Rotational **Marks: 30**

ANSWER KEY

Physics

Q.1 B	Q.2 D	Q.3 A	Q.4 C	Q.5 D	Q.6 D	Q.7 D	Q.8 C
Q.9 B	Q.10 B	Q.11 A	Q.12 C	Q.13 B	Q.14 A	Q.15 D	Q.16 B
Q.17 B	Q.18 B	Q.19 A	Q.20 C	Q.21 D	Q.22 A	Q.23 D	Q.24 A
Q.25 D	Q.26 D	Q.27 B	Q.28 A	Q.29 D	Q.30 B		

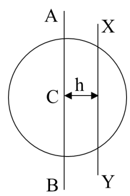
Physics

Q.1 A body crosses the topmost point of a vertical circle with critical speed. Its centripetal acceleration, when the string is horizontal will be

Correct option: (B) 3 g

v = $\sqrt{3gr}$ and a = $\frac{v^2}{r}$ = $\frac{3gr}{r}$ = 3g

Q.2 The radius of gyration K of a hollow sphere of mass M and radius R about an axis XY is equal to R. The distance of that axis from the centre of the sphere is h. The value of h is



Correct option: (D)

Given: K = R

∴ I = MK^2 = MR^2 …(i)

Applying parallel axis theorem,

I = I_{CM} + Mh^2

For a hollow sphere, $I_{CM} = \frac{2}{3}MR^2$

∴ I = $\frac{2}{3}MR^2 + Mh^2$

Using equation (i),

$MR^2 = \frac{2}{3}MR^2 + Mh^2$

∴ $\frac{1}{3}R^2 = h^2$

∴ h = $\frac{R}{\sqrt{3}}$

Q.3 A disc and a solid sphere having same mass and radius roll down on the same inclined plane. The ratio of their linear speeds is

Correct option: (A)

For disc, $K_1 = \frac{R^2}{2}$; For sphere, $K_2 = \frac{2}{5}R^2$

∴

$$\frac{v_1^2}{v_2^2} = \frac{1+\frac{K_1^2}{R^2}}{1+\frac{K_2^2}{R^2}} = \frac{1+\frac{R^2}{2R^2}}{1+\frac{\frac{2}{5}R^2}{R^2}} = \frac{1+\frac{1}{2}}{1+\frac{2}{5}} = \frac{3/2}{7/5} = \frac{3}{2}\times\frac{5}{7} = \frac{15}{14}$$

∴ $\frac{v_1}{v_2} = \sqrt{\frac{15}{14}}$

Q.4 A flywheel at rest is reached to an angular velocity of 36 rad/s in 6 s with a constant angular acceleration. The total angle turned during this interval is

Correct option: (C)

Using,

α = $\frac{\omega-\omega_0}{t} = \frac{36-0}{6}$ = 6 rad/s^2

θ = $\omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}$ × 6 × 6 × 6 = 108 rad

Q.5 A wheel initially at rest, begins to rotate about its axis with constant angular acceleration. If it rotates through an angle θ_1 in first 2 s and a further angle θ_2 in the next 2 s, the ratio $\theta_1 : \theta_2$ is

Correct option: (D)

For constant angular acceleration

θ = $\omega_0 t + \frac{1}{2}\alpha t^2$

Wheel is initially at rest

∴ $\omega_0 = 0$

θ = $\frac{1}{2}\alpha t^2$

First 2 seconds (t = 2):

$\theta_1 = \frac{1}{2}\alpha\left(2^2\right)$

Next 2 seconds (t =4 total, so angle from D to 4 sec):

$\theta_{total} = \frac{1}{2}\alpha\left(4^2\right)$

$= 8\alpha$

So, angle in next 2 seconds:

$\theta_2 = \theta_{total} - \theta_1 = 8\alpha - 2\alpha = 6\alpha$

$\therefore \ \frac{\theta_1}{\theta_2} = \frac{2\alpha}{6\alpha} = \frac{1}{3}$

$\therefore \ \theta_1 : \theta_2 = 1:3$

Q.6 A body of mass 'M' rotating with angular velocity 'ω' has angular momentum 'L' about its own axis. Its radius of gyration is

Correct option: (D)

Radius of gyration is given by,

$$K = \sqrt{\frac{I}{M}}$$

$$L = I\omega$$

$$\therefore \ K = \sqrt{\frac{L}{M\omega}}$$

$$\therefore \ K = \left(\frac{L}{M\omega}\right)^{\frac{1}{2}}$$

Q.7 A solid sphere of radius r is rolling without sliding. The ratio of rotational kinetic energy and total kinetic energy associated with the sphere is

Correct option: (D)

Rotational Kinetic energy in case of sphere,

$$K_{rot} = \frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right) = \frac{1}{2}mv^2\left(\frac{2}{5}\right)$$

Total kinetic energy,

K_{total} =

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right) = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$= \frac{1}{2}mv^2\left(\frac{7}{5}\right)$$

$$\therefore \ \frac{K_{rot}}{K_{total}} = \frac{2/5}{7/5} = \frac{2}{7}$$

Q.8 A body is just revolved in a vertical circle of radius 'R'. When the body is at highest point, the string breaks. The horizontal distance covered by the body after the string breaks is

Correct option: (C)

The velocity of the body at the highest point is given by

$$V = \sqrt{gR}$$

After the string breaks, it will fall under gravity with initial horizontal velocity $V = \sqrt{gR}$. Its initial velocity in vertical direction is zero.

If it falls through a height h, in time t, then

$$h = \frac{1}{2}gt^2 \text{ or } t = \sqrt{\frac{2h}{g}}$$

In horizontal direction its velocity will remain constant (V).

Hence, distance covered in horizontal direction is, $x = Vt = \sqrt{gR} \times \sqrt{\frac{2h}{g}} = \sqrt{2hR}$

If h = 2R, then $x = \sqrt{4R^2} = 2R$

Q.9 A can filled with water is revolved in a vertical circle of radius 'r' and water just does not fall down. The time period of revolution is (g = acceleration due to gravity)

Correct option: (B)

When a bucket full of water is rotated in a vertical circle, water will not spill only if velocity of bucket at the highest point is $\geq \sqrt{gr}$.

$$v = \sqrt{gr}$$

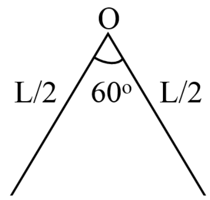
$$\therefore \ \omega = \frac{v}{r} = \sqrt{\frac{g}{r}}$$

$$\therefore \ \frac{2\pi}{T} = \sqrt{\frac{g}{r}} \quad \Rightarrow \ T = 2\pi\sqrt{\frac{r}{g}}$$

Thinking Hatke

Amongst all the options, only the expression given in option (B) has the dimensions of Time period (i.e., $[T^1]$). Hence, (B) is the only correct option possible.

Q.10 A thin rod of length 'L' and mass 'M' is bent at the middle point 'O' at an angle of 60°. The moment of inertia of the rod about an axis passing through point 'O' and perpendicular to the plane of the rod will be



Correct option: (B)

Moment of inertia of rod about an axis passing through one end is given by, I = $\frac{ML^2}{3}$

$\therefore$ Moment of inertia of each half of the rod about the midpoint is given by,

$$I = \frac{\frac{M}{2}\left(\frac{L}{2}\right)^2}{3} = \frac{ML^2}{24}$$

Total moment of inertia = I = $2I_1$ = $\frac{ML^2}{12}$

Q.11 Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is

Correct option: (A) $\frac{r_2}{r_1}$

F = $\frac{mv^2}{r}$

If m and v are constants, then F $\propto \frac{1}{r}$

$\therefore \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$

Q.12 Select the WRONG statement.

Correct option: (C)

Centrifugal force is a *pseudo force*, which is a *fictitious force* that appears to act on an object in a non-inertial frame of reference. It is not a real force, but rather an effect of the object's inertia in a rotating frame of reference. This means that the centrifugal force *doesn't exist in an inertial frame of reference*, because in an inertial frame, objects at rest stay at rest and objects in motion continue in motion at a constant velocity. So, the centrifugal force is *not* an actual force like gravity or electromagnetic force, but rather an apparent force that arises due to the rotation of the frame of reference.

Q.13 Two discs of same thickness but of different radii are made up of two different materials such that their masses are same. The densities of the material are in the ratio 1 : 3. The moment of inertia of these discs about the respective axis passing through their centre and perpendicular to plane will be in the ratio

Correct option: (B)

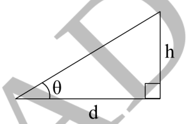
The ratio of moments of inertia of two discs of the same mass and same thickness but of different densities is given by

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$$

$$\therefore \frac{I_1}{I_2} = \frac{3}{1}$$

Q.14 A railway track is banked for a speed 'v' by elevating outer rail by a height 'h' above the inner rail. The distance between two rails is 'd' then the radius of curvature of track is (g = gravitational acceleration)

Correct option: (A)



From figure,

tan θ = $\frac{h}{d}$

$\therefore \frac{v^2}{rg} = \frac{h}{d}$ ....($\because$ tanθ = $\frac{v^2}{rg}$)

$\therefore$ r = $\frac{v^2 d}{gh}$

Q.15 The moment of inertia of circular disc of mass 'm' and radius 'r' about an axis passing through centre of mass is 'I_0'. The moment of inertia of another circular disc of same mass and thickness but half the density about the same axis is

Correct option: (D)

The ratio of moments of inertia of two discs of the same mass and same thickness but of different densities is given by,

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$$

$\therefore \frac{I_1}{I_2} = \frac{1}{2}$ .... ($\because$ $d_2 = d_1/2$)

$\therefore$ $I_2 = 2I_0$

Q.16 The new dimensional formula for the moment of inertia of a body is

Correct option: (B)

The moment of inertia of a body is defined as the resistance of a body to changes in its angular velocity. It is given by the formula: $I = mr^2$ where m is the mass of the body and r is the distance from the axis of rotation. Hence, the dimensional formula for the moment of inertia is: $[I] = [M^1L^2T^0]$

Q.17 Two uniform thin rods each of mass M and length *l* are placed along X and Y-axis with one end of each at the origin. M.I. of the system about Z–axis is

Correct option: (B)

According to theorem of parallel axes, moment of inertia of a rod about one of its ends,

$$I = \frac{ML^2}{12} + M\frac{L^2}{4} = \frac{ML^2}{3} = I_x = I_y$$

∴ Moment of inertia of two rods about Z-axis

$= I_z = I_x + I_y$

= Moment of inertia of 2 rods placed along X and Y-axis $= \frac{2ML^2}{3}$

Q.18 A wheel has circumference C. If it makes f r.p.s., the linear speed of a point on the circumference is

Correct option: (B) fC

$C = 2\pi r$

$\therefore r = \frac{C}{2\pi}$

$\therefore v = r(2\pi n) = \frac{C}{2\pi} \times 2\pi \times f = fC$ ….[∵ $\omega = 2\pi n$]

Q.19 A wheel having a diameter of 3 m starts from rest and accelerates uniformly to an angular velocity of 210 r.p.m. in 5 seconds. Angular acceleration of the wheel is

Correct option: (A) 4.4 rad s^{-2}

$n_1 = 0$, $n_2 = 210$ r.p.m. $= \frac{210}{60}$ r.p.s.

$d\omega = 2\pi(n_2 - n_1) = 2\pi\left(\frac{210}{60} - 0\right) = 7\pi$ rad/s

$$\alpha = \frac{d\omega}{dt} = \frac{2\pi \times 210}{60 \times 5} = 4.4 \text{ rad/s}^2$$

Q.20 A body slides down a smooth inclined plane of inclination 'θ' and reaches the bottom with velocity 'V'. If the same body is a ring which rolls down the same inclined plane the linear velocity at the bottom of plane is

Correct option: (C)

Case 1: $\frac{1}{2}mV^2 = mgh$ ….(i)

Case 2: $\frac{1}{2}m\left(v'\right)^2 + \frac{1}{2}I\omega^2 = mgh$

$$\frac{1}{2}m\left(v'\right)^2 + \frac{1}{2}(mR^2)\frac{\left(v'\right)^2}{R^2} = mgh$$

$m\left(v'\right)^2 = mgh$ ….(ii)

$\frac{1}{2}mV^2 = mv'^2$ ….[from (i) and (ii)]

$$\therefore v' = \frac{1}{\sqrt{2}}V$$

Q.21 A flywheel at rest is set into rotation about an axis passing through its centre and perpendicular to its plane, to have an angular velocity of 24 rad/s in 6 second with constant angular acceleration. The total angle made by the flywheel during this interval is

Correct option: (D)

Using, $\alpha = \frac{\omega - \omega_0}{t} = \frac{24 - 0}{6} = 4$ rad/s^2

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 4 \times 6^2 = 72$ rad

Q.22 The moment of inertia of a hollow cylinder of mass M, length 2R and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of cylinder is I_2. Then,

Correct option: (A)

$I_1 = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

$\therefore I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M(4R^2)$

$= \frac{1}{2}MR^2 + \frac{1}{3} MR^2 = \frac{5}{6} MR^2$

$\therefore I_2 = \frac{1}{2}MR^2 + \frac{1}{3}M(4R^2)$

$= \frac{1}{2}MR^2 + \frac{4}{3}MR^2 = \frac{11}{6}MR^2$

$\therefore \frac{I_1}{I_2} = \frac{5}{11}$ and $I_2 > I_1$

$\therefore I_2 - I_1 = \frac{11}{6}MR^2 - \frac{5}{6}MR^2 = MR^2$

Q.23 Two identical rings A and B of same mass and radii are revolving, ring A around its own diameter and ring B about tangential axis in its own plane. Both the rings A and B have same rotational kinetic energy. The ratio of angular velocity of ring B to that of ring A is

Correct option: (D)

Both the rings have same rotational kinetic energy,

$\frac{1}{2}I_B\omega_B^2 = \frac{1}{2}I_A\omega_A^2$ …(i)

Moment of inertia of ring A around its own diameter,

$I_A = \frac{MR^2}{2}$

Moment of inertia of ring B about tangential axis is,

$I_B = \frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}$

From equation (i),

$\frac{\omega_B^2}{\omega_A^2} = \frac{I_A}{I_B}$

$\therefore \frac{\omega_B^2}{\omega_A^2} = \frac{MR^2/2}{3MR^2/2} = \frac{1}{3}$

$\therefore \frac{\omega_B}{\omega_A} = \frac{1}{\sqrt{3}}$

Q.24 A woman weighing 600 N is sitting in a car which is travelling at a constant speed on a straight road. The car suddenly goes over a hump in the road (hump may be regarded as an arc of a circle of radius 12.1 m). If the woman experiences weightlessness, calculate the speed of the car. [Take g = 10 m/s^2]

Correct option: (A)

Using,

$\frac{mv^2}{r} = mg$

$\therefore v^2 = gr$

$v = \sqrt{gr} = \sqrt{10 \times 12.1} = \sqrt{121} = 11$ m/s

Q.25 The ratio of the radii of gyration of a circular disc to that of circular ring, each of same mass and radius, around their respective axes is

Correct option: (D)

Radius of gyration of circular disc $k_{disc} = \frac{R}{\sqrt{2}}$

Radius of gyration of circular ring $k_{ring} = R$

$\therefore$ Ratio $= \frac{k_{disc}}{k_{ring}} = \frac{1}{\sqrt{2}}$.

Q.26 Eight identical small solid spheres, each of moment of inertia 'I' are recast to form a big solid sphere. M.I. of the big solid sphere is

Correct option: (D)

The moment of inertia of a solid sphere is given by $(2/5)MR^2$. When eight identical spheres are

melted and recast into a single sphere, the mass of the new sphere will be eight times the mass of each individual sphere, and the volume of the new sphere will also be eight times the volume of each individual sphere. Since volume is proportional to R^3, the radius of the new sphere will be twice the

radius of each individual sphere. Therefore, the moment of inertia of the new sphere will be: $(2/5)(8M)(2R)^2 = 32(2/5)MR^2 = 32I$ Hence,

the correct answer is 32I.

Q.27 A mass tied to a string is whirled in a horizontal circular path with a constant angular velocity A mass tied to a string is whirled in a horizontal circular path with a constant angular velocity and its angular momentum is L. If the string is now halved, keeping angular velocity same, then the angular momentum will be

Correct option: (B)

Angular momentum, $L = mr^2\omega$

Let $L_1 = mr_1^2\omega$

When string is halved,

$L_2 = mr_2^2\omega = m\left(\frac{r_1}{2}\right)^2\omega$ ….(Given: $r_2 = \frac{r_1}{2}$)

$= \frac{1}{4}mr_1^2\omega = \frac{1}{4}L_1$

Q.28 Two bodies having moments of inertia I_1 and I_2 ($I_1 > I_2$) have same angular momentum. If E_1 and E_2 are their rotational kinetic energies, then

Correct option: (A)

$E = \frac{L^2}{2I} \Rightarrow E \propto \frac{1}{I}$ when L is constant

$\therefore$ As $I_1 > I_2 \Rightarrow E_1 < E_2$

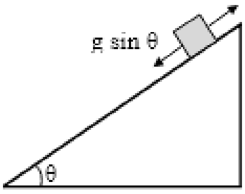
Q.29 A particle moves in a circular path with decreasing speed. Choose the correct statement.

Correct option: (D)

Angular momentum is an axial vector. It is directed always in a fixed direction (perpendicular to the plane of rotation either outward or inward), if the sense of rotation remains same.

Q.30 A body at rest starts from top of a smooth inclined plane and requires 4 second to reach bottom. How much time does it take, starting from rest at top, to cover one-fourth of a distance?

Correct option: (B)



Using $S = \frac{1}{2}at^2$

$S = \frac{1}{2} g \sin\theta.(4)^2$ ….(i)

$\therefore \frac{S}{2} = \frac{1}{2} g \sin\theta.(t)^2$ ….(ii)

Dividing equation (ii) by (i),

$\frac{1}{4} = \frac{t^2}{16}$ or $t^2 = 4 \Rightarrow t = 2$ s