}{4-k^2}$

Since, the line is parallel to Y-axis.

$\therefore \frac{1}{m} = 0$

$\Rightarrow \frac{4-k^2}{k-3} = 0$

$\Rightarrow 4 - k^2 = 0$

$\Rightarrow k = \pm 2$

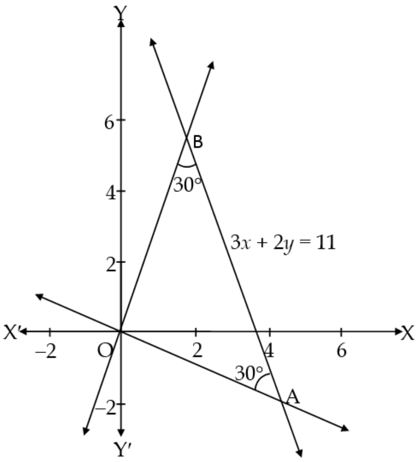
Q.49 The combined equation of lines passing through the origin each of which making an angle of 30º with the line $3x + 2y - 11 = 0$ is

Correct option: (B)

Let OA and OB be two lines through the origin, each making an angle of 30° with the line $3x + 2y - 11 = 0$.

Let slope of OA (or OB) be m.

Slope of the line $3x + 2y - 11 = 0$ is $-\dfrac{3}{2}$ and $\theta = 30°$



$\therefore \tan 30° = \left|\dfrac{m - \left(-\frac{3}{2}\right)}{1 + \left(-\frac{3}{2}\right)m}\right|$

$\therefore \dfrac{1}{\sqrt{3}} = \left|\dfrac{2m+3}{2-3m}\right|$

By taking square of both sides, we get

$(2 - 3m)^2 = 3(2m + 3)^2$

$\therefore 4 - 12m + 9m^2 = 12m^2 + 36m + 27$

$\therefore 3m^2 + 48m + 23 = 0 \quad \ldots(i)$

Equation of OA (or OB) is $y = mx$, since it passes through the origin.

$\therefore m = \dfrac{y}{x}$

Substituting the value of m in (i), we get

$3\left(\dfrac{y}{x}\right)^2 + 48\left(\dfrac{y}{x}\right) + 23 = 0$

$\therefore \dfrac{3y^2}{x^2} + \dfrac{48y}{x} + 23 = 0$

$\therefore 3y^2 + 48xy + 23x^2 = 0$

$\therefore 23x^2 + 48xy + 3y^2 = 0$ is the required combined equation.

Q.50 If one of the lines given by the equation $x^2 + kxy + 2y^2 = 0$ is $x + 2y = 0$, then k =

Correct option: (A)

Given equation of pair of lines is

$x^2 + kxy + 2y^2 = 0$

$\Rightarrow 2\left(\dfrac{y}{x}\right)^2 + k\left(\dfrac{y}{x}\right) + 1 = 0$

$\Rightarrow 2m^2 + km + 1 = 0 \quad \ldots(i) \left[\text{for } m = \dfrac{y}{x}\right]$

Given that $x + 2y = 0$ is one of the line.

$\Rightarrow \text{slope} = m = \dfrac{-1}{2}$

$\therefore$ Substituting in equation (i), we get

k = 3

Q.51 If m_1 and m_2 are the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$ satisfying the condition $16h^2 = 25ab$, then ….

Correct option: (B)

If the slopes of lines

$ax^2 + 2hxy + by^2 = 0$ are in the ratio $m_1 : m_2$ then

$(m_1 + m_2)^2\, ab = 4m_1m_2\, h^2 \quad \ldots(i)$

Comparing (i) with $16h^2 = 25ab$, we get

$(m_1 + m_2)^2 = 25$ and $m_1m_2 = 4$

$\Rightarrow m_1 + m_2 = \pm 5 \quad \ldots(ii)$

and $m_1m_2 = 4 \quad \ldots(iii)$

Solving (ii) and (iii), we get

$m_1 = 4$ or $m_1 = -4$

$m_2 = 1$ or $m_2 = -1$

$\therefore m_1 = 4m_2$

Q.52 The equation of the locus of a point whose distance from (a, 0) is equal to its distance from Y-axis is

Correct option: (C)

Let the point be (h, k).

$\therefore$ distance of a point from Y-axis = h

According to the given condition,

$\sqrt{(h-a)^2 + (k-0)^2} = h$

$\Rightarrow (h - a)^2 + (k - 0)^2 = h^2$

$\Rightarrow h^2 + a^2 - 2ah + k^2 = h^2$

Hence, locus of (h, k) is $y^2 - 2ax + a^2 = 0$.

Q.53 If two sides of a square are $4x + 3y - 20 = 0$ **and** $4x + 3y + 15 = 0$

, then the area of the square is

Correct option: (D)

Given equations of lines are $4x + 3y - 20 = 0$ and

$4x + 3y + 15 = 0$

Slope of $4x + 3y - 20 = 0$ is $\frac{-4}{3}$.

Slope of $4x + 3y + 15 = 0$ is $\frac{-4}{3}$

∴ Lines are parallel.

∴ Distance between two parallel lines

$= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

$= \left|\frac{-20 - 15}{\sqrt{4^2 + 3^2}}\right| = \frac{35}{5} = 7$ units

∴ Area of square $= 7^2 = 49$ sq. units

Q.54 The measure of the angle between the lines $x^2 + 2xy \operatorname{cosec} \alpha + y^2 = 0$ **is**

Correct option: (B)

Here, a = 1, h = cosec α, b = 1

∴ $\tan\theta = \left|\frac{2\sqrt{h^2 - \mathrm{ab}}}{\mathrm{a} + \mathrm{b}}\right|$

$= \left|\frac{2\sqrt{\operatorname{cosec}^2\alpha - 1}}{1 + 1}\right|$

$= \left|\sqrt{\cot^2\alpha}\right|$

∴ $\tan\theta = \cot\alpha$

∴ $\tan\theta = \tan\left(\frac{\pi}{2} - \alpha\right)$

∴ $\theta = \frac{\pi}{2} - \alpha$

Q.55 If the pair of lines $3x^2 - 5xy + py^2 = 0$ **and** $6x^2 - xy - 5y^2 = 0$ **have one line common, then p =**

Correct option: (C)

$6x^2 - xy - 5y^2 = 0$

$\Rightarrow 6x^2 - 6xy + 5xy - 5y^2 = 0$

$\Rightarrow 6x(x - y) + 5y(x - y) = 0$

$\Rightarrow (x - y)(6x + 5y) = 0$

$\Rightarrow x = y$ or $x = \frac{-5}{6}y$

$3x^2 - 5xy + py^2 = 0$

$x = \frac{5y \pm \sqrt{(-5y)^2 - 4(3)(py^2)}}{2(3)}$

$= \frac{5y \pm \sqrt{25y^2 - 12py^2}}{6}$

∴ $x = \frac{5y \pm y\sqrt{25 - 12p}}{6}$ …(i)

If $x = y$, then

$y = \frac{5y \pm y\sqrt{25 - 12p}}{6}$ …[From (i)]

$\Rightarrow 1 = \frac{5 \pm \sqrt{25 - 12p}}{6}$

$\Rightarrow 1 = \pm\sqrt{25 - 12p}$

$\Rightarrow 1 = 25 - 12p$

$\Rightarrow 12p = 24$

$\Rightarrow p = 2$

If $x = \frac{-5}{6}y$, then

$\frac{-5}{6}y = \frac{5y \pm y\sqrt{25 - 12p}}{6}$ …[From (i)]

$\Rightarrow -5 = 5 \pm \sqrt{25 - 12p}$

$\Rightarrow -10 = \pm\sqrt{25 - 12p}$

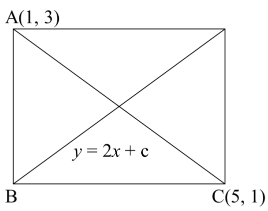
$\Rightarrow 100 = 25 - 12p$

$\Rightarrow 12p = -75$

$\Rightarrow p = \frac{-25}{4}$

Q.56 The points (1, 3), (5, 1) are opposite vertices of a diagonal of a rectangle. If the other two vertices lie on the line $y = 2x + c$**, then one of the vertex on the other diagonal is**

Correct option: (C)



Diagonals of rectangle bisect each other.

∴ Midpoint of (1, 3) and (5, 1) is (3, 2).

Also, $y = 2x + c$ passes through (3, 2).

∴ $2 = 2(3) + c$

∴ $c = -4$

∴ Other two vertices lie on $y = 2x - 4$.

Let co-ordinates of B be (x, y) i.e., $(x, 2x - 4)$

slope of AB × slope of BC = –1

$$\Rightarrow \left(\frac{2x-4-3}{x-1}\right)\left(\frac{2x-4-1}{x-5}\right) = -1$$

$$\Rightarrow \left(\frac{2x-7}{x-1}\right)\left(\frac{2x-5}{x-5}\right) = -1$$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow x = 4, 2$

When $x = 4$, $y = 4$

When $x = 2$, $y = 0$

∴ Vertex of the other diagonal is (2, 0).

Q.57 The line $2x - y = 5$ is rotated about a point on it, whose ordinate and abscissae are equal, through an angle of 45°. The equation of the new line is

Correct option: (C)

Suppose the point through which the line is rotated is A(α, α).

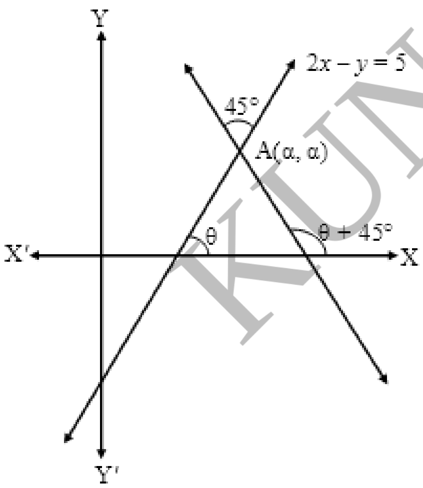
Given equation of line is

$2x - y = 5$

$\Rightarrow 2\alpha - \alpha = 5$

$\Rightarrow \alpha = 5$

$\therefore A \equiv (5, 5)$



Slope of line (tan θ) = 2

Since, the line is rotated through 45°.

∴ Angle made by new line with X-axis = (θ + 45°)

∴ Slope of new line = tan (θ + 45°)

$$= \frac{\tan\theta + \tan 45^\circ}{1 - \tan\theta \tan 45^\circ}$$

$$= \frac{2+1}{1-2\times 1} = -3$$

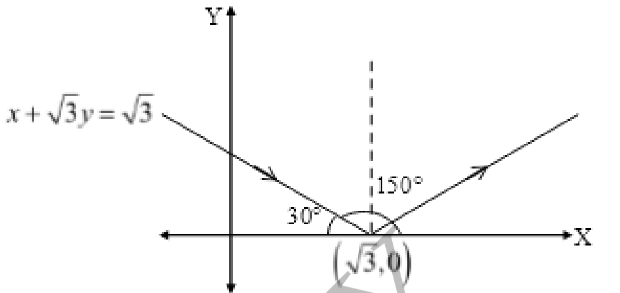
∴ Equation of new line passing through (5, 5) and having slope –3 is

$y - 5 = -3(x - 5)$

$\Rightarrow 3x + y = 20$

Q.58 A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching the X-axis. The equation of the reflected ray is

Correct option: (D)



Equation of incident ray is

$x + \sqrt{3}y = \sqrt{3}$

$\Rightarrow y = \frac{-1}{\sqrt{3}}x + 1$

∴ Slope of incident ray is $\frac{-1}{\sqrt{3}}$.

$\Rightarrow \tan\theta = \frac{-1}{\sqrt{3}}$

$\Rightarrow \theta = 150^\circ$

∴ Incident ray makes an angle of 150° with the positive direction of X-axis.

∴ Angle made by reflected ray with positive direction of X-axis is 30°.

∴ Slope = tan 30° = $\frac{1}{\sqrt{3}}$

∴ Equation of reflected ray passing through $\left(\sqrt{3}, 0\right)$ and having slope $\frac{1}{\sqrt{3}}$ is

$y - 0 = \frac{1}{\sqrt{3}}\left(x - \sqrt{3}\right)$

$\Rightarrow x - \sqrt{3}y - \sqrt{3} = 0$

Q.59 Two lines represented by equations $x + y = 1$ and $x + ky = 0$ are mutually orthogonal if k is

Correct option: (B)

Here, $m_1 = -1$, $m_2 = -\frac{1}{k}$.

For orthogonal lines,

$m_1 m_2 = -1 \Rightarrow \frac{1}{k} = -1 \Rightarrow k = -1$

Q.60 Measure of angle between the two lines $3xy - 4y = 0$ is

Correct option: (C)

Given equation of pair of lines is

$3xy - 4y = 0$

∴ a = b = 0

Now a + b = 0

∴ The lines are perpendicular to each other.

Q.61 The equation of lines passing through the origin and parallel to the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is

Correct option: (A)

According to the given conditions, equations of the lines are $y = m_1x$ and $y = m_2x$

∴ Combined equation is $(y - m_1x)(y - m_2x) = 0$

i.e., $y^2 - m_1xy - m_2xy + m_1m_2x^2 = 0$

i.e., $m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0$

Q.62 The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - kx - 2y + 2 = 0$ then the value of k is

Correct option: (C)

The line $5x + y - 1 = 0$ is coincides

$5x^2 + xy - kx - 2y + 2 = 0$

∴ $a = 5, b = 0, c = 2, f = -1, g = -\frac{k}{2}, h = \frac{1}{2}$

$m_1 + m_2 = \frac{-2h}{b}$

As b = 0, this case is not defined

Slope of line $5x + y - 1 = 0$ is $m = -5$

∴ Slope of another line must be infinite

∴ equation of another line is $x = k_1$

∴ Combine equation is $(5x + y - 1)(x - k_1) = 0$

$\Rightarrow 5x^2 - 5xk_1 + xy - yk_1 - x + k_1 = 0$

$\Rightarrow 5x^2 + xy - (5k_1 + 1)x - yk_1 + k_1 = 0$

Comparing this equation with the given equation, we get k = 11

Q.63 The joint equation of two lines through the origin, each making an angle with measure of 30° with the positive Y-axis, is

Correct option: (B)

Slopes of the required lines are $m_1 = \sqrt{3}$, $m_2 = -\sqrt{3}$

∴ Required lines are

$\left(y - \sqrt{3}x\right)\left(y + \sqrt{3}x\right) = 0$

$\Rightarrow 3x^2 - y^2 = 0$

Q.64 The distance between the line $3x + 4y = 9$ and $6x + 8y = 15$ is

Correct option: (C)

Given parallel lines are

$3x + 4y = 9 \Rightarrow 6x + 8y = 18$ and $6x + 8y = 15$

Required distance = $\frac{|18-15|}{\sqrt{36+64}} = \frac{3}{10} = 0.3$ units.

Q.65 The co-ordinates of the points on the line $2x - y = 5$ which are the distance of 1 unit from the line $3x + 4y = 5$ are

Correct option: (C)

Let (x_1, y_1) be the required point

∴ $2x_1 - y_1 = 5$ …(i)

Also, (x_1, y_1) is at the distance of 1 unit from line $3x + 4y = 5$

∴ $1 = \left|\frac{3x_1 + 4y_1 - 5}{\sqrt{9+16}}\right|$

∴ $\pm 5 = 3x_1 + 4y_1 - 5$

∴ $3x_1 + 4y_1 - 5 = 5$ or $3x_1 + 4y_1 - 5 = -5$

∴ $3x_1 + 4y_1 = 10$ …(ii)

or

$3x_1 + 4y_1 = 0$ …(iii)

Solving equations (i) and (ii), we get

$x_1 = \frac{30}{11}$ and $y_1 = \frac{5}{11}$

Solving equations (i) and (iii), we get

$x_1 = \frac{20}{11}$ and $y_1 = \frac{-15}{11}$

∴ $\left(\frac{30}{11}, \frac{5}{11}\right)$ and $\left(\frac{20}{11}, \frac{-15}{11}\right)$ are the required points.

Q.66 If $6x + 8y + 21 = 0$ and $3x + 4y + 7 = 0$ are the tangents of same circle, then its radius will be

Correct option: (B)

The diameter of the circle is perpendicular distance between the parallel lines (tangents)

$6x + 8y + 21 = 0$ and $3x + 4y + 7 = 0$ and so it is equal to $\frac{\frac{21}{2} - 7}{\sqrt{3^2+4^2}} = \frac{\frac{7}{2}}{5} = \frac{7}{10}$

∴ radius = $\frac{7}{20}$

Q.67 A variable line through the point $\left(\frac{1}{5}, \frac{1}{5}\right)$ cuts the coordinate axes in the points A and B. If the point P divides AB

internally in the ratio 3 : 1, then the locus of P is

Correct option: (B)

Equation of line in double intercept form is

$\frac{x}{a}+\frac{y}{b}=1.$

This line passes through $\left(\frac{1}{5},\ \frac{1}{5}\right)$.

$\therefore$ a + b = 5ab ….(i)

Point P(x, y) divides AB joining A(a, 0) and B(0, b) internally in the ratio 3 : 1.

$\therefore x = \frac{3\times0+1\times a}{3+1}, y = \frac{3\times b+1\times0}{3+1}$

$\Rightarrow x = \frac{a}{4}, y = \frac{3b}{4}$

$\Rightarrow$ a = 4x, b = $\frac{4y}{3}$

Substituting the values of a and b in (i), we get

$4x + \frac{4y}{3} = 5(4x)\left(\frac{4y}{3}\right) \Rightarrow 3x + y = 20xy$

Q.68 If the sum of slopes of the pair of lines given by $4x^2 + 2hxy - 7y^2 = 0$ is equal to the product of the slopes, then h is

Correct option: (D)

Given equation of pair of lines is

$4x^2 + 2hxy - 7y^2 = 0$

$\therefore$ A = 4, H = h, B = –7

$\therefore\ m_1 + m_2 = \frac{-2H}{B}$ and $m_1m_2 = \frac{A}{B}$

$\therefore\ m_1 + m_2 = \frac{2h}{7}$ and $m_1m_2 = \frac{-4}{7}$

Given that $m_1 + m_2 = m_1m_2$

$\Rightarrow \frac{2h}{7} = \frac{-4}{7}$

$\Rightarrow$ h = –2

Q.69 The angle between the straight lines $x^2 + 4xy + y^2 = 0$ is

Correct option: (C)

Given equation of pair of lines is

$x^2 + 4xy + y^2 = 0$

$\therefore$ a = 1, h = 2, b = 1

$\therefore \tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{(2)^2-(1)(1)}}{1+1}\right| =$

$\sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\right) = 60°$

Q.70 If lines represented by $(1 + \sin^2\theta)x^2 + 2hxy + 2\sin\theta\, y^2 = 0$, $\theta \in [0, 2\pi]$ are perpendicular to each other then θ =

Correct option: (D)

Given equation of pair of lines is

$(1 + \sin^2\theta)x^2 + 2hxy + 2\sin\theta y^2 = 0$

$\therefore$ a = 1 + $\sin^2\theta$, b = 2 sinθ

If the given lines are perpendicular, we get

a + b = 0

$\Rightarrow 1 + \sin^2\theta + 2\sin\theta = 0$

$\Rightarrow (1 + \sin\theta)^2 = 0$

$\Rightarrow \sin\theta = -1$

$\Rightarrow \theta = \frac{3\pi}{2}$

Q.71 If the lines $x + 3y - 9 = 0$, $4x + by - 2 = 0$, and $2x - y - 4 = 0$ are concurrent, then the equation of the line passing through the point (b,0) and concurrent with the given lines, is

Correct option: (D)

Since, lines $x + 3y - 9 = 0$, $4x + by - 2 = 0$, and $2x - y - 4 = 0$ are concurrent

$\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$

$\therefore$ 1 (–4b – 2) –3 (–16 + 4) –9 (–4 – 2b) = 0

$\Rightarrow$ b = –5

$\therefore$ the required line passes through (–5, 0)

Now, consider option [D] and $x + 3y - 9 = 0$, $4x - 5y - 2 = 0$

$\therefore \begin{vmatrix} 1 & 3 & -9 \\ 4 & -5 & -2 \\ 1 & -4 & 5 \end{vmatrix} = 0$

$\therefore$ option [D] is correct

Q.72 A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

Correct option: (C)

The required equation of line is $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$

Q.73 A line passes through the point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ and makes equal intercepts with axes. The equation of the line is

Correct option: (A)

The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ are $\left(\frac{-4}{5}, \frac{7}{5}\right)$. The equation of line which makes equal intercepts with the axes is $x + y = a$.

$\therefore -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$

$\therefore$ the required equation of the line is

$x + y - \frac{3}{5} = 0$ i.e., $5x + 5y - 3 = 0$

Q.74 Distance between the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ is

Correct option: (B)

Given lines are $3x - 4y + 4 = 0$ ...(i)

and $6x - 8y - 7 = 0$

$\Rightarrow 3x - 4y - \frac{7}{2} = 0$ ...(ii)

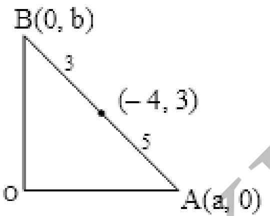
Lines (i) and (ii) are parallel.

$\therefore$ Required distance = $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{4 + \frac{7}{2}}{\sqrt{3^2 + 4^2}}\right|$

$= \frac{3}{2}$

Q.75 Equation of the line passing through the point (– 4, 3) and the portion of the line intercepted between the axes which is divided internally in the ratio 5 : 3 by this point, is

Correct option: (C)



By the section formula, we get $a = -\frac{32}{3}$ and $b = \frac{24}{5}$

Hence, the required equation is given by

$\frac{x}{-(32/3)} + \frac{y}{(24/5)} = 1$

$\Rightarrow 9x - 20y + 96 = 0$

Q.76 The equation of the line perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ and passing through the point at which it cuts X–axis, is

Correct option: (D)

The given line is $bx - ay = ab$ …(i)

It cuts X-axis at (a, 0).

The equation of a line perpendicular to (i) is

$ax + by = k$.

Since, the line passes through (a, 0) $\Rightarrow k = a^2$

Hence, required equation of line is $ax + by = a^2$

i.e., $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$

Q.77 PS is the median of the triangle with vertices at P(2, 2), Q(6, –1) and R(7, 3), then the intercepts on the co-ordinate axes of the line passing through point (1, –1) and parallel to PS are respectively

Correct option: (C)

S = midpoint of QR = $\left(\frac{6+7}{2}, \frac{-1+3}{2}\right) = \left(\frac{13}{2}, 1\right)$

$\therefore$ 'm' of $PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

$\therefore$ The required equation is $y + 1 = \frac{-2}{9}(x - 1)$

$\Rightarrow 2x + 9y + 7 = 0$

Here, intercept on X-axis is $-\frac{7}{2}$ and intercept on Y-axis is $-\frac{7}{9}$.

Q.78

If the angle between the lines given by $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$; $\lambda \geq 0$ is $\tan^{-1}\left(\frac{1}{3}\right)$, then the value of λ is

Correct option: (B)

Given equation of pair of lines is

$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$

Here, $a = 1$, $b = \lambda$, $c = 2$, $f = \frac{-5}{2}$, $g = \frac{3}{2}$, $h = \frac{-3}{2}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan\theta = \frac{1}{3}$

Since $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

$$\Rightarrow \frac{1}{3} = \left| \frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - \lambda}}{\lambda + 1} \right|$$

$\Rightarrow (\lambda + 1)^2 = 9(9 - 4\lambda) \Rightarrow \lambda^2 + 38\lambda - 80 = 0$

$\Rightarrow (\lambda + 40)(\lambda - 2) = 0 \Rightarrow \lambda = 2$ …[$\because \lambda \geq 0$]

Q.79 The equation of line whose midpoint (x_1, y_1) is in between the axes, is

Correct option: (A)

Intersection point on X-axis is $(2x_1, 0)$ and on Y-axis is $(0, 2y_1)$. Thus, equation of line passing through these points is $\frac{x}{x_1} + \frac{y}{y_1} = 2$.

Q.80 The slope of a straight line which does not intersect X-axis is equal to [Kerala (Engg.) 2011]

Correct option: (D)

Here, the straight line is parallel to X-axis.

So, the slope of such a line = 0.