integration **Marks: 120**

ANSWER KEY

Maths

Q.1 A	Q.2 C	Q.3 D	Q.4 D	Q.5 B	Q.6 C	Q.7 D	Q.8 C
Q.9 B	Q.10 C	Q.11 B	Q.12 B	Q.13 C	Q.14 C	Q.15 B	Q.16 D
Q.17 D	Q.18 C	Q.19 C	Q.20 A	Q.21 B	Q.22 D	Q.23 C	Q.24 A
Q.25 C	Q.26 C	Q.27 D	Q.28 B	Q.29 B	Q.30 B	Q.31 C	Q.32 C
Q.33 C	Q.34 A	Q.35 B	Q.36 C	Q.37 C	Q.38 D	Q.39 B	Q.40 C
Q.41 D	Q.42 A	Q.43 D	Q.44 C	Q.45 C	Q.46 A	Q.47 D	Q.48 D
Q.49 B	Q.50 A	Q.51 A	Q.52 C	Q.53 D	Q.54 A	Q.55 D	Q.56 D
Q.57 B	Q.58 A	Q.59 B	Q.60 D				

Maths

Q.1 **If A(*x*) =** $\begin{vmatrix} x & x^2-x & x^3-x^2 \\ 2x & 2x^2-x & 2x^3-x^2 \\ 3x & 3x^2-x & 3x^3-x^2 \end{vmatrix}$**, then** $\int_0^1 A(x)\,dx =$

Correct option: (A)

A(x) = $\begin{vmatrix} x & x^2-x & x^3-x^2 \\ 2x & 2x^2-x & 2x^3-x^2 \\ 3x & 3x^2-x & 3x^3-x^2 \end{vmatrix}$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, we get

A(x) = $\begin{vmatrix} x & x^2-x & x^3-x^2 \\ 0 & x & x^2 \\ 0 & 2x & 2x^2 \end{vmatrix}$

Applying $R_3 \to R_3 - 2R_2$, we get

A(x) = $\begin{vmatrix} x & x^2-x & x^3-x^2 \\ 0 & x & x^2 \\ 0 & 0 & 0 \end{vmatrix}$ = $x(0-0) - (x^2-x)(0-0) + (x^3-x^2)(0-0) = 0$

$\therefore \int_0^1 A(x)\,dx = 0$

Q.2 $\int \frac{\sin x\ dx}{3+4\cos^2 x} =$

Correct option: (C)

Put $\cos x = t$

$\Rightarrow -\sin x\,dx = dt$

$\therefore \int \frac{\sin x}{3+4\cos^2 x} dx = \int \frac{-dt}{3+4t^2} = \frac{-1}{4}$

$\int \frac{dt}{t^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= -\frac{1}{4.\frac{\sqrt{3}}{2}}.\tan^{-1}\frac{t}{\left(\frac{\sqrt{3}}{2}\right)} + c$

$= \frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\,t}{\sqrt{3}}\right) + c$

$= \frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c$

Q.3 **If [*x*] denotes the greatest integer function, then** $\int_0^5 x^2[x]\ \mathrm{d}x$ **=**

Correct option: (D)

$\int_0^5 x^2[x]\ \mathrm{d}x =$

$\int_0^1 x^2[x]\mathrm{d}x + \int_1^2 x^2[x]\mathrm{d}x + \int_2^3 x^2[x]\mathrm{d}x + \int_3^4 x^2[x] + \int_4^5 x^2[x]\mathrm{d}x$

$=$

$\int_0^1 x^2(0)\mathrm{d}x + \int_1^2 x^2(1)\mathrm{d}x + \int_2^3 x^2(2)\mathrm{d}x + \int_3^4 x^2(3)\mathrm{d}x + \int_4^5 x^2(4)\mathrm{d}x$

$= \left[\frac{x^3}{3}\right]_1^2 + 2\left[\frac{x^3}{3}\right]_2^3 + 3\left[\frac{x^3}{3}\right]_3^4 + 4\left[\frac{x^3}{3}\right]_4^5$

$= \frac{1}{3}(8-1) + \frac{2}{3}(27-8) + (64-27) +$

$\frac{4}{3}(125-64)$

$= \frac{400}{3}$

Q.4 $\int \frac{\cos x}{\sqrt{1+\sin x}}\ dx =$

Correct option: (D)

$\int \frac{cos\ x}{\sqrt{1+sin\ x}} dx = 2\sqrt{1+sinx} + c$ ...[

$\int \frac{f'(x)}{\sqrt{f\ (x)}} dx = 2\sqrt{f(x)} + c$]

$= 2\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + c$

$= 2\left[\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right] + c$

Q.5 $\int_0^7 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}$**d*x* =**

Correct option: (B)

Let I = $\int_0^7 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}} dx$ …(i)

$\therefore$ I = $\int_0^7 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}} dx$ …(ii)

$\left[\because \int_0^a f(x)dx = \int_0^a f(\mathrm{a}-x)dx\right]$

Adding (i) and (ii), we get

2I = $\int_0^7 dx$

$\therefore$ I = $\frac{1}{2}[x]_0^7 = \frac{7}{2}$

Q.6 *int***limits_{-1}^{1}{{{x}^{17}}}cos^4 *x* d*x* =**

Correct option: (C)

Let f(x) = $x^{17} \cos^4 x$

$\therefore$ f($-x$) = $(-x)^{17}$ {cos $(-x)\}^4 = -$ f(x)

$\therefore$ f(x) is an odd function.

$\therefore$ *int*limits_{-1}^{1}{{{x}^{17}}} cos^4 xdx = 0

Q.7 $\int_{-2}^{2} |x|\, dx$ **is equal to**

Correct option: (D)

Let I = $\int_{-2}^{2} |x|\ dx$

$= 2\int_0^2 x\ dx\ \ldots \left[\begin{array}{l}\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \\ \text{if f}(x)\text{is an even function}\end{array}\right]$

$= 2\left[\frac{x^2}{2}\right]_0^2 = 2\left(\frac{4}{2}\right) = 4$

Q.8 $\int_0^{\pi/2} \frac{\sqrt{cotx}}{\sqrt{cotx}+\sqrt{tanx}}$ **d*x* =**

Correct option: (C)

Let I = $\int_0^{\pi/2} \frac{\sqrt{cotx}}{\sqrt{cotx}+\sqrt{tanx}} dx$ ...(i)

$= \int_0^{\pi/2} \frac{\sqrt{cot\left(\frac{\pi}{2}-x\right)}}{\sqrt{cot\left(\frac{\pi}{2}-x\right)}+\sqrt{tan\left(\frac{\pi}{2}-x\right)}} dx$ ...

$\left[\because \int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx\right]$

$= \int_0^{\pi/2} \frac{\sqrt{tanx}}{\sqrt{tanx}+\sqrt{cotx}} dx$

...(ii)

Adding (i) and (ii), we get

2I = $\int_0^{\pi/2} \frac{\sqrt{cotx}+\sqrt{tanx}}{\sqrt{tanx}+\sqrt{cotx}} dx = [x]_0^{\frac{\pi}{2}} \Rightarrow$ I = $\frac{\pi}{4}$

Q.9 $\int_{\pi/4}^{\pi/2} cosec^2$**xdx =**

Correct option: (B)

$\int_{\pi/4}^{\pi/2} cosec^2 x$ dx $= [-cotx]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\left[cot\frac{\pi}{2} - cot\frac{\pi}{4}\right] =$

1

Q.10 $\int \frac{(\log x - 1)^2}{\left[1+(\log x)^2\right]^2} = dx$ **, (where C is a constant of integration.)**

Correct option: (C)

Let I = $\int \left[\frac{1-\log x}{1+(\log x)^2}\right]^2 dx$

Put log x = t

$\therefore$ $x = e^t$

dx = e^tdt

$\therefore$ I $= \int \left(\frac{1-t}{1+t^2}\right)^2 e^t$dt

$= \int \left(\frac{1-2t+t^2}{(1+t^2)^2}\right) e^t$dt

$= \int e^t \left(\frac{1+t^2-2t}{(1+t^2)^2}\right)$

$= \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]$dt

$= e^t \left(\frac{1}{1+t^2}\right) + c$ …

$\left[\begin{array}{l}\because \int e^x \left[f\ (x) + f'(x)\right] dx \\ = e^x f(x) + c\end{array}\right]$

$= \frac{x}{1+(\log x)^2} + C$

Q.11 If

$\int \frac{dx}{x^4+5x^2+4} = A\tan^{-1}x + B\tan^{-1}\frac{x}{2} + c$

where c is a constant of integration, then

Correct option: (B)

Let I = $\int \frac{dx}{x^4+5x^2+4}$

$\therefore$ I = $\int \frac{1}{(x^2+1)(x^2+4)}dx$

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{A}{x^2+1} + \frac{B}{x^2+4}$$

$\Rightarrow$ 1 = A(x^2 + 4) + B (x^2 + 1)

$\Rightarrow$ 1 = (A + B) x^2 + 4A + B

$\Rightarrow$ A + B = 0 and 4A + B = 0

On solving, we get A = $\frac{1}{3}$, B = $\frac{-1}{3}$

$\therefore$ I = $\frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$

= $\frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c$

On comparing with the given expression, we get

A = $\frac{1}{3}$, B = $\frac{-1}{6}$

Q.12 $\int_0^1 \frac{e^{-2x}}{1+e^{-x}}$ **dx =**

Correct option: (B)

Put $1 + e^{-x} = t \Rightarrow -e^{-x}\, dx = dt$

When x = 0, t = 2 and when x = 1, t = 1 + $\frac{1}{e}$

$\therefore \int_0^1 \frac{e^{-2x}}{1+e^{-x}}\, dx = \int_2^{1+\frac{1}{e}} \frac{(t-1)(-dt)}{t}$

$= \int_2^{1+\frac{1}{e}} \left(\frac{1}{t} - 1\right) dt$

$= [\log t - t]_2^{1+\frac{1}{e}}$

$= \log\left(1+\frac{1}{e}\right) - \left(1+\frac{1}{e}\right) - \log 2 + 2$

$= \log\left(\frac{e+1}{2e}\right) - \frac{1}{e} + 1$

Q.13 $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(5+\sin x)(4+\sin x)}$ **dx =**

Correct option: (C)

Let I = $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(5+\sin x)(4+\sin x)}$ dx

Put sin x = t $\Rightarrow$ cos x dx = dt

$\therefore$ I = $\int_0^1 \frac{dt}{(5+t)(4+t)}$

= $-\int_0^1 \frac{1}{5+t}$ dt + $\int_0^1 \frac{1}{4+t}$ dt

= $-[\log|5+t|]_0^1 + [\log|4+t|]_0^1$

= – (log 6 – log 5) + (log 5 – log 4)

= – log $\frac{6}{5}$ + log $\frac{5}{4}$

$\therefore$ I = log $\left(\frac{25}{24}\right)$

Q.14 $\int \mathbf{e}^{(\mathbf{e}^x+x)}$ **d*x* =**

Correct option: (C)

Let I = $\int \mathbf{e}^{\mathbf{e}^x+x}dx$

= $\int \mathbf{e}^{\mathbf{e}^x}$. e^x dx

Put e^x = t

e^x dx = dt

I = $\int e^t$dt

= e^t + c

$\therefore$ I = e^{e^x} + c

Q.15 $\int \frac{dx}{\cot^2 x - 1} = \frac{1}{A}\log|\sec 2x + \tan 2x| - \frac{x}{B} + c$

, (where c is constant of integration), then A + B =

Correct option: (B)

$\int \frac{dx}{\cot^2 x - 1} = \int \frac{dx}{\frac{\cos^2 x - \sin^2 x}{\sin^2 x}}$

$= \int \frac{\sin^2 x}{\cos 2x}dx$

$= \int \frac{\frac{1-\cos 2x}{2}}{\cos 2x}dx$

$= \frac{1}{2}\int (\sec 2x - 1)dx$

$= \frac{1}{2}\left(\frac{\log|\sec 2x + \tan 2x|}{2} - x\right) + c$

$= \frac{1}{4}\log|\sec 2x + \tan 2x| - \frac{x}{2} + c$

$\therefore$ A = 4, B = 2

$\Rightarrow$ A + B = 6

Q.16 **If** $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \ cosec\ x} dx$ **= m(π + n),**

then (m·n) equals

Correct option: (D)

Let I = $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + cosec\ x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + 1} dx$

$= \int_0^{\frac{\pi}{2}} \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}} dx$

$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(2 - \sec^2 \frac{x}{2}\right) dx$

$= \frac{1}{2} \left[2x - 2\tan \frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= \left[\left(\frac{\pi}{2} - 1\right) - 0\right]$

$\therefore$ I = $\frac{1}{2}(\pi - 2)$

Comparing with m(π + n), we get

m = $\frac{1}{2}$ and n = –2

$\Rightarrow$ (m · n) = –1

Q.17 $\int_{-4}^{4} \log\left(\frac{8 - x}{8 + x}\right) dx =$

Correct option: (D)

Let f(x) = $\log\left(\frac{8 - x}{8 + x}\right)$

$\therefore$ f(–x) = $\log\left(\frac{8 + x}{8 - x}\right)$

$= -\log\left(\frac{8 - x}{8 + x}\right)$ = –f(x)

$\therefore$ f(x) is an odd function.

$\therefore \int_{-4}^{4} \log\left(\frac{8 - x}{8 + x}\right) dx = 0$

Q.18 $\int_0^{\frac{\pi}{4}} \sin x \cdot \sec^2 x dx =$

Correct option: (C)

Let I = $\int_0^{\frac{\pi}{4}} \sin x \cdot \sec^2 x dx$

$= \int_0^{\frac{\pi}{4}} \sec x \cdot \tan x dx$

$= [\sec x]_0^{\frac{\pi}{4}}$

$= \sec \frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$

Q.19 **If** $\int \frac{2x + 3}{x^2 - 5x + 6}$ **dx = 9 log** $|x - 3|$ **– 7 log** $|x - 2|$ **+ A, then A =**

Correct option: (C)

$\int \frac{2x + 3}{x^2 - 5x + 6} dx = \int \frac{2x + 3}{(x - 3)(x - 2)} dx$

$= \int \left(\frac{9}{x - 3} - \frac{7}{x - 2}\right) dx$

$= 9 \log|x - 3| - 7 \log|x - 2| + c$

Given, $\int \frac{2x + 3}{x^2 - 5x + 6} dx = 9\log|x - 3| - 7 \log |x - 2| + A$

$\therefore$ A = constant

Q.20 $\int_0^{\frac{\pi}{2}} \frac{300 \sin x + 100 \cos x}{\sin x + \cos x} dx = \ldots$

Correct option: (A)

Since $\int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx = \frac{\pi}{4}(a + b),$

$\int_0^{\frac{\pi}{2}} \frac{300 \sin x + 100 \cos x}{\sin x + \cos x} dx$

$= \frac{\pi}{4}(300 + 100)$

= 100 π

Q.21 $\int \frac{\text{cosec}\, x}{\cos^2\left(1 + \log \tan \frac{x}{2}\right)} dx =$

Correct option: (B)

Put 1 + log tan $\frac{x}{2}$ = t

$$\Rightarrow \left(\frac{1}{\tan\left(\frac{x}{2}\right)} \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2}\right)dx = dt$$

$\Rightarrow$ cosec x dx = dt

$$\therefore \int \frac{\cos ec\, x}{\cos^2\left(1+\log\ \tan\frac{x}{2}\right)}\, dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t\, dt$$

= tan t + c

=

$$\tan\left(1+\log\tan\frac{x}{2}\right)+c$$

Q.22 $\displaystyle\int_{-8}^{8} \frac{x^5+x^3}{4-x^2}dx =$

Correct option: (D)

Let f(x) = $\dfrac{x^5+x^3}{4-x^2}$

$\therefore$ f(–x) = $\dfrac{(-x)^5+(-x)^3}{4-(-x)^2} = \dfrac{-\left(x^5+x^3\right)}{4-x^2}$

= –f(x)

$\therefore$ f(x) is an odd function.

$\therefore \displaystyle\int_{-8}^{8} \frac{x^5+x^3}{4-x^2}dx = 0$

Q.23 $\int \frac{dx}{\sin^2 x \cos^2 x} =$

Correct option: (C)

$$\int \frac{dx}{\sin^2 x\cos^2 x} = \int \frac{(\cos^2 x+\sin^2 x)}{\cos^2 x \sin^2 x}\, dx$$

$$= \int\left(\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}\right)dx$$

=

$\int \text{cosec}^2 x\, dx + \int \sec^2 x\, dx$

= – cot x + tan x + c

Q.24 $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x$**dx =**

Correct option: (A)

Let f(x) = $x^3\sin^4 x$

$\therefore$ f(– x) = $(-x)^3$ [sin (– x)]^4

= – x^3 (– sin x)^4

= – x^3 $\sin^4 x$

= – f(x)

$\therefore$ f(x) is an odd function.

$\therefore \displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3\sin^4 x\,\text{d}x = 0$

Q.25 $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\sin x+\cos x}$**dx =**

Correct option: (C)

$\displaystyle\int_a^b \frac{f(x)}{f(x)\ +\ f(\text{a}\ +\ \text{b}\ -\ x)}\ dx = \frac{1}{2}$ (b – a)

$\therefore \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\sin x+\cos x}\text{d}x = \frac{1}{2}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{12}$

Q.26 $\displaystyle\int_2^3 \frac{x}{x^2-1}dx =$

Correct option: (C)

$$\int_2^3 \frac{x}{x^2-1}dx = \int_2^3\left(\frac{x}{(x-1)(x+1)}\right)dx$$

$$= \frac{1}{2}\int_2^3\left(\frac{1}{x-1}+\frac{1}{x+1}\right)dx$$

$$= \frac{1}{2}[\log|x-1|+\log|x+1|]_2^3$$

$$= \frac{1}{2}[\log 2+\log 4-\log 1-\log 3]$$

$$= \frac{1}{2}\log\left(\frac{8}{3}\right)$$

Q.27 **If f(x) =**

$$\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$$

Then $\displaystyle\int_0^{\frac{\pi}{2}} f(x)dx =$

Correct option: (D)

$$f(x) = \begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4\sin x \\ 0 & \sin x & 1 \end{vmatrix} \ (C_1 \to C_1 - C_2 - C_3)$$

= sin$x$ (3 – 4 $\sin^2 x$) = 3sin$x$ – 4 $\sin^3 x$ = sin 3x

$$\therefore \int_0^{\frac{\pi}{2}} f(x)dx = \int_0^{\frac{\pi}{2}} \sin 3x dx$$

$$= -\frac{1}{3}[\cos 3x]_0^{\pi/2}$$

$= -\frac{1}{3}\left[\cos\frac{3\pi}{2} - \cos 0\right]$

$= -\frac{1}{3}(0-1) = \frac{1}{3}$

Q.28 **The integral** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x cosec^{\frac{4}{3}} x dx$ **is equal to**

Correct option: (B)

Let I = $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \ \ \text{cosec}^{\frac{4}{3}} x \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\cos^{\frac{2}{3}} x \cdot \sin^{\frac{4}{3}} x}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{\sin^{\frac{4}{3}} x}{\cos^{\frac{4}{3}} x} \cdot \cos^2 x}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^{\frac{4}{3}} x} dx$

Put tan x = t ⇒ $\sec^2 x$ dx = dt

∴ I = $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dt}{t^{\frac{4}{3}}} = \left[-3t^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$= -3\left[\left(\sqrt{3}\right)^{\frac{-1}{3}} - \left(\frac{1}{\sqrt{3}}\right)^{\frac{-1}{3}}\right]$

$= -3\left(3^{-\frac{1}{6}} - 3^{\frac{1}{6}}\right)$

$= 3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

Q.29 $\int x^3 \ \log\ x\, dx =$

Correct option: (B)

$\int x^3 \ \log\ x\, dx = \log\ x\, .\ \frac{x^4}{4} - \int \frac{x^4}{4} . \frac{1}{x}\, dx$

$= \frac{x^4}{4} \log x - \int \frac{x^3}{4} dx$

$= \frac{x^4}{4} \log x - \frac{x^4}{16} + c$

$= \frac{1}{16}(4x^4 \ \log\ x\ -\ x^4) + c$

Q.30 $\int e^{x\log a} \cdot e^x\, dx$ **is equal to**

Correct option: (B)

$\int e^{x\log a} \cdot e^x\, dx = \int e^{\log a^x} \cdot e^x\, dx = \int a^x e^x\, dx$

$= \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + c$

Q.31 **If** $\int_0^{\frac{\pi}{2}} \log(\ \cos x) dx = -\frac{\pi}{2}$ **log 2, then**

$\int_0^{\frac{\pi}{2}} \log(cosec\ x) dx =$

Correct option: (C)

$\int_0^{\frac{\pi}{2}} \log(cosec\ x) dx$

$= \int_0^{\frac{\pi}{2}} \log(\ \sec x) dx \qquad \ldots$

$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$

$= \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{\cos x}\right) dx$

$= \int_0^{\frac{\pi}{2}} [\log 1 - \log(\cos x)] dx$

$= -\int_0^{\frac{\pi}{2}} \log(\cos x) dx = \frac{\pi}{2} \log 2$

Q.32 **The value of** $\int \frac{x^3}{\sqrt{1\ +\ x^4}} dx$ **is**

Correct option: (C)

$\int \frac{x^3}{\sqrt{1\ +\ x^4}} dx = \frac{1}{4} \int \frac{4x^3}{\sqrt{1\ +\ x^4}} dx$

$= \frac{1}{4} \int \frac{dt}{t^{1/2}}$ …[Put 1 + x^4 = t ⇒ 4x^3 dx = dt]

$= \frac{1}{4} \cdot \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= \frac{1}{2}\sqrt{t} + c = \frac{1}{2}\sqrt{1\ +\ x^4} + c$

Q.33 $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\, dx$=

Correct option: (C)

$\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\, dx = \int \frac{x^5 - x^4}{x^3 - x^2}\, dx$

$= \int \frac{x^4(x-1)}{x^2(x-1)}\, dx = \int x^2 dx = \frac{x^3}{3} + c$

Q.34 $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx =$

Correct option: (A)

$\int \frac{e^x(1+\sin x)}{1+\cos x} dx =$

$\int e^x \left[\frac{1 + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right] dx$

$= \int e^x \left(\frac{1}{2}\sec^2 \frac{x}{2} + \tan \frac{x}{2}\right) dx$

$= e^x \tan \frac{x}{2} + c$

Q.35 **The value of** $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}$ **d*x* is**

Correct option: (B)

Let I = $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

Put cos*x* = t ⇒ – sin *x* d*x* = dt

$\therefore$ I = $\int_1^0 \frac{-dt}{1+t^2} = \int_0^1 \frac{dt}{1+t^2} = [\tan^{-1} t]_0^1 = \frac{\pi}{4}$

Q.36 $\int \frac{4x^2 \cot^{-1}(x^3)}{1+x^6} dx =$ **(where C is a constant of integration)**

Correct option: (C)

Let I = $\int \frac{4x^2 \cot^{-1}(x^3)}{1+x^6}\, dx$

Put $\cot^{-1}(x^3)$ = t

$\frac{-1}{1+x^6}(3x^2)dx = dt$

$\therefore$ I = $\int t\left(\frac{-4}{3}\right) dt = \frac{-4}{3}\left[\frac{t^2}{2}\right]$ + C =

$\frac{-2}{3}\left(\cot^{-1}(x^3)\right)^2$ + C

Q.37 $\int (e^{a\log x} + e^{x\log a}) dx =$

Correct option: (C)

$\int (e^{a\log x} + e^{x\log a})\, dx = \int (e^{\log_e x^a} + e^{\log_e a^x})\, dx$

$= \int (x^a + a^x)\, dx$

$= \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a}$ + c

Q.38 **If** $f(x) = \frac{1}{\log x}, g(x) = \frac{1}{(\log x)^2}$**, then** $\int (f(x) - g(x)) dx =$ **, (where C is a constant of integration.)**

Correct option: (D)

$\int [f(x) - g(x)] dx$=

$\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx$

Put log *x* = t

$\therefore\ x = e^t$

$dx = e^t dt$

$\therefore \int [f(x) - g(x)] dx = \int \left[\frac{1}{t} - \frac{1}{t^2}\right] e^t dt$

$= e^t \frac{1}{t}$ + C

$= e^{\log x} \frac{1}{\log x}$ + C

$= \frac{x}{\log x} + C$ …

$\left[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + C\right]$

Q.39 $\int e^{3\log x}(x^4+1)^{-1}$ **dx=**

Correct option: (B)

$\int e^{3\log x}(x^4+1)^{-1}$ dx

$= \int e^{\log x^3}(x^4+1)^{-1}$ dx

$= \int \frac{x^3}{x^4+1}$ dx

$= \frac{1}{4} \int \frac{4x^3}{x^4+1}$ dx

$= \frac{1}{4} \log |x^4+1|$ +c …

$\left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right]$

Q.40 **If** $\int \frac{1}{1-\cot x}$ **d*x* = A*x* + B log |sin *x* – cos *x*| + c, then A + B =**

Correct option: (C)

Let I = $\int \frac{1}{1-\cot x}\, dx$

$= \int \frac{\sin x}{\sin x - \cos x}\, dx$

$= \frac{1}{2}\left[\int \frac{(\sin x - \cos x) + (\cos x + \sin x)}{(\sin x - \cos x)}\right]$

dx

$= \frac{1}{2}\left[\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \int \frac{\cos x + \sin x}{\sin x - \cos x} dx\right]$

$= \frac{1}{2}\left[\int 1 dx + \int \frac{\cos x + \sin x}{\sin x - \cos x} dx\right]$

$= \frac{1}{2}[x + \log|\sin x - \cos x|] + c$

$= \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

Comparing with given expression, we get

A = $\frac{1}{2}$ and B = $\frac{1}{2}$

$\therefore$ A + B = 1

Q.41 $\int_0^{\pi/2} log\, tan\, x\, dx =$

Correct option: (D)

$\int_0^{\pi/2} log\, tan\, x\, dx = \int_0^{\pi/2} log\left(\frac{sinx}{cosx}\right)$dx

$= \int_0^{\pi/2}$ log sin x dx – $\int_0^{\pi/2}$ log cos x dx = 0 … $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

Q.42 **The value of** $\int \frac{dx}{x\sqrt{x^4 - 1}}$ **is**

Correct option: (A)

Put x^2 = t

$\Rightarrow$ 2x dx = dt $\Rightarrow$ dx = $\frac{dt}{2x} = \frac{dt}{2\sqrt{t}}$

$\therefore \int \frac{dx}{x\sqrt{x^4 - 1}} = \int \frac{dt}{2t\sqrt{t^2 - 1}} = \frac{1}{2}\sec^{-1}t + c$

$= \frac{1}{2}\sec^{-1}x^2 + c$

Q.43 $\int \frac{dx}{(\sin x + \cos x)(2\cos x + \sin x)}$

Correct option: (D)

Let I = $\int \frac{dx}{(\sin x + \cos x)(2\cos x + \sin x)}$

$= \int \frac{dx}{\cos^2 x(\tan x + 1)(2 + \tan x)}$

I = $\int \frac{\sec^2 x dx}{(\tan x + 1)(2 + \tan x)}$

put tan x = t

$\therefore$ $\sec^2 x$ dx = dt

$\therefore$ I $= \int \frac{dt}{(t+1)(2+t)}$

$= \int \frac{(2+t) - (t+1)}{(t+1)(2+t)}$

$= \int \frac{dt}{(t+1)} - \int \frac{dt}{(2+t)}$

= log |t + 1| – log |2+t| + c

= log $\left|\frac{\tan x + 1}{2 + \tan x}\right|$ + c

= log $\left|\frac{\tan x + 1}{\tan x + 2}\right| + c$

Q.44 $\int_0^3 \{x\}dx =$ _______, **{·} is fractional part of function.**

Correct option: (C)

I = $\int_0^1 (x - [x])dx + \int_1^2 (x - [x])dx + \int_2^3 (x - [x])dx$

$= \int_0^1 (x - 0)dx + \int_1^2 (x - 1)dx + \int_2^3 (x - 2)dx$

$= \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{(x-1)^2}{2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^3$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

Q.45 $\int \frac{e^x}{\sqrt{x}}(1 + 2x)dx$ =

Correct option: (C)

Let I = $\int \frac{e^x}{\sqrt{x}}(1 + 2x)dx$

$= \int e^x\left[\frac{1}{\sqrt{x}} + 2\sqrt{x}\right]dx$

$= \int e^x\left[2\sqrt{x} + \frac{1}{\sqrt{x}}\right]dx$

$\therefore$ I = $e^x\left(2\sqrt{x}\right)+c$

= $2\sqrt{x}e^x + c$

Q.46 **If $I_1 = \int_e^{e^2} \frac{dx}{logx}$ and $I_2 = \int_1^2 \frac{e^x}{x}$ dx, then**

Correct option: (A)

$I_1 = \int_e^{e^2} \frac{dx}{\log x}$

Put log x = t

$\Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$\therefore I_1 = \int_1^2 \frac{e^t}{t}dt$

$= \int_1^2 \frac{e^x}{x}dx \ldots \left[\because \int_a^b f(x)\,dx = \int_a^b f(t)\,dt\right]$

$\therefore I_1 = I_2$

Q.47 $\int \frac{1}{x^2-x^3}$ **dx =**

Correct option: (D)

$\int \frac{dx}{x^2-x^3} = \int \frac{(1-x)\,dx}{x^2(1-x)} + \int \frac{x\,dx}{x^2(1-x)}$

$= \int \frac{1}{x^2}dx + \int \frac{dx}{x(1-x)}$

$= -\frac{1}{x} + \int \frac{dx}{x} + \int \frac{dx}{1-x}$

$= -\frac{1}{x} + \log|x| - \log|1-x| + c$

$= \log\left|\frac{x}{1-x}\right| - \frac{1}{x} + c$

Q.48 $\int_{-4}^{4} \log\left(\frac{9-x}{9+x}\right) dx$ **equals**

Correct option: (D)

Let $f(x) = \log\left(\frac{9-x}{9+x}\right)$

$\therefore f(-x) = \log\left(\frac{9-x}{9+x}\right)^{-1}$

$= -\log\left(\frac{9-x}{9+x}\right) = -f(x)$

$\therefore f(x)$ is an odd function.

$\therefore \int_{-4}^{4} \log\left(\frac{9-x}{9+x}\right) dx = 0$

Q.49 $\int_0^1 \frac{1}{2+\sqrt{x}}dx =$

Correct option: (B)

Let I = $\int_0^1 \frac{1}{2+\sqrt{x}}dx =$

Put $2+\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}\,dx = dt$

$\Rightarrow dx = 2(t-2)\,dt$

$\therefore I = \int_2^3 \frac{2(t-2)}{t}dt$

$= 2\int_2^3 \left(1-\frac{2}{t}\right)dt$

$= 2[t-2\log t]_2^3$

$= 2[1-2(\log 3-\log 2)]$

$= 2-4\log\frac{3}{2}$

$= 2+4\log\frac{2}{3}$

$= 2\log e + 2\log\frac{4}{9}$

$= 2\log\left(\frac{4e}{9}\right)$

Q.50 $\int \cos(\log_e x)\,dx$ **is equal to**

Correct option: (A)

Let $I = \int \cos(\log_e x)\,dx$

Put $\log_e x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$\therefore$ I = $\int \cos t . e^t dt$

= $\cos t . e^t - \int (-\sin t) . e^t dt$

= $\cos t . e^t + \left[\sin t . e^t - \int \cos t . e^t dt\right]$

$\therefore$ I = $\cos t . e^t + \sin t . e^t - I + c_1$

$\Rightarrow$ 2I = $\cos t . e^t + \sin t . e^t + c_1$

$\Rightarrow I = \frac{x}{2}\left[\cos\ (\log_e x) + \sin\ (\log_e x)\right] + c,$

where c = $\frac{c_1}{2}$

Q.51 **If** $\int \frac{\sin\ 2x}{\sin\ 5x\ \sin\ 3x}$ **dx =** $\frac{1}{3}$ **log |sin 3x |**

– $\frac{1}{5}$ **log | f (x) | + c , then f (x) = _______.**

Correct option: (A)

Let I = $\int \frac{\sin\ 2x}{\sin\ 5x\ \sin\ 3x}$ dx

= $\int \frac{\sin\ (5x\ -\ 3x)}{\sin\ 5x\ \sin\ 3x}$ dx

= $\int \frac{\sin\ 5x\ \cos\ 3x\ -\ \cos\ 5x\ \sin\ 3x}{\sin\ 5x\ \sin\ 3x}$ dx

= $\int (\cot 3x\ -\ \cot\ 5x)$ dx

= $\frac{1}{3}$ log |sin 3x| – $\frac{1}{5}$ log | sin 5x | + c

Q.52 $\int x^3$**tan**$^{-1}$ **x dx =**

Correct option: (C)

$\int x^3\tan^{-1} x$ dx = $\tan^{-1} x . \frac{x^4}{4} - \frac{1}{4}\int \frac{x^4}{x^2\ +\ 1}$ dx

= $\frac{x^4}{4}\tan^{-1} x - \frac{1}{4}\int \frac{x^4\ -\ 1\ +\ 1}{x^2\ +\ 1}$ dx

= $\frac{x^4}{4}\tan^{-1} x - \frac{1}{4}\int \left[(x^2\ -\ 1)\ +\ \frac{1}{x^2\ +\ 1}\right]$

dx

= $\frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\left[\frac{x^3}{3}\ -\ x\ +\ \tan^{-1} x\right]$ + c

= $\frac{1}{4}\left[(x^4 - 1)\tan^{-1}x\ -\ \frac{x^3}{3}\ + x\right]$ + c

Q.53 $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x}$ =

Correct option: (D)

$\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x} = \int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2 \frac{x}{2}}$

= $\frac{1}{2}\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2} dx$

= $\frac{1}{2}\left[\frac{\tan \frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$

= $\tan \frac{\pi}{4} - \tan 0$

= 1

Q.54 $\int \frac{e^{\log\left(1+\frac{1}{x^2}\right)}}{x^2 + \frac{1}{x^2}}$ **dx =**

Correct option: (A)

Let I= $\int \frac{e^{\log\left(1+\frac{1}{x^2}\right)}}{x^2 + \frac{1}{x^2}} dx = \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 2}\ dx$

$\ldots[\because\ e^{\log a} = a]$

Put $x - \frac{1}{x}$ = t

$\Rightarrow \left(1 + \frac{1}{x^2}\right)$ dx = dt

$\therefore$ I = $\int \frac{dt}{t^2 + 2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$ + c

= $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right)$ + c

= $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right)$ + c

Q.55 **If**

$\int tan\ (x - \alpha)\ tan\ (x\ +\ \alpha).\ tan\ 2x\ dx$

= p log |sec 2x| + q log |sec(x + α)| + r log|sec(x – α)| + c, then p + q + r =

Correct option: (D)

$\tan 2x = \tan[(x + \alpha) + (x - \alpha)]$

$$\tan 2x = \frac{\tan(x+\alpha)+\tan(x-\alpha)}{1-\tan(x+\alpha)\cdot\ \tan(x-\alpha)}$$

$\Rightarrow \tan 2x - \tan 2x \cdot \tan(x + \alpha) \cdot \tan(x - \alpha) = \tan(x + \alpha) + \tan(x - \alpha)$

$\Rightarrow \tan 2x - \tan(x + \alpha) - \tan(x - \alpha) = \tan 2x \cdot \tan(x - \alpha) \cdot \tan(x + \alpha)$

$$\therefore \int \tan(x - \alpha) \cdot \tan(x + \alpha) \cdot \tan 2x\, dx$$

$$= \int \tan 2x - \tan(x + \alpha) - tan(x - \alpha) dx$$

$$= \int \tan 2x\, dx - \int \tan(x + \alpha) dx - \int \tan(x - \alpha) dx$$

$$= \frac{\log|\sec 2x|}{2} - \log|\sec(x + \alpha)| - \log|\sec(x - \alpha)| + c$$

Comparing with given expression, we get

$p = \frac{1}{2}$, q = –1, r = –1

$$\therefore p + q + r = \frac{1}{2} - 1 - 1 = \frac{-3}{2}$$

Q.56 $$\int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\tan x)^{\sqrt{2018}}} =$$

Correct option: (D)

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\tan x)^{\sqrt{2018}}} \quad \ldots(i)$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\cot x)^{\sqrt{2018}}} \quad \ldots(ii)$$

$$\left[\because \int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx\right]$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} \left[\frac{1}{1 + (\tan x)^{\sqrt{2018}}} + \frac{1}{1 + (\cot x)^{\sqrt{2018}}}\right] dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{1 + (\tan x)^{\sqrt{2018}}} + \frac{1}{1 + \left(\frac{1}{\tan x}\right)^{\sqrt{2018}}} dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$\Rightarrow I = \frac{\pi}{4}$$

Q.57 $\int 2^x 3^{x+1} 4^{x+2} dx =$

Correct option: (B)

$\int 2^x 3^{x+1} 4^{x+2} dx = 16 \times 3 \int 2^x 3^x 4^x dx$

$$= 48 \int (24)^x dx = \frac{48(24)^x}{log 24} + c$$

$$= \frac{2^x 3^{x+1} 4^{x+2}}{log 2 + log 4 + log 3} + c$$

Q.58 **If** $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ **and** $g(x) = \log_e x$, **(x > 0), then the value of the integral** $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x)) dx$ **is**

Correct option: (A)

$$g(f(x)) = g\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = log\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$$

$$\text{Let } h(x) = \log\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$$

$$\Rightarrow h(-x) = \log\left(\frac{2 + x\cos x}{2 - x\cos x}\right) = -\log\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$$

$\Rightarrow h(-x) = -h(x)$

$\Rightarrow h(x)$ is an odd function.

$$\Rightarrow \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = 0 = \log_e 1$$

Q.59 **If $\int_{-1}^{1} f(x)$ dx = 0, then**

Correct option: (B)

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

In 1st integral, put $x = -t \Rightarrow dx = -dt$

$$\therefore \int_{-1}^{0} f(x)\,dx = -\int_{1}^{0} f(-t)\,dt$$

$$= \int_{0}^{1} f(-t)\,dt$$

$$= \int_{0}^{1} f(-x)\,dx$$

$$\therefore = \int_{-1}^{1} f(x)\,dx = \int_{0}^{1} f(-x)\,dx + \int_{0}^{1} f(x)\,dx$$

$= 0$, if $f(-x) = -f(x)$

Q.60 $\int \frac{ax^{-2} + bx^{-1} + c}{x^{-3}}\,dx =$

Correct option: (D)

$$\int \frac{ax^{-2} + bx^{-1} + c}{x^{-3}}\,dx =$$

$$\int (ax + bx^2 + cx^3)\,dx$$

$$= \frac{1}{2}ax^2 + \frac{1}{3}bx^3 + \frac{1}{4}cx^4 + k$$